Real Time System Design

Analog Computer Components

Analog Computer: a computer which operates with numbers represented by some physically measurable quantity, such as weight, length, voltage, etc. This quantity is (usually) continuous and real-valued (not discrete, not digital). So **analogue** has also come to mean “continuous, not digital” in the world of computation.

Why analog computation? With the interest in analog computing equipment rapidly increasing in our digitally oriented Agency, this is a question many of us must ask. The majority of digital computing equipment in this Agency would prevent analog computation from consideration if the two types of computers performed the same operations equally well; but this is not the case. A comparison of digital and analog computer applications reveals in basic difference in their operation. The digital computer performs numerical operations on discrete signals; in contrast, the analog computer performs algebraic and integro-differential operations upon continuous signals. Therefore certain operations, which are difficult to program on a digital computer, are available inherently on the analog machine. In order to gain where an analog computer can be advantageously applied, one must become more familiar with what it is and how it is used.

The procedure for simulating an analog system:

The mathematical model of an analog computer programmed to simulate a specific physical system is identical to the mathematical model of the system. The normal procedure for simulating a system starts with determining the mathematical model describing the physical quantities of interest. An analog block diagram is made to relate the sequence of arithmetical operations and to aid in scaling the variables. From the analog block diagram the electrical components are connected together (patched). The computer is operated and the computer variables observed on a recorder or oscilloscope. Since the output is a computer variable (voltage wave form) it is necessary to convert the output variable back to the original problem variable.

Solving Differential Equation with Analog Computer:

A typical simulation of a physical system involves a mathematical model consisting of a set of one or more differential equations and initial conditions on the variables. If the system is linear, the differential equations are linear and the operations required are:

1) Summation,

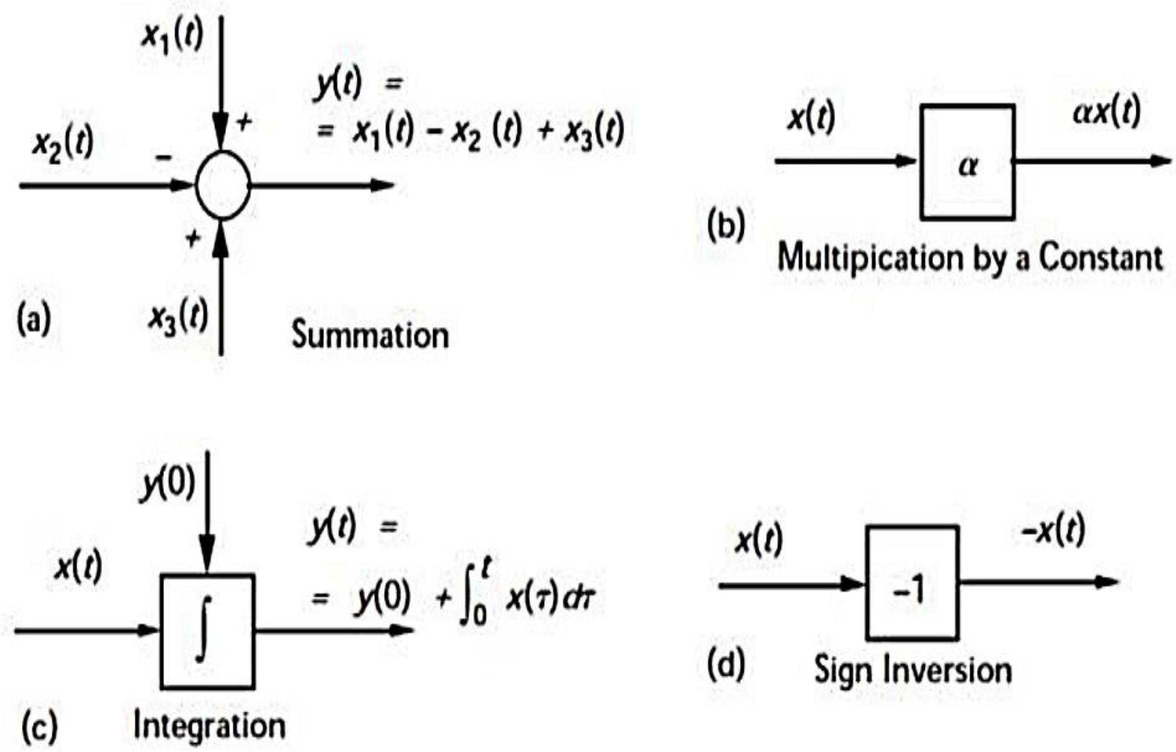
2) Sign inversion,

3) Multiplication by a constant,

4) Integration

5) differentiation.

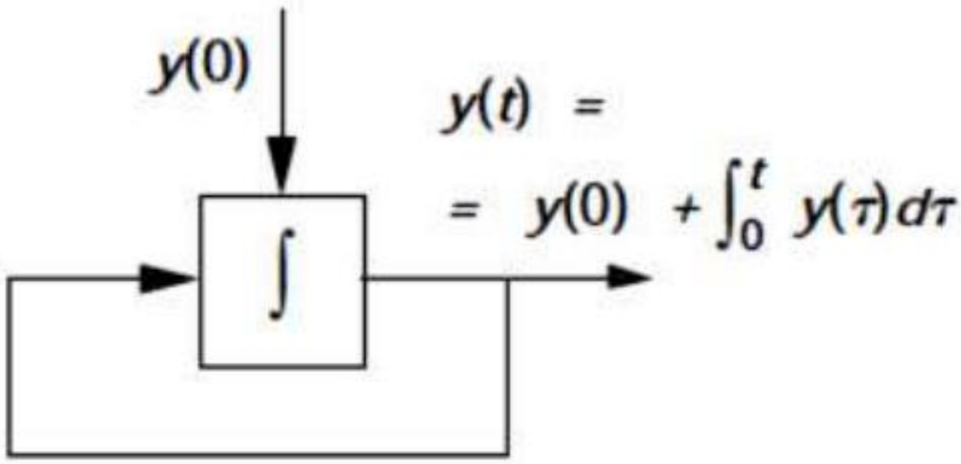
For practical reasons, the integration operation is easier to implement than the differentiation operation. The reason lies in the fact that computer signals are real voltages and, therefore, are corrupted by noise to some extent. Since integration has a tendency to average out the effects of noise (while differentiation will accentuate it), a more precise solution can be obtained using integration techniques.



As an example, consider the computer solution of the differential equation:

$$\frac{dy}{dt} = y, \quad y(0) = 1$$

$$y(t) = y(0) + \int_0^t y(\tau)\, d\tau$$



A higher order linear differential equation may be handled by reducing it to a set of first–order equations and following a similar procedure. For example:

may be turned into the set of equations:

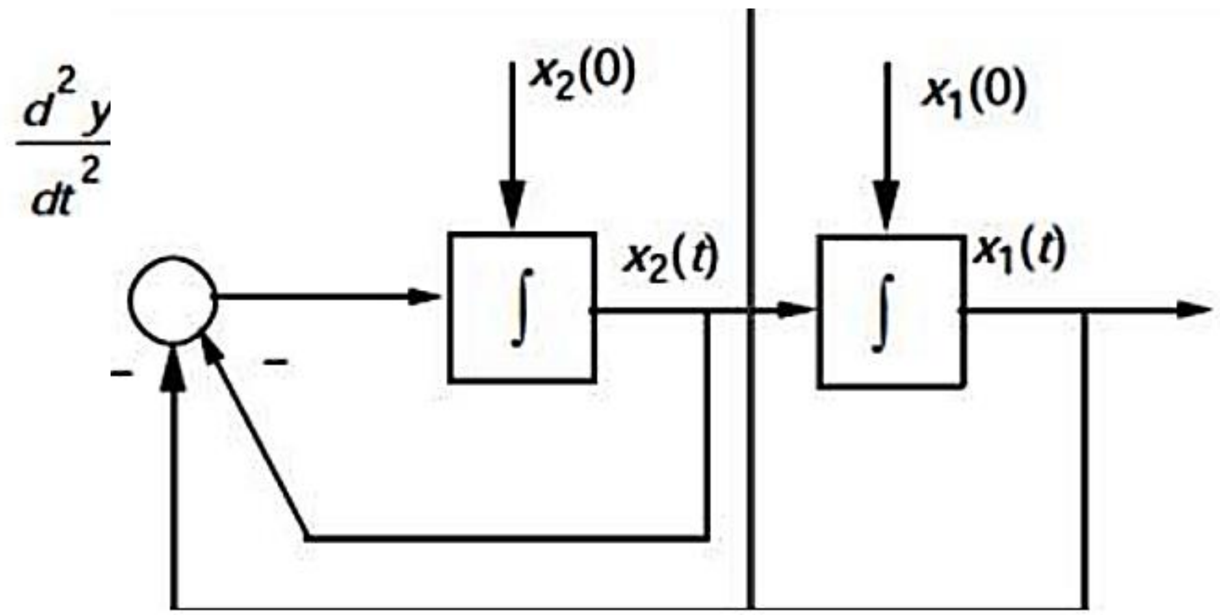
$$\dot{x}_1 = x_2, \quad x_1(0) = 1 \qquad \text{............ 3.4a}$$

$$\dot{x}_2 = -x_1 - x_2, \quad x_2(0) = 0 \qquad \text{............ 3.4b}$$

where $x_1 = y$, and $x_2 = dy/dt$. The equivalent integral forms are:

$$x_1(t) = x_1(0) + \int_0^t x_2(\tau)\,d\tau, \qquad \text{............ 3.5a}$$

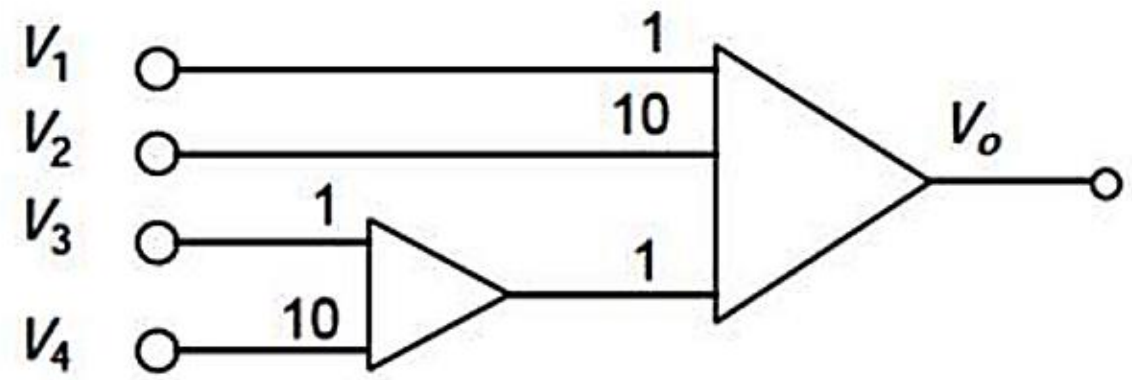
$$x_2(t) = x_2(0) - \int_0^t \left[x_1(\tau) + x_2(\tau)\right]d\tau \qquad \text{............ 3.5b}$$

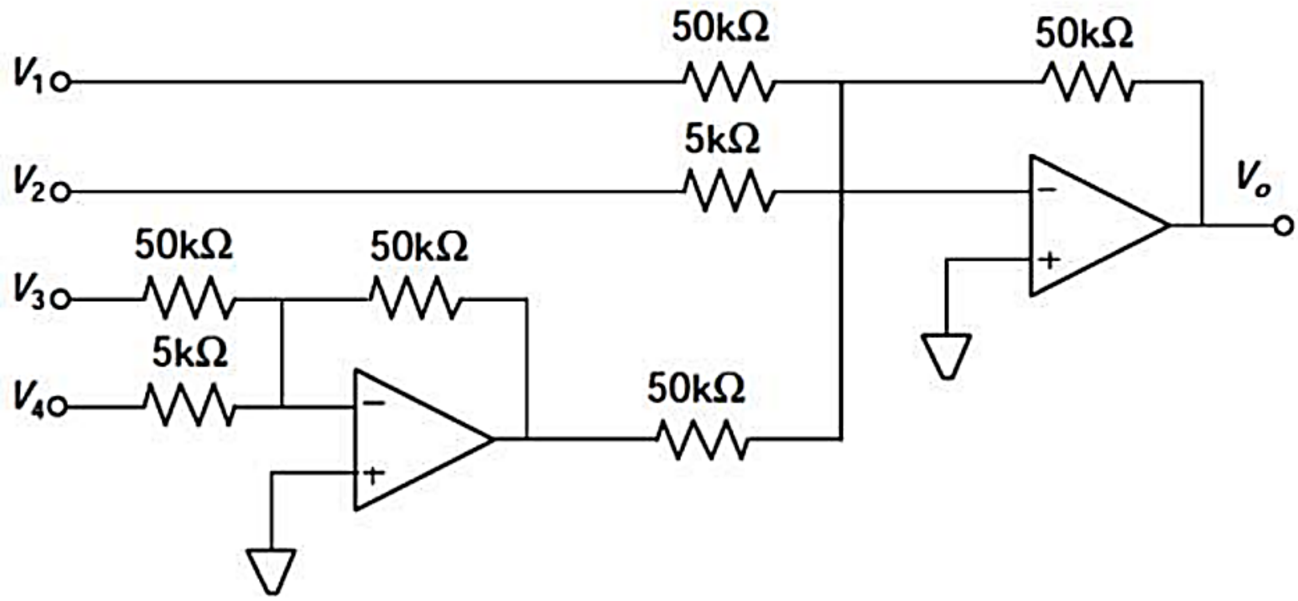


Physical Realization of Linear Operations on an Analog Computer

Example: Using the OP-Amp as summer. Determine the circuit to produce the output voltage given by:

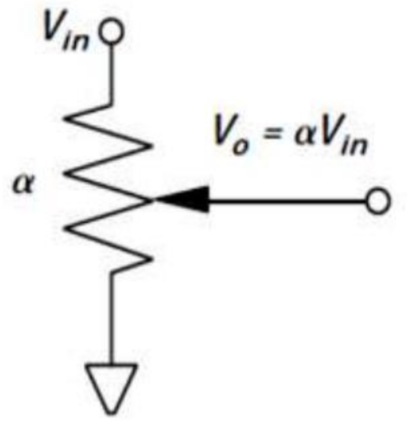
$$V_o = -V_1 - 10V_2 + V_3 + 10V_4$$

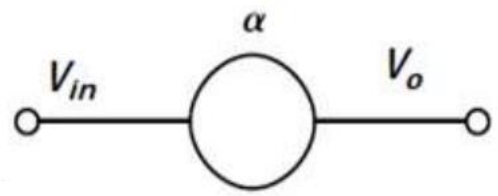




Example: Multiplication by a constant, using the POT. The coefficient potentiometer (pot) is a voltage divider which allows the output voltage to be some fraction of the input voltage. A pot thus has a gain of less than unity.

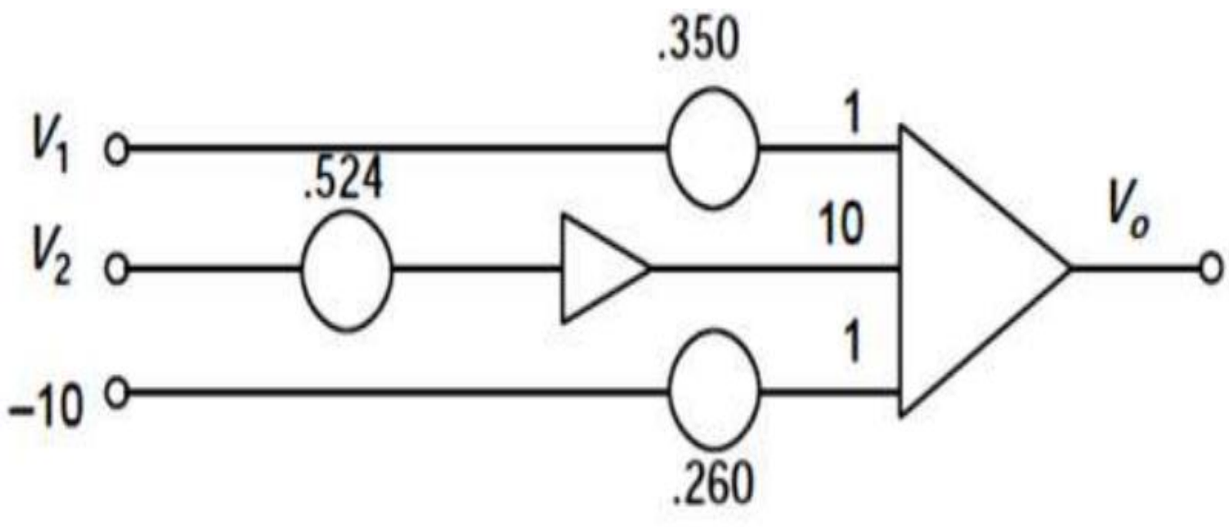
$$V_o = \alpha V_{in}, \quad 0 \le \alpha \le 1.$$

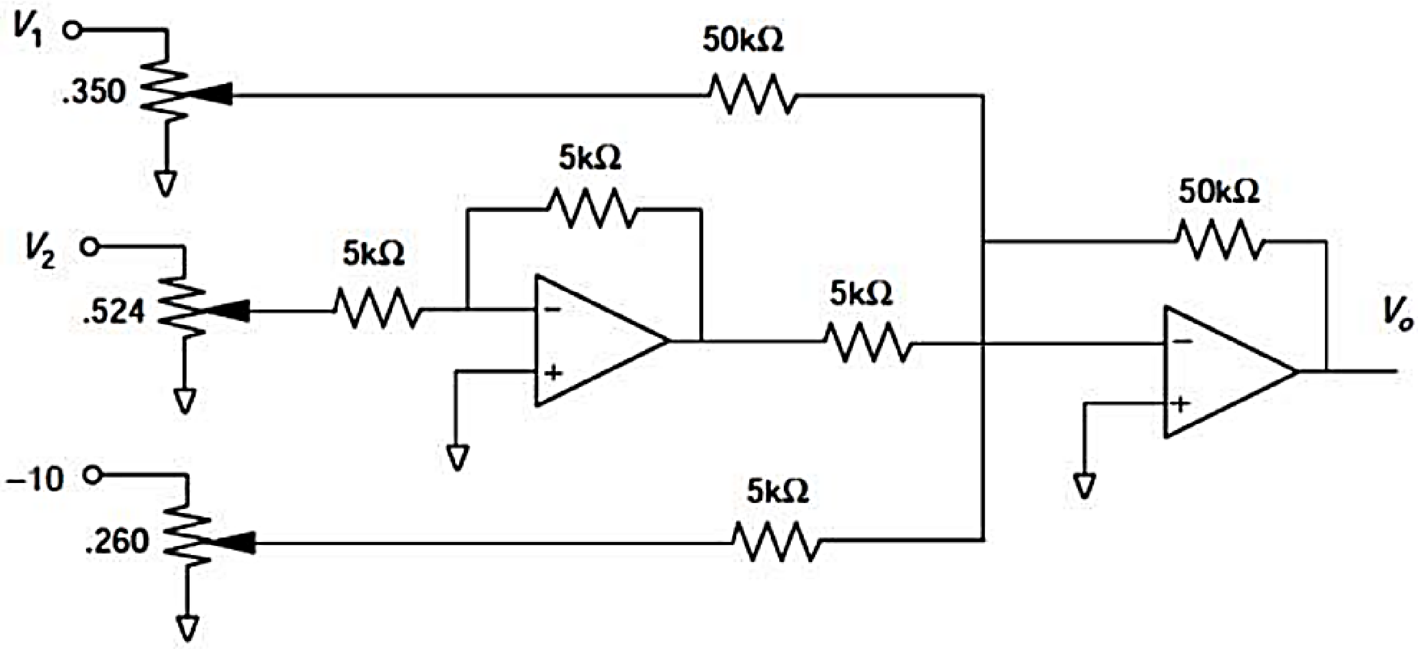




Determine the analog diagram and circuit to implement the equation:

$$V_o = -0.35 V_1 + 5.24 V_2 + 2.6$$

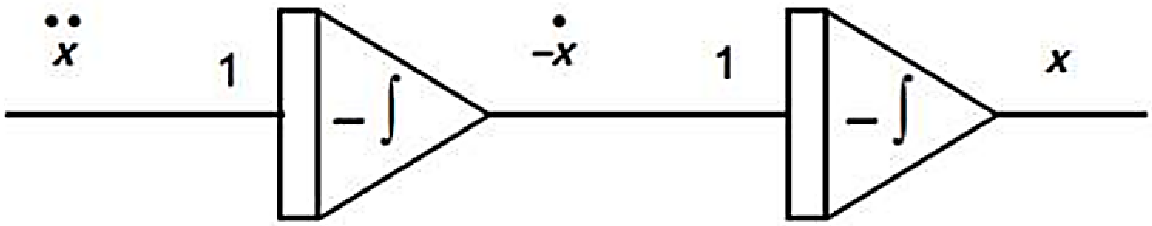




Example: Using the OP-Amp as integrator:

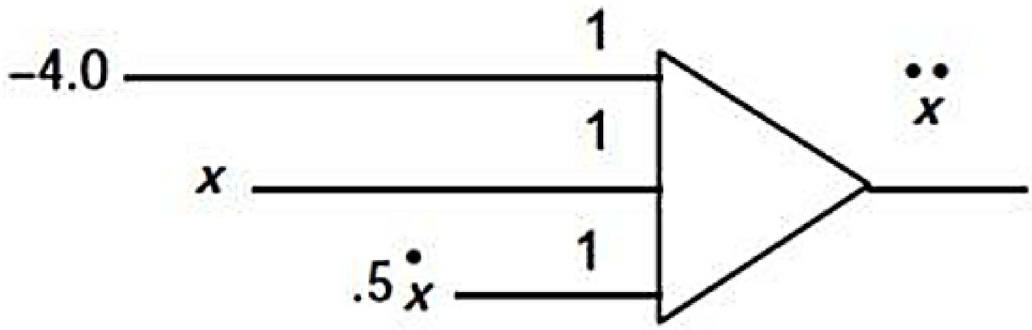
$$\frac{d^2x}{dt^2} + 0.5\frac{dx}{dt} + x = 4, \qquad x(0) = 0, \dot{x}(0) = 1$$

(a) Assume that the highest order derivative ($d^2x/\ dt^2$ in this case) is known and generate all lower order derivatives. Note that the output should always be x, not -x, since x is the desired solution.



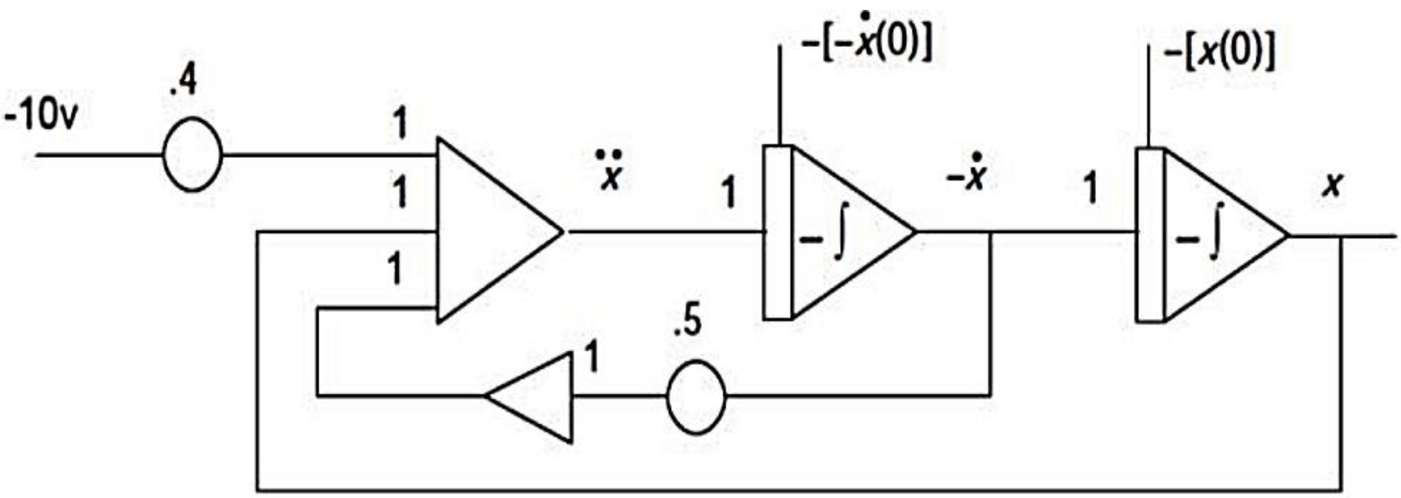
(b) Solve the differential equation for the input to the integrator string and form the indicated sum.

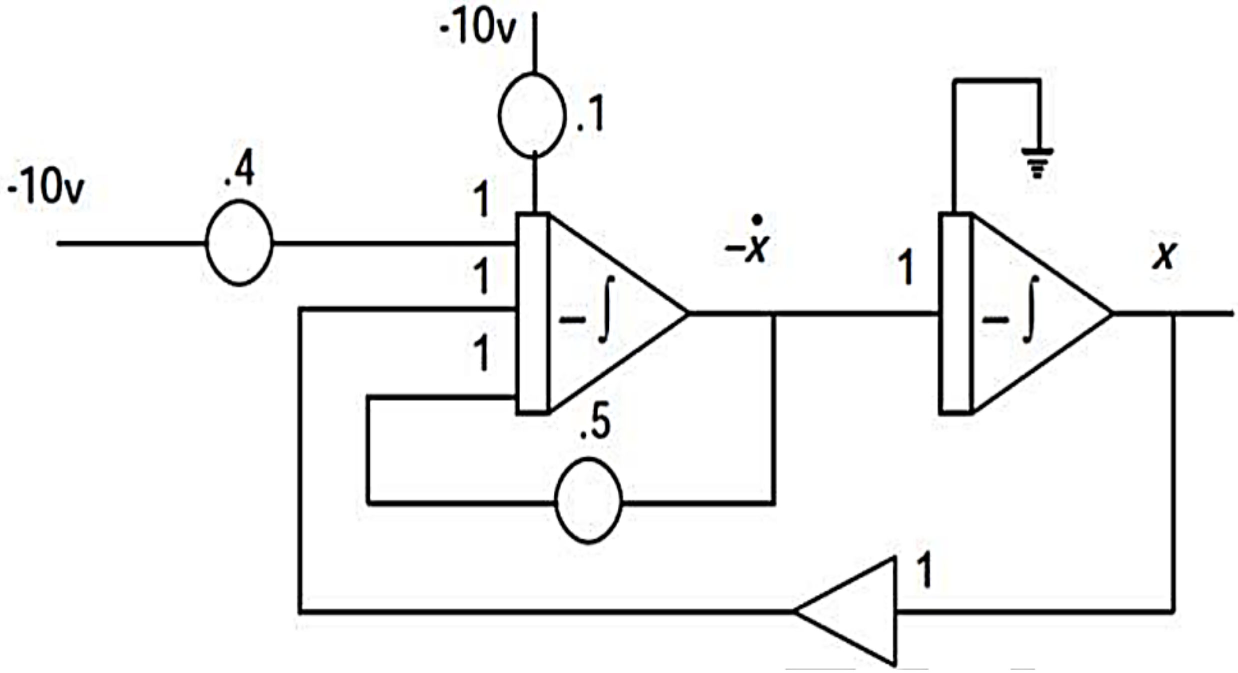
$$\frac{d^2x}{dt^2} = -0.5\frac{dx}{dt} - x + 4$$



(c) Combine the results of (a) and (b) using pots, summers and inverters where required.

(d) Add all the initial conditions. Recall that the applied initial condition should be the negative of the integrator output at t = 0.




-10v
.1
-10v
.4
1
1
1
$-\dot{x}$
1
x
.5
1