هندسة اتصالات الحاسبات
المرحلة الثالثة
نظرية المجالات

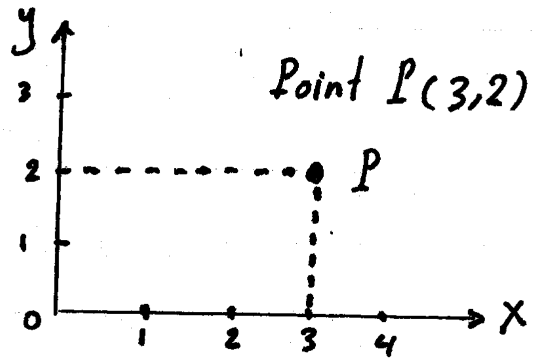
- EM.1 -

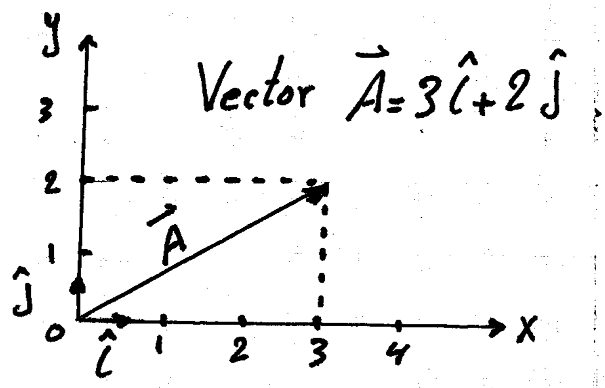
Electromagnetics

*- **Vectors :** $\vec{A}, \vec{E}, \vec{H}, \vec{r}, \ldots$
Unit vectors : $\hat{a}, \hat{i}, \hat{r}, \hat{a}_x, \hat{a}_r, \ldots$
Scalars : $c, b, V, \phi, \ldots$

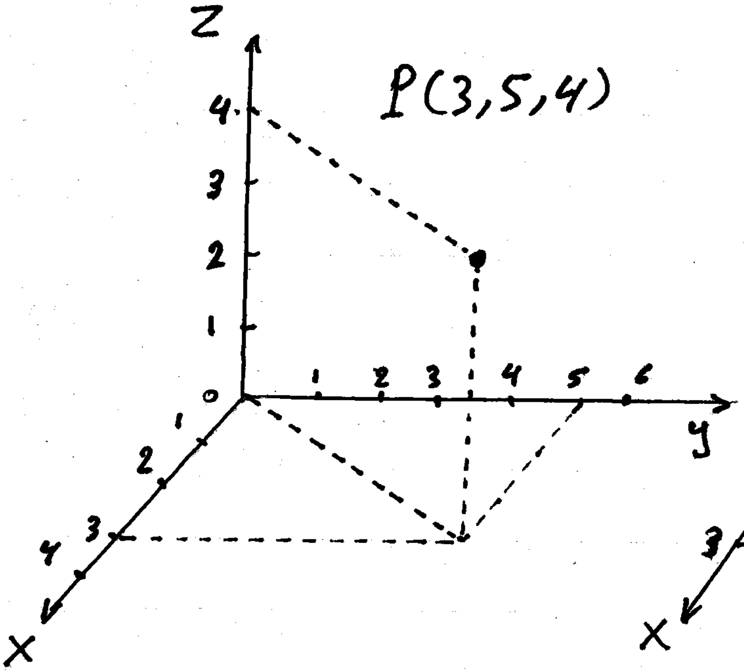
Cartezian Coordinates

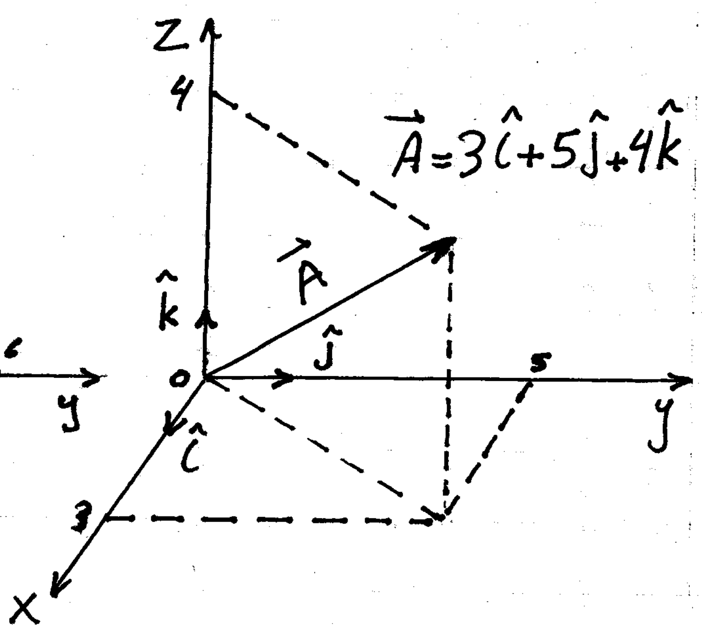
*- Two- Dimensional Coordinates (x, y) :-





Three-Dimensional Coordinates (x, y, z)





Vectors

$$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

or, $\vec{A} = \hat{i} A_x + \hat{j} A_y + \hat{k} A_z$

$$\left.\begin{aligned} \vec{A_x} &= \hat{i} A_x \\ \vec{A_y} &= \hat{j} A_y \\ \vec{A_z} &= \hat{k} A_z \end{aligned}\right\} \text{Vectors}$$

*- Example: $\vec{A} = 3\hat{i} + 5\hat{j} + 4\hat{k}$

$$\vec{A_x} = 3\hat{i}$$
$$\vec{A_y} = 5\hat{j}$$
$$\vec{A_z} = 4\hat{k}$$

$$A_x = 3$$
$$A_y = 5$$
$$A_z = 4$$

*- Magnitude of a vector: $|\vec{A}| \equiv A$

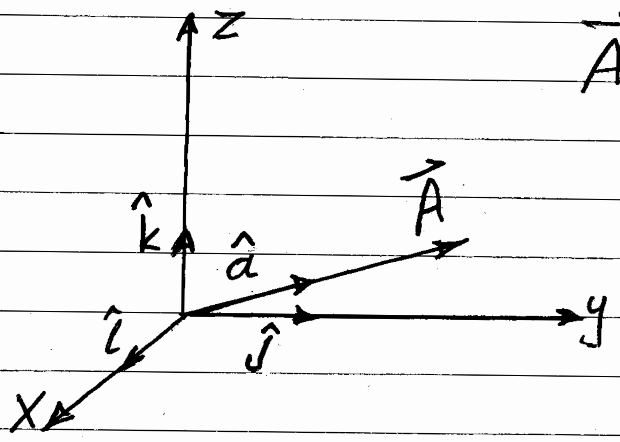
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

*- for: $\vec{A} = 3\hat{i} + 5\hat{j} + 4\hat{k}$

$$|\vec{A}| = \sqrt{9 + 25 + 16} = \sqrt{50}$$

$$|\vec{A}| = 7.071$$

*-Every vector has it's unit vector :



$$\vec{A} = A\,\hat{a}$$

$$\text{or, } \vec{A} = |\vec{A}|\,\hat{a}$$

$$|\hat{a}| = 1$$

$$|\hat{a}| = |\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = |\hat{n}| = 1$$

Ex.: Prove that the vector :

$$\vec{A} = \frac{1}{3}\hat{\imath} - \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}, \text{ is a unit vector.}$$

Sol.:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{A}| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{4}{9}}$$

$$|\vec{A}| = \sqrt{\frac{9}{9}} = 1$$

$\therefore$ $\vec{A}$ is a unit vector $(\vec{A} \equiv \hat{a})$.

Ex.: Find the vector whose magnitude is (12 m) directed along the unit vector $\left(\frac{1}{3}\hat{\imath} - \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}\right)$.

Sol.: Let the vector is $\vec{B}$.

$$\therefore \vec{B} = |\vec{B}|\,\hat{a}_B \quad , \quad \hat{a}_B = \frac{1}{3}\hat{\imath} - \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$$

$$|\vec{B}| = 12 \text{ m}.$$

$$\therefore \vec{B} = 12\left(\frac{1}{3}\hat{\imath} - \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}\right)$$

$$\vec{B} = 4\hat{\imath} - 8\hat{\jmath} + 8\hat{k} \quad , \text{ (m)}.$$

*- Vectors Addition
→ Vectors Summation

$$\vec{A} = \hat{i} A_x + \hat{j} A_y + \hat{k} A_z$$
$$\vec{B} = \hat{i} B_x + \hat{j} B_y + \hat{k} B_z$$

let $\vec{A} + \vec{B} = \vec{C}$

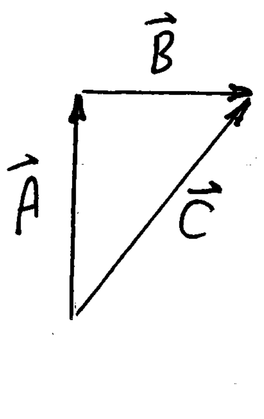
where, $\vec{C} = \hat{i} C_x + \hat{j} C_y + \hat{k} C_z$

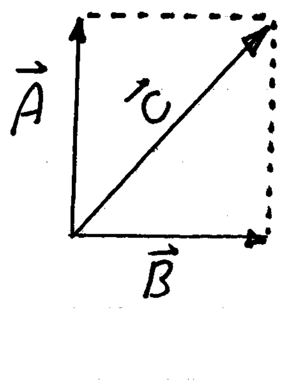
$$\vec{A} + \vec{B} = \hat{i}(A_x + B_x) + \hat{j}(A_y + B_y) + \hat{k}(A_z + B_z)$$
$$= \hat{i} \; C_x + \hat{j} \; C_y + \hat{k} \; C_z$$

$\therefore$

$C_x = A_x + B_x$	$\vec{C}_x = (A_x + B_x)\,\hat{i}$
$C_y = A_y + B_y$	$\vec{C}_y = (A_y + B_y)\,\hat{j}$
$C_z = A_z + B_z$	$\vec{C}_z = (A_z + B_z)\,\hat{k}$

*- For example :

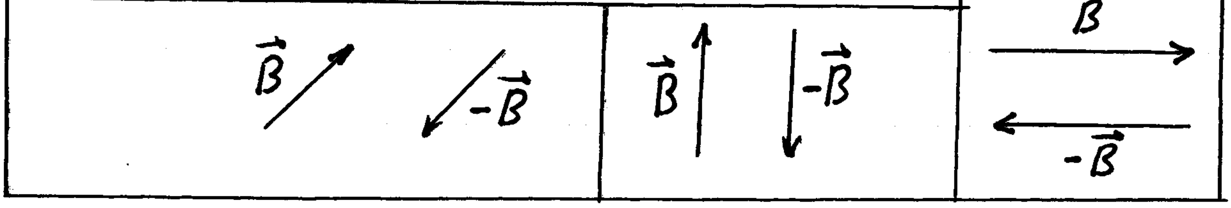




→ Vectors Subtraction

Let $\vec{A} - \vec{B} = \vec{D}$

Note: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

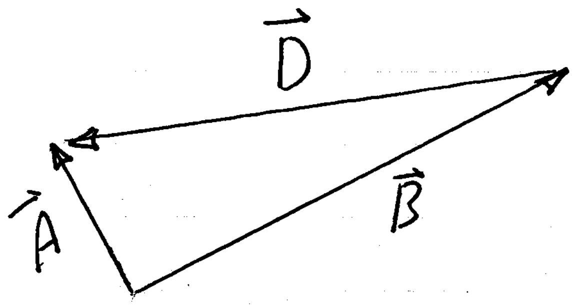


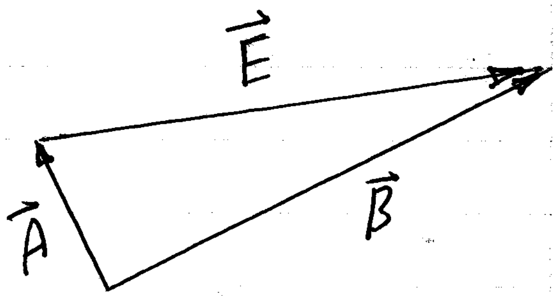
$$\vec{A} - \vec{B} = \hat{i}(A_x - B_x) + \hat{j}(A_y - B_y) + \hat{k}(A_z - B_z)$$
$$= \hat{i}\, D_x + \hat{j}\, D_y + \hat{k}\, D_z.$$

$\therefore$ $D_x = A_x - B_x$, $\vec{D}_x = (A_x - B_x)\,\hat{i}$

$D_y = A_y - B_y$, $\vec{D}_y = (A_y - B_y)\,\hat{j}$

$D_z = A_z - B_z$, $\vec{D}_z = (A_z - B_z)\,\hat{k}$





$\vec{D} = \vec{A} - \vec{B}$

$\vec{E} = \vec{B} - \vec{A}$

$\vec{E} = -\vec{D}$ (or, $\vec{D} = -\vec{E}$)

$|\vec{E}| = |\vec{D}|$

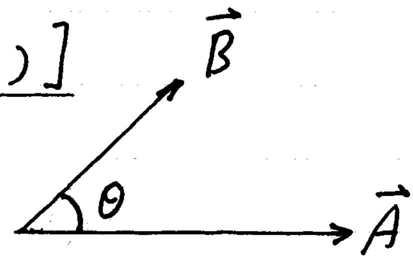
Multiplication

*- Multiplication of vectors with scalars

$$\phi(\vec{A}+\vec{B}) = \phi\vec{A} + \phi\vec{B} = \vec{E}$$

*- Multiplication of vectors

1- Scalar product [Dot (.)]



*- $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$

*- $\underline{\vec{A}\cdot\vec{B}}$: a scalar quantity.

*- If $\vec{A} \perp \vec{B} \Longrightarrow \vec{A}\cdot\vec{B} = 0 \quad (\theta = 90^\circ)$.

*- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \qquad (\theta = 0)$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0 \qquad (\theta = 90^\circ)$

*- $\vec{A}\cdot\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z)\cdot(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z)$

$= A_x B_x + A_y B_y + A_z B_z$

*- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

*- $\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2 = |\vec{A}|^2$ or A^2

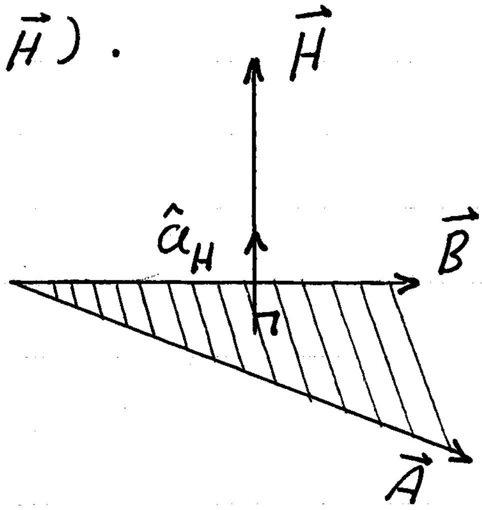
$\therefore \quad |\vec{A}| \equiv A = \sqrt{\vec{A}\cdot\vec{A}}$.

⊛ $\vec{A}\cdot\hat{i} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z)\cdot\hat{i} = A_x$

$\vec{A}\cdot\hat{j} = A_y \quad , \quad \vec{A}\cdot\hat{k} = A_z$

2- Vector product [Cross (x)]

* $\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}|\,\sin(\theta)\,\hat{a}_H = \vec{H}$
*- $\vec{A} \times \vec{B}$: a vector quantity $(\vec{H})$.
*- $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin(\theta)$
*- $\vec{H} \perp$ on both $\vec{A}$ and $\vec{B}$.



*- $\vec{A} \times \vec{B} = \vec{H}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}\,(A_y B_z - A_z B_y) - \hat{j}\,(A_x B_z - A_z B_x) + \hat{k}\,(A_x B_y - A_y B_x)$$
$$\vec{A} \times \vec{B} = \hat{i}\,(A_y B_z - A_z B_y) + \hat{j}\,(A_z B_x - A_x B_z) + \hat{k}\,(A_x B_y - A_y B_x)$$

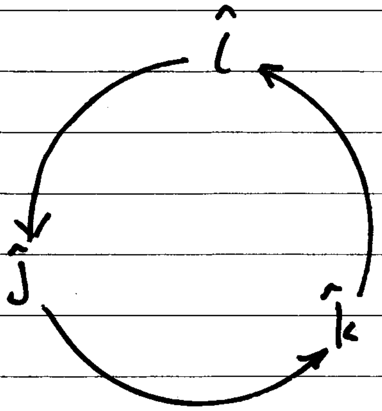
$$\vec{A} \times \vec{B} = \vec{H} = \hat{i} H_x + \hat{j} H_y + \hat{k} H_z$$

$\therefore$
$$H_x = A_y B_z - A_z B_y$$
$$H_y = A_z B_x - A_x B_z$$
$$H_z = A_x B_y - A_y B_x$$

*- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

*- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 \qquad (\theta = 0)$

In general: $\vec{A} \times \vec{A} = 0$



$\hat{i} \times \hat{j} = \hat{k} \quad , \quad \hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i} \quad , \quad \hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j} \quad , \quad \hat{i} \times \hat{k} = -\hat{j}$

Vector Differential Operator ($\vec{\nabla}$)

$$\boxed{\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}}$$

*-1- $\vec{\nabla}$ - Operation on Scalars :

*- Gradient (grad), (Grad)

Let ϕ is a scalar quantity, then the gradient of ϕ is given as :

$$\vec{\nabla}\phi \equiv \text{grad}\,\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi$$

$$\vec{\nabla}\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z} = \vec{F}$$

*- The gradient of a scalar is a vector quantity -

*- 2 - $\vec{\nabla}$- Operations on Vectors :

a- Divergence (Div), (div) [Scalar Product]

Let: $\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$, $Div\vec{F} \equiv \vec{\nabla}\cdot\vec{F} \equiv$ Scalar

then, $\vec{\nabla}\cdot\vec{F} = (\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z})\cdot(\hat{i}F_x + \hat{j}F_y + \hat{k}F_z)$

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \equiv \Phi$$

*- The divergence of a vector is a scalar quantity.

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b- Curl [Vector Product]

$$Curl\,\vec{F} \equiv \vec{\nabla}\times\vec{F} \equiv \text{Vector}$$

$$\vec{\nabla}\times\vec{F} = (\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z})\times(\hat{i}F_x + \hat{j}F_y + \hat{k}F_z)$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{\nabla}\times\vec{F} = \hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{j}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$\therefore\ \vec{\nabla}\times\vec{F} = \vec{H}$, a vector quantity.

*- The Laplacian Operator (∇^2)

$$\nabla^2 = \vec{\nabla} \cdot \vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \text{ : Scalar.}$$

*- The Laplacian may operate on scalars and vectors.

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \quad : \text{ Scalar}$$

$$\nabla^2 \vec{F} = \frac{\partial^2 \vec{F}}{\partial x^2} + \frac{\partial^2 \vec{F}}{\partial y^2} + \frac{\partial^2 \vec{F}}{\partial z^2} \quad : \text{ Vector}$$

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*- Some $\vec{\nabla}$- Relations :

$$\vec{\nabla} \cdot \vec{\nabla} \phi = \nabla^2 \phi$$

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} = 0$$

$$\vec{\nabla} \times \vec{\nabla} \phi = 0$$

$$\vec{\nabla} (\phi \psi) = \phi \vec{\nabla} \psi + \psi \vec{\nabla} \phi$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{F} = \vec{\nabla} (\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}$$

Summery on the $\vec{\nabla}$ - Operations

Operations on Scalars

* Gradient (grad)
* $\vec{\nabla}\phi$ (grad ϕ)
* $\vec{\nabla}\phi$ is a Vector
* $\vec{\nabla}\phi = \hat{i}\dfrac{\partial\phi}{\partial x} + \hat{j}\dfrac{\partial\phi}{\partial y} + \hat{k}\dfrac{\partial\phi}{\partial z}$

Operations on Vectors

* ① Dot Product (Scaler Product)
* Divergence (Div)
* $\vec{\nabla}\cdot\vec{A}$ (Div $\vec{A}$)
* $\vec{\nabla}\cdot\vec{A}$ is a scaler.
* $\vec{\nabla}\cdot\vec{A} = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$

* ② Cross Product (Vector Product)
* Curl
* $\vec{\nabla}\times\vec{A}$ (Curl $\vec{A}$)
* $\vec{\nabla}\times\vec{A}$ is a Vector
* $\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

Examples

Ex. 1: For the two vectors :

$\vec{A} = 4\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} - 2\hat{k}$,

find: $\vec{A}+\vec{B}$, $\vec{B}-\vec{A}$, $\vec{A}\cdot\vec{B}$, $\vec{B}\cdot\vec{A}$,
$\vec{B}\times\vec{A}$, and $\vec{A}\times\vec{B}$.

Sol. * $\vec{A}+\vec{B} = 6\hat{i} - \hat{j} - \hat{k}$

$\vec{B}-\vec{A} = -2\hat{i} + 3\hat{j} - 3\hat{k}$

* $\vec{A}\cdot\vec{B} = (4\hat{i} - 2\hat{j} + \hat{k})\cdot(2\hat{i} + \hat{j} - 2\hat{k}) = 8 - 2 - 2$

$\vec{A}\cdot\vec{B} = 4$ (Scalar)

$\vec{B}\cdot\vec{A} = \vec{A}\cdot\vec{B} = 4$

* $$\vec{B}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 4 & -2 & 1 \end{vmatrix} = \hat{i}(1-4) + \hat{j}(-8-2) + \hat{k}(-4-4)$$

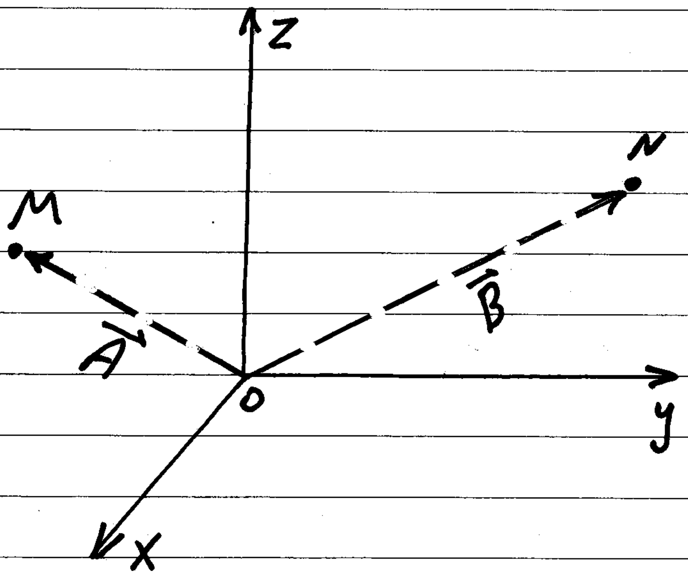
$\vec{B}\times\vec{A} = -3\hat{i} - 10\hat{j} - 8\hat{k}$, (Vector)

$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$\vec{A}\times\vec{B} = 3\hat{i} + 10\hat{j} + 8\hat{k}$

Ex. 2:

Given points:
$M(4,-2,5)$ and
$N(3,10,7)$.



Find:

a- Vectors $\vec{A}$ and $\vec{B}$.

b- Use $\vec{A}$ and $\vec{B}$ to find the vector directed from M to N.

Sol.:

a- $\vec{A} = 4\hat{i} - 2\hat{j} + 5\hat{k}$
$\vec{B} = 3\hat{i} + 10\hat{j} + 7\hat{k}$

b- Let $\vec{C} \equiv$ The vector directed from M to N.

$$\vec{C} = \vec{B} - \vec{A} = (3\hat{i} + 10\hat{j} + 7\hat{k}) - (4\hat{i} - 2\hat{j} + 5\hat{k})$$

$$\vec{C} = -\hat{i} + 12\hat{j} + 2\hat{k}$$

$$\boxed{\text{Note: We may find } \vec{C} \text{ from the coordinates of } M \text{ and } N.}$$

Ex. 3: Find the vector directed from $(2,-4,1)$ to $(0,-2,0)$ in cartesian coordinates and find the unit vector along that vector.

Sol. Let the vector is $\vec{A}$.

$$\vec{A} = (0-2)\hat{i} + (-2+4)\hat{j} + (0-1)\hat{k}$$

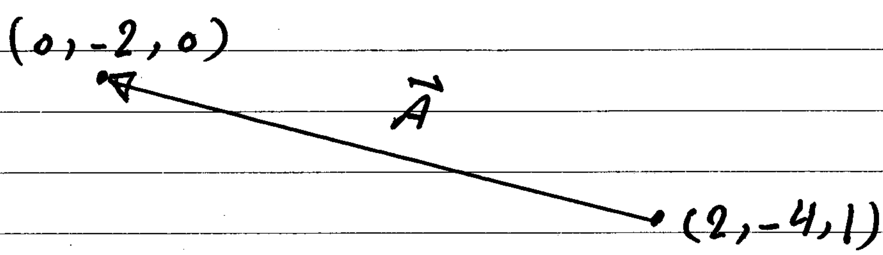
$$\boxed{\vec{A} = -2\hat{i} + 2\hat{j} - \hat{k}}$$

$$\hat{a} = \vec{A} / |\vec{A}|$$

$$|\vec{A}| \equiv A = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$\hat{a} = \frac{1}{3}(-2\hat{i} + 2\hat{j} - \hat{k})$$

$$\boxed{\hat{a} = \frac{-2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}}$$



H.w.1: Show that $\vec{A} = 4\hat{i} - 2\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + 4\hat{j} - 4\hat{k}$ are perpendicular.

H.w.2: Given $\vec{A} = 2\hat{i} + 4\hat{j}$ and $\vec{B} = 6\hat{j} - 4\hat{k}$,

find the angle between them using:
(a)- the cross product, (b)- the dot product.

Ans.: $\theta \simeq 41.9°$ {Put your calculator on the Deg. mode}.

Ex.4: Given vector $\vec{A}$ as:

$$\vec{A} = (3x^2y)\hat{i} - (4ze^{-2y})\hat{j} - (2xy^2z^3)\hat{k}$$

(a)- find the divergence of $\vec{A}$.
(b)- Determine the value of Div $\vec{A}$ at location (2, 0, 1).

Sol.: (a): $\vec{\nabla}\cdot\vec{A} = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$

$\therefore \vec{\nabla}\cdot\vec{A} = 6xy + 8ze^{-2y} - 6xy^2z^2$, (Scalar)

(b): At (2, 0, 1):

$$(\vec{\nabla}\cdot\vec{A})_{(2,0,1)} = 0 + 8 - 0 = 8 .$$

Class

Ex.5 : Given : $\Phi = \dfrac{-4y^3\cos(2x)}{e^{-2z}}$, find:

(a)- The gradient of Φ , (b)- The value of this gradient at (0,2,1).

Sol. (a):

$$\vec{\nabla}\Phi = \hat{i}\frac{\partial \Phi}{\partial x} + \hat{j}\frac{\partial \Phi}{\partial y} + \hat{k}\frac{\partial \Phi}{\partial z}$$

$$\Phi = -4y^3 e^{2z}\cos(2x)$$

$$\vec{\nabla}\Phi = \hat{i}\left[8y^3 e^{2z}\sin(2x)\right] - \hat{j}\left[12y^2 e^{2z}\cos(2x)\right] - \hat{k}\left[8y^3 e^{2z}\cos(2x)\right] \equiv \text{Grad.}\,\Phi$$

(b): At (0,2,1) :

$$\vec{\nabla}\Phi = -\hat{j}(12\times 4\times e^2\times 1) - \hat{k}(8\times 8\times e^2\times 1)$$

$$\vec{\nabla}\Phi = -473.2608\,\hat{j} - 631.0144\,\hat{k}$$, (Vector).

Class

H.w.3:

Given $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = \hat{i} + 2\hat{k}$, and $\vec{C} = 2\hat{j} + \hat{k}$, find $(\vec{A} \times \vec{B}) \times \vec{C}$ and compare it with $\vec{A} \times (\vec{B} \times \vec{C})$.

Ans.: $(\vec{A} \times \vec{B}) \times \vec{C} = -2\hat{j} + 4\hat{k}$, $\vec{A} \times (\vec{B} \times \vec{C}) = 2\hat{i} - 2\hat{j} + 3\hat{k}$

H.w.4: Using the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ of (H.w.3), find $\vec{A} \cdot \vec{B} \times \vec{C}$ and compare it with $\vec{A} \times \vec{B} \cdot \vec{C}$.

Ans.: (-5) for both.