هندسة اتصالات الحاسبات
المرحلة الثالثة
نظرية المجالات

- EM.3 -

Some Concepts

a- Line Representation of Electric Fields

(Streamlines), (Lines of Force), (Flux Lines)

*- They are used to describe the distribution of the electric fields throughout space (mapping).

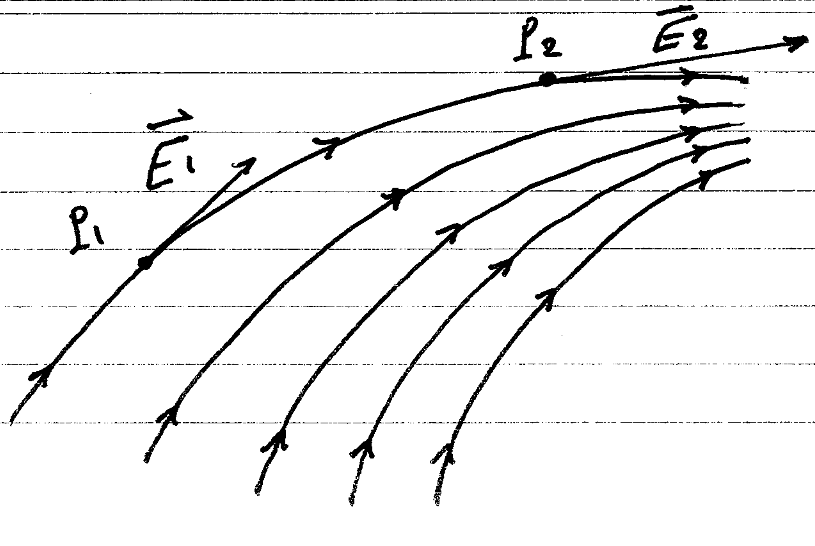
*- These lines have directions which are directed away from the positive charges and toward the negative charges.

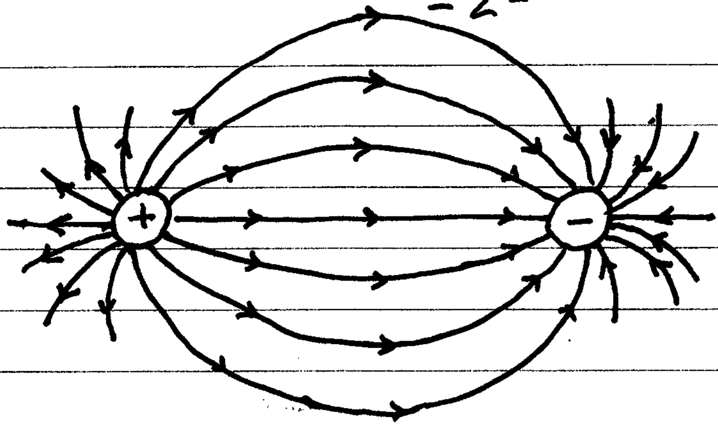
*- The density of these lines is a measure of the magnitude of the electric field intensity ($|\vec{E}|$)

[Higher density means higher field intensity].

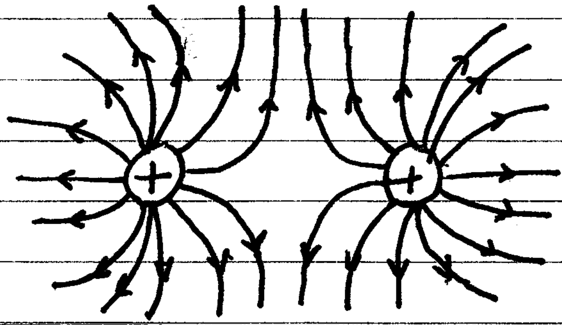
*- The electric field intensity at any point in space is represented by the tangent of the flux line at that point.

$|\vec{E}_2| > |\vec{E}_1|$



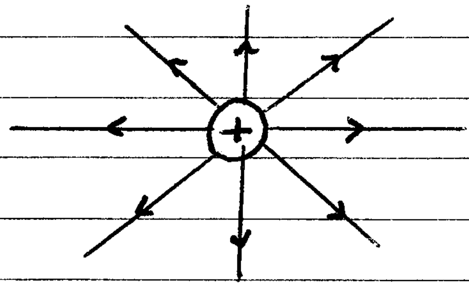


Opposite spherical (or point) charges

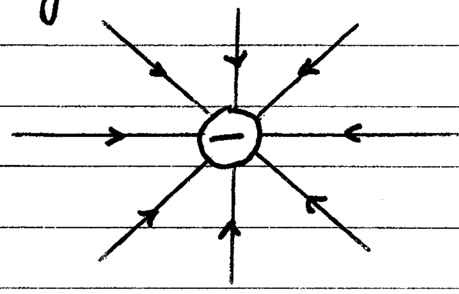


Similar spherical (or point) charges

Some special cases of straight flux lines

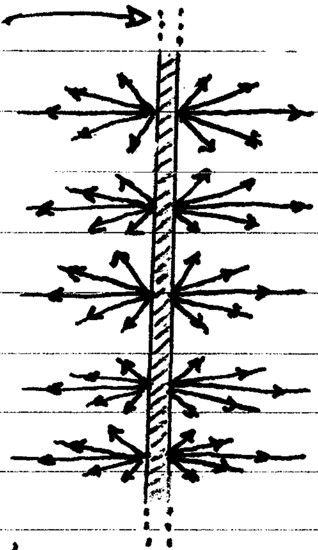


Isolated, Spherical (or point)

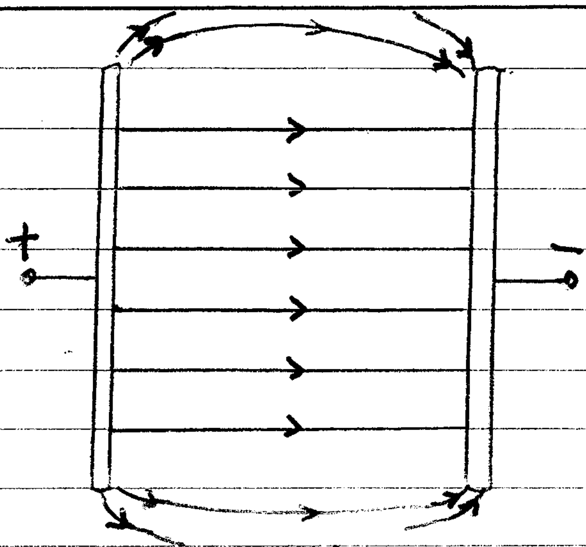


Isolated, spherical (or point)

+ve line charge



Isolated, Infinitly long.



Flat plates (large areas) (neglecting the edges).

b- Vectors Integration

*- Vectors may be integrated over geometrical shapes.

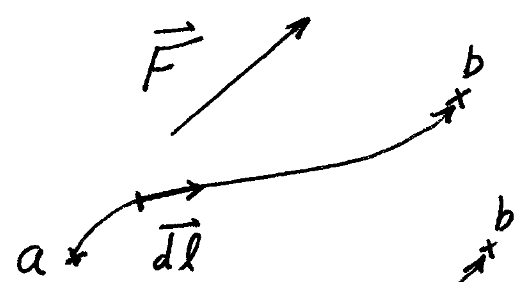
*- The geometry of those shapes could be:

→ 1- Dim. (Linear).

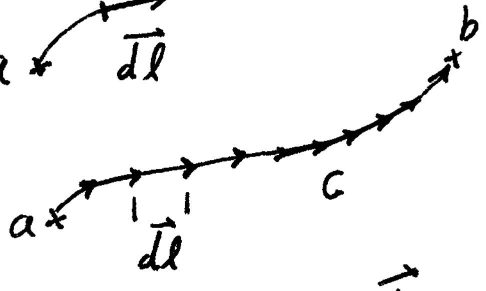
→ 2- Dim. (Surfaces)

→ 3- Dim. (Volumes).

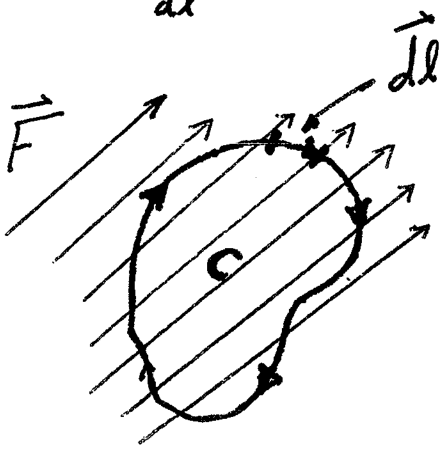
1- Line Integral



$$\int_a^b \vec{F}\cdot\vec{dl}$$

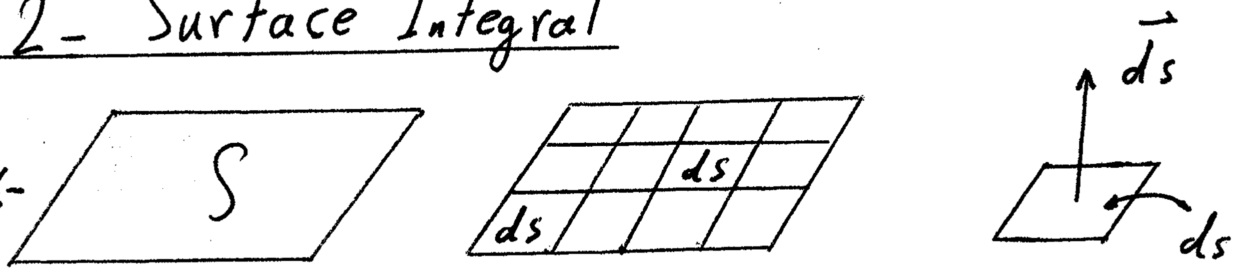


$\vec{dl}$: Line Element

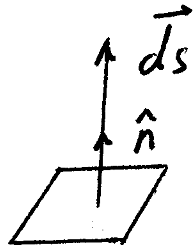


$$\oint_c \vec{F}\cdot\vec{dl}$$

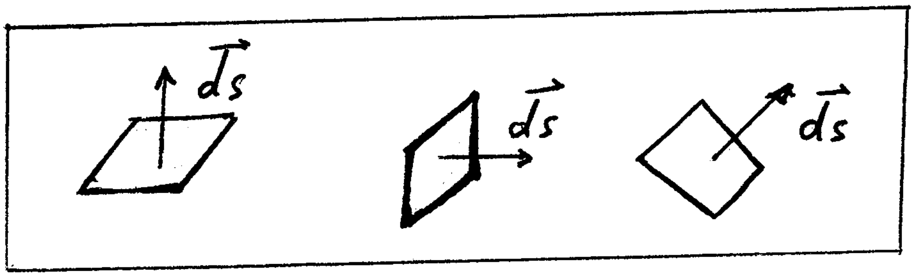
2- Surface Integral

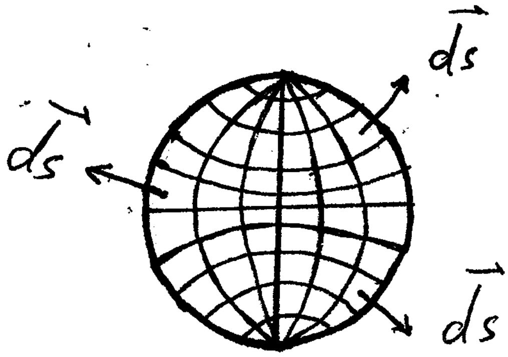
*- 

$\vec{ds}$: Surface Element.



$\vec{ds} = \hat{n}\, ds$



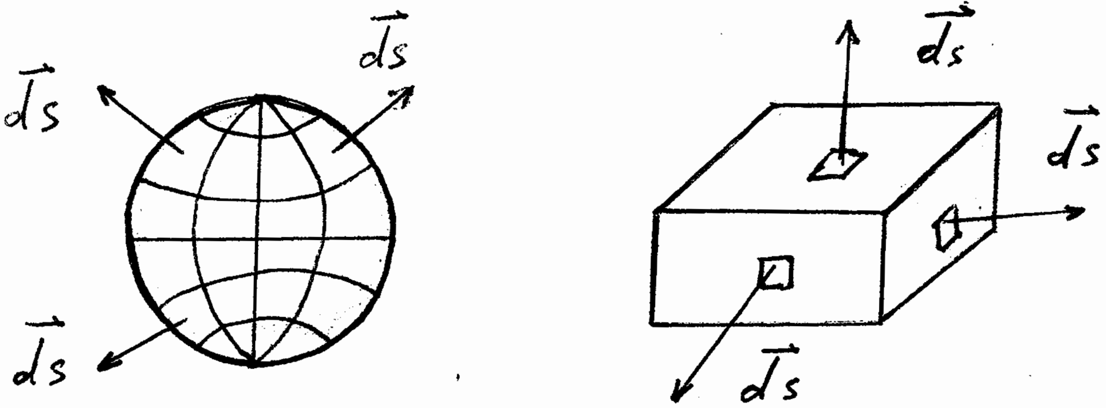
*- 

*- $\int_s \vec{F} \cdot \vec{ds} \equiv \int_s \vec{F} \cdot \hat{n}\, ds$

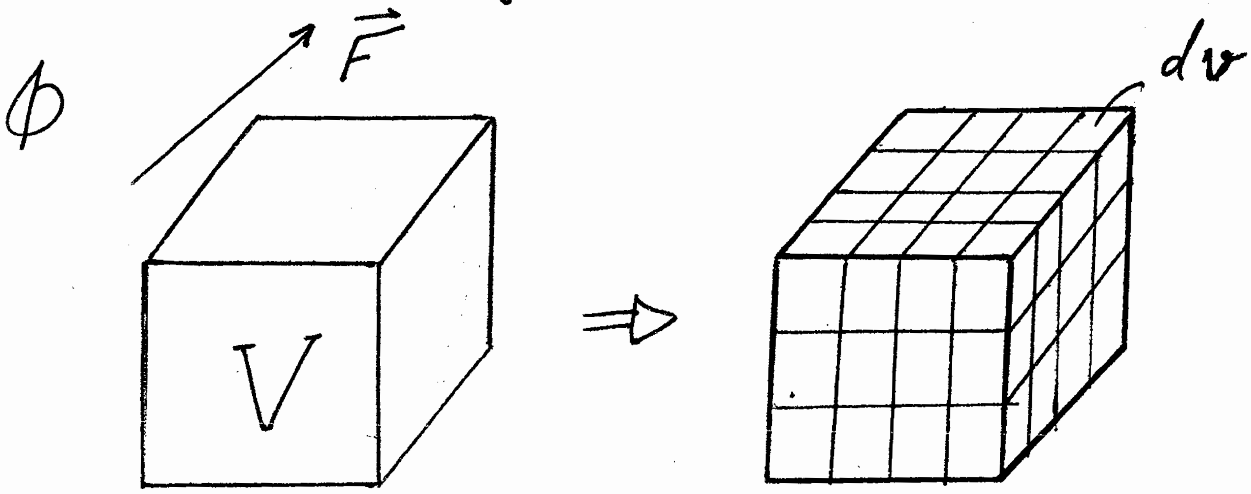
For closed surfaces : $\oint_s \vec{F} \cdot \vec{ds}$

Or $\oint_s \vec{F} \cdot \hat{n}\, ds$

For closed surfaces, the direction of the surface normal vector ($\vec{ds}$) is considered outwards :



3- Volume Integral



*- dv : Volume Element .

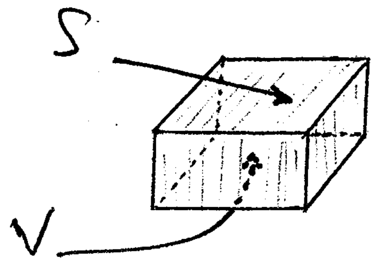
$$\rho = \int_V \phi \, dv \quad , \quad \vec{J} = \int_V \vec{F} \, dv$$

C- Divergence & Stockes' Theorems

1- Divergence Theorem

$$\int_V \vec{\nabla}\cdot\vec{F}\, dv = \oint_s \vec{F}\cdot\vec{ds} = \oint_s \vec{F}\cdot\hat{n}\, ds$$

S : Closed surface bounding volume V :

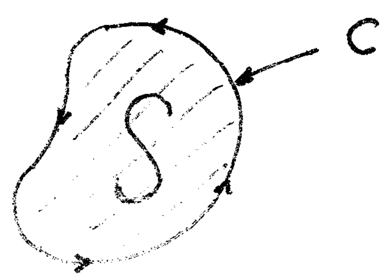


2- Stockes' Theorem

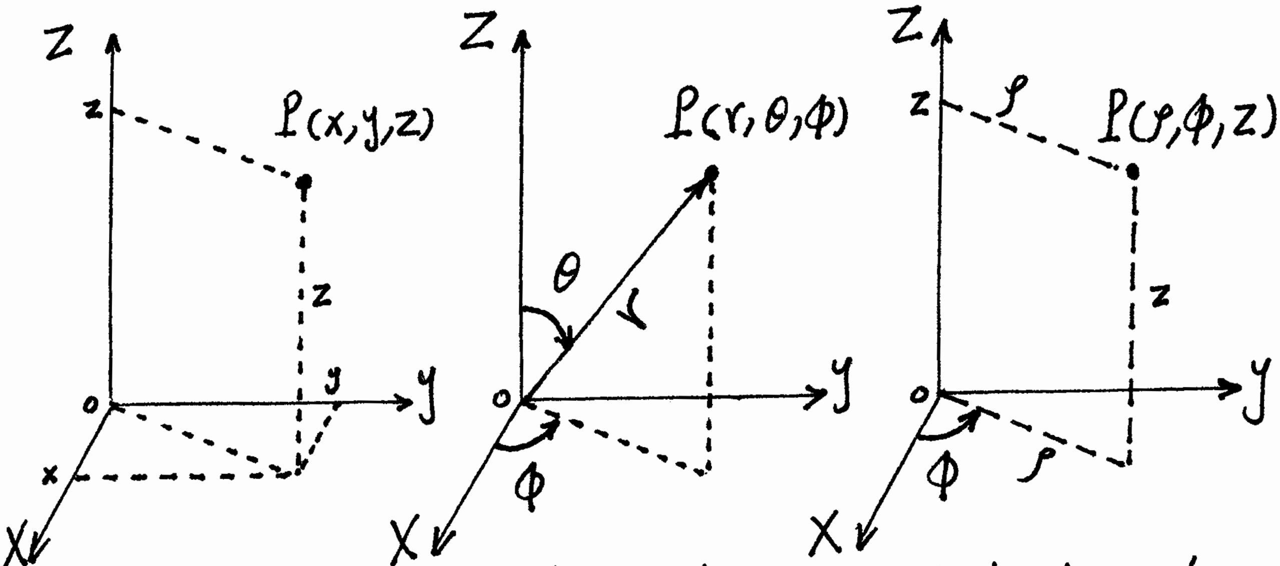
$$\oint_c \vec{F}\cdot\vec{dl} = \int_s (\vec{\nabla}\times\vec{F})\cdot\vec{ds}$$

$$= \int_s (\vec{\nabla}\times\vec{F})\cdot\hat{n}\, ds$$

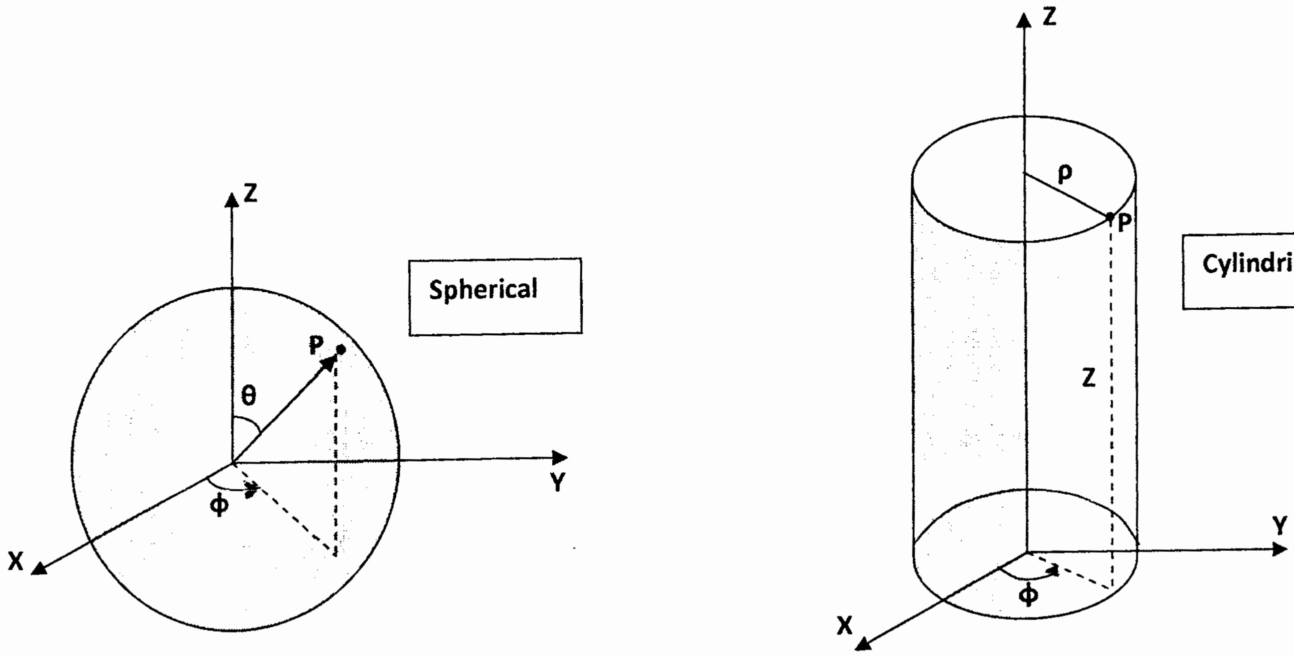
C : Closed curve bounding surface S :



d- Coordinate Systems



Cartesian coordinates   Spherical coordinates   Cylindrical coordinates

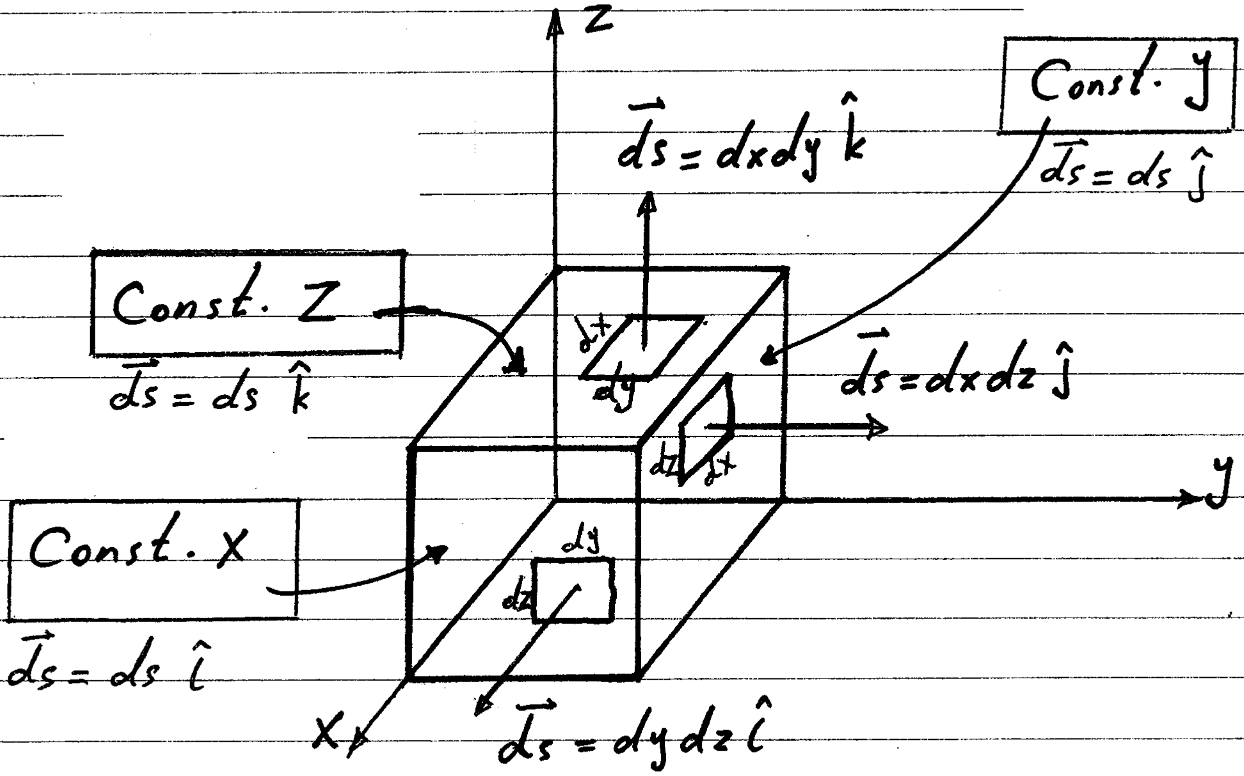


Coordinates	Axis	Unit Vector
Cartesian	x , y , z	$\hat{\imath}, \hat{\jmath}, \hat{k}$
Spherical	r, θ , ϕ	$\hat{a}_r , \hat{a}_\theta , \hat{a}_\varphi$
Cylindrical	ρ , ϕ , z	$\hat{a}_\rho , \hat{a}_\varphi , \hat{a}_z$

e- Surface Element ($\vec{ds}$) in Coordinate Systems

1- Cartesian Coordinates

*- Axis : x, y, z
*- Unit Vectors : $\hat{i}, \hat{j}, \hat{k}$ {or: $\hat{a}_x, \hat{a}_y, \hat{a}_z$}.



*- Try to write the expressions of the rest of the surface elements for the above figure.

2- Cylindrical Coordinates

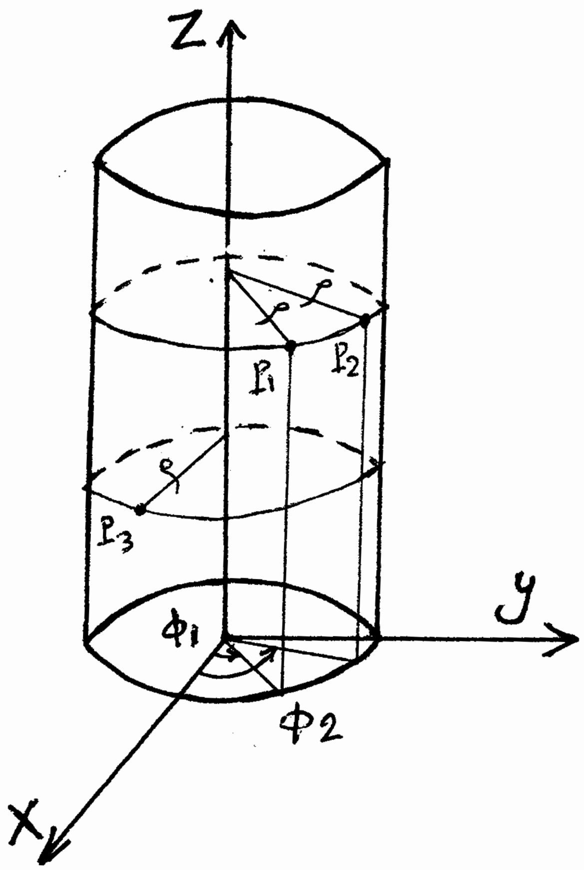
Axis : ρ , ϕ , z

Unit Vectors: $\hat{a}_\rho$, $\hat{a}_\phi$, $\hat{a}_z$

?: P_1 and P_2 are different in the ____ axis.

?: P_2 and P_3 are different in the ____ and ____ axis.

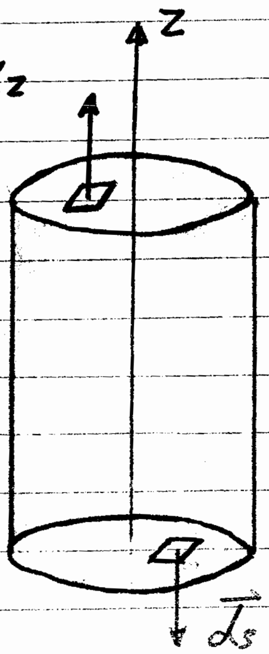
?: P_1, P_2, and P_3 are common in the ____ axis.



*- Important Surface Elements

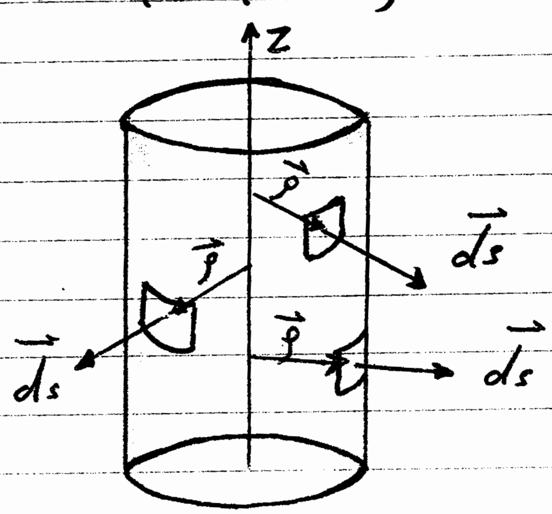
Const. z

$\vec{ds} = \rho \, d\rho \, d\phi \, \hat{a}_z$



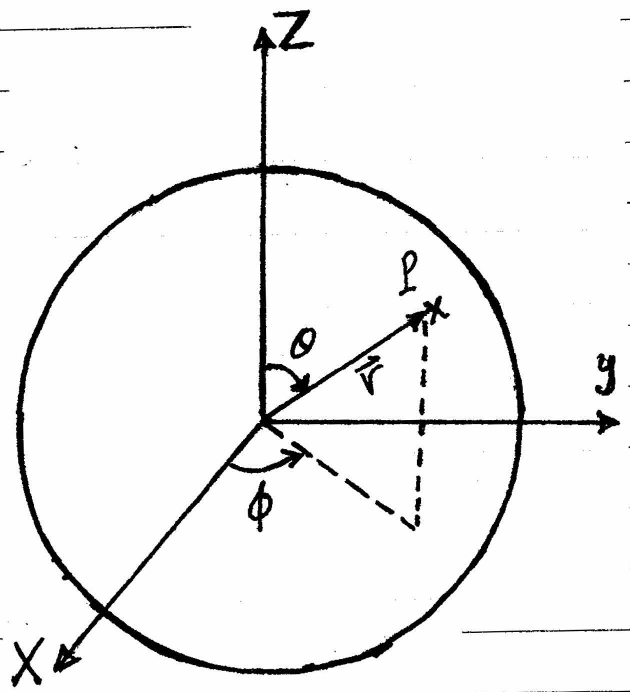
Const. ρ

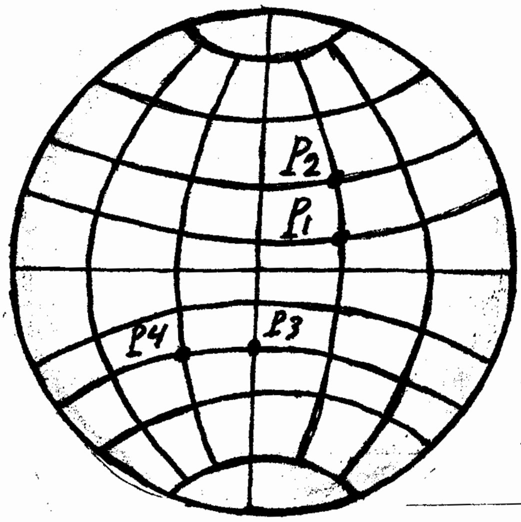
$\vec{ds} = \rho \, d\phi \, dz \, \hat{a}_\rho$



3- Spherical Coordinates

Axis : r, θ, Φ , Unit Vectors : $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$





?: P_1 and P_2 are different in ____ axis.

?: P_3 and P_4 are different in ____ axis.

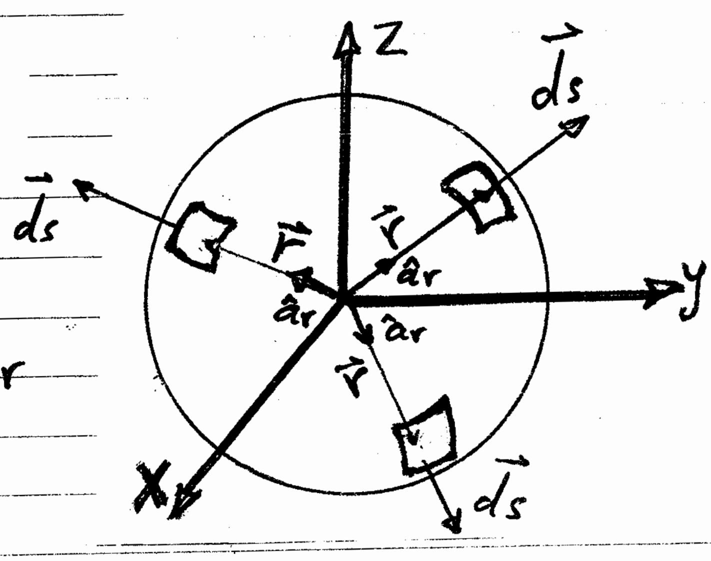
?: The four points are similar in ____ axis.

*- Important Surface Element : Const. r

$$\vec{ds} = ds\ \hat{n}$$

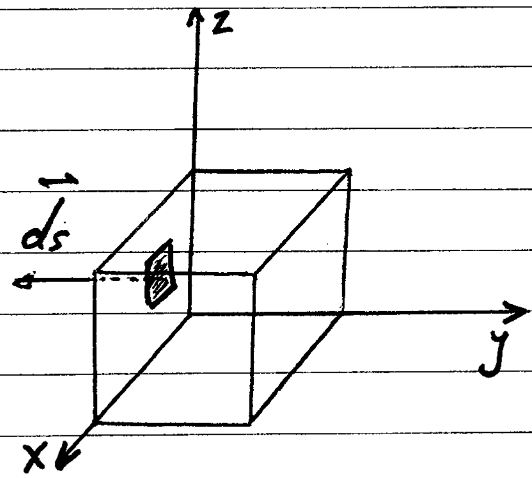
$$\vec{ds} = ds\ \hat{a}_r$$

$$\vec{ds} = r^2 \sin(\theta)\, d\theta\, d\Phi\ \hat{a}_r$$



Review

$\vec{ds} =$

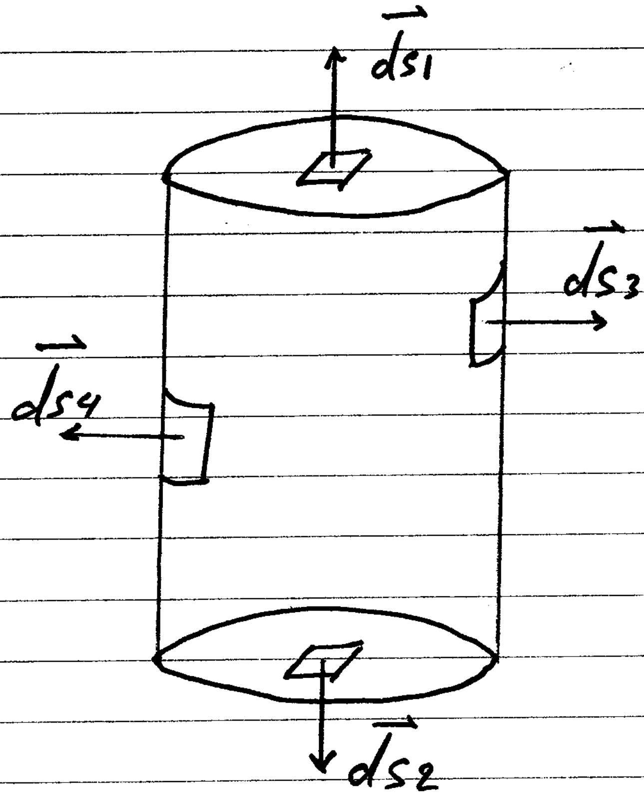


$\vec{ds_1} =$

$\vec{ds_2} =$

$ds_3 =$

$ds_4 =$

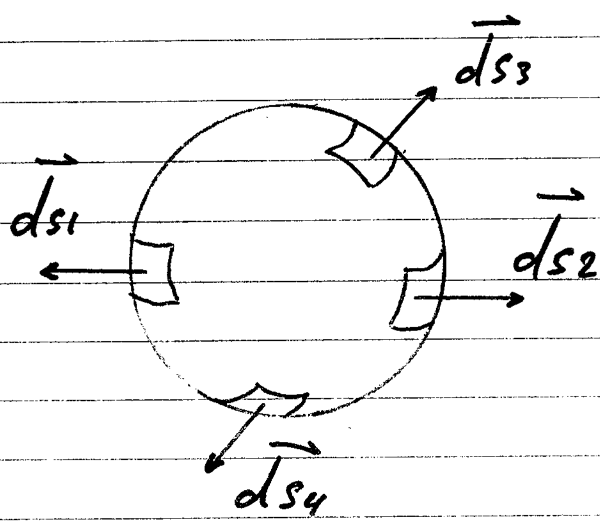


$\vec{ds_1} =$

$\vec{ds_2} =$

$\vec{ds_3} =$

$\vec{ds_4} =$



Examples (Vectors Integration)

Ex. 1: Integrate the vector: $\vec{F} = 3x\hat{i} + 6y^2\hat{j} - 2e^{-2z}\hat{k}$ over the path joining points $A(1,3,1)$m and $B(2,1,2)$m.

Sol. Let the integral is I:

$$I = \int_A^B \vec{F} \cdot \vec{dl} \quad , \quad \vec{dl} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$$

$$I = \int_A^B (3x\hat{i} + 6y^2\hat{j} - 2e^{-2z}\hat{k}) \cdot (\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz)$$

$$I = \int_A^B (3x\,dx + 6y^2\,dy - 2e^{-2z}\,dz)$$

$$I = 3\int_1^2 x\,dx + 6\int_3^1 y^2\,dy - 2\int_1^2 e^{-2z}\,dz$$

$$I = \frac{3}{2}x^2\Big|_1^2 + \frac{6}{3}y^3\Big|_3^1 - \left(\frac{2}{-2}\right)e^{-2z}\Big|_1^2$$

$$I = 1.5(4-1) + 2(1-27) + (e^{-4} - e^{-2})$$

$$I = 4.5 - 52 + (.0183 - .1353)$$

$$I = -47.617$$

$$\therefore \int_A^B \vec{F} \cdot \vec{dl} = -47.617 \,.$$

Ex. 2: Integrate the vector: $\vec{G} = 6\hat{i} + 2x\sqrt{y}\,\hat{j} + 3xz\,\hat{k}$ along the path given as: $y = x^2$, $z = \sqrt{x}$ from $A(1,3,2)$ to $B(2,2,1)$.

Sol.: Let the integral is I:

$$I = \int_A^B \vec{G} \cdot \vec{dl} \quad , \quad \vec{dl} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$$

$$I = \int_A^B (6\hat{i} + 2x\sqrt{y}\,\hat{j} + 3xz\hat{k}) \cdot (\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz)$$

$$I = \int_A^B (6\,dx + 2x\sqrt{y}\,dy + 3xz\,dz)$$

$$I = 6\int_1^2 dx + 2\int_3^2 x\sqrt{y}\,dy + 3\int_2^1 xz\,dz$$

Using the path eqs.: $y = x^2 \Longrightarrow x = \sqrt{y}$
and, $z = \sqrt{x} \Longrightarrow x = z^2$

$$\therefore I = 6x\Big|_1^2 + 2\int_3^2 \sqrt{y}\,\sqrt{y}\,dy + 3\int_2^1 z^2 \cdot z\,dz$$

$$I = 6(2-1) + 2\int_3^2 y\,dy + 3\int_2^1 z^3\,dz$$

$$I = 6 + \frac{2}{2}\,y^2\Big|_3^2 + \frac{3}{4}\,z^4\Big|_2^1$$

$$I = 6 + (4-9) + \frac{3}{4}(1-16)$$

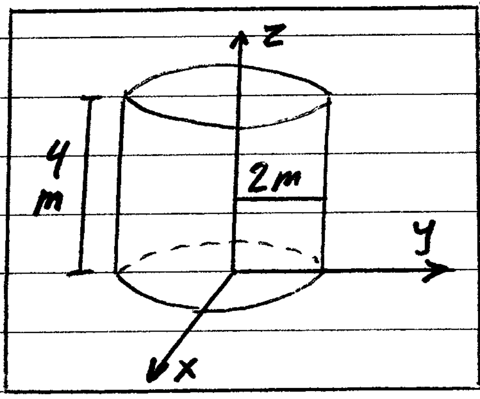
$$I = 6 - 5 - 11.25 = -10.25$$

$$\therefore \int_A^B \vec{G} \cdot \vec{dl} = -10.25$$

Ex.3: Integrate the vector :

$$\vec{A} = \rho\, e^{\phi/2} z^2 \hat{a}_\rho + \rho^2 e^{-\phi} z\, \hat{a}_\phi + \sqrt{\rho}\sqrt{z}\, \hat{a}_z$$

over the side surface of a cylinder of radius 2m and hight 4m.



Sol. Let the integral is I :

$$I = \int_{side} \vec{A} \cdot \vec{ds}$$

At the side of the cylinder :

$$\vec{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$$

$$\therefore I = \int_{side} \left(\rho e^{\phi/2} z^2 \hat{a}_\rho + \rho^2 e^{-\phi} z\, \hat{a}_\phi - \sqrt{\rho}\sqrt{z}\, \hat{a}_z\right) \cdot \rho\, d\phi\, dz\, \hat{a}_\rho$$

$$I = \int_{side} \rho\, e^{\phi/2} z^2 \rho\, d\phi\, dz = \rho^2 \int_0^{2\pi} e^{\phi/2} d\phi \int_0^4 z^2 dz$$

$$I = 2^2 \times 2\, e^{\phi/2} \Big|_0^{2\pi} \times \frac{1}{3} z^3 \Big|_0^4$$

$$I = 8\,(e^{\pi} - e^{0}) \times \frac{1}{3}(4^3 - 0)$$

$$I = \frac{8}{3}(e^{3.14} - 1)\, 64 = \frac{8}{3} \times 64\,(23.1038 - 1)$$

$$I = 3772.3818$$

$$\therefore \int_{side} \vec{A} \cdot \vec{ds} = 3772.3818$$

Hw.: Find the integration over the upper surface in this example.

Ans.: $I = 28.42$

Note: $\pi \approx 3.14$

Ex. 4: Integrate the vector:

$$\vec{A} = \frac{e^{\phi}}{100\,r}\,\hat{a}_r + 50\,r\sin\theta\,\hat{a}_\theta + 20\,r\,e^{\phi}\,\hat{a}_\phi$$

over the spherical surface of radius $R = 1\,m$.

Sol.: $I = \oint_{sph.} \vec{A}\cdot\vec{ds}$, $\vec{ds} = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$

$$I = \oint_{sph} \frac{e^{\phi}}{100\,r}\,r^2\sin\theta\,d\theta\,d\phi$$

$$I = \frac{r}{100}\int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}e^{\phi}\,d\phi$$

$$I = \frac{-1}{100}\cos\theta\Big|_0^{\pi}\;e^{\phi}\Big|_0^{2\pi}$$

$$I = \frac{-1}{100}(-1-1)(e^{2\pi}-e^{0}) = \frac{2}{100}(533.7886-1)$$

$$I = 10.6557$$

$$\therefore \int_{sph.} \vec{A}\cdot\vec{ds} = 10.6557$$