نظرية المجالات
هندسة اتصالات الحاسبات
المرحلة الثانية

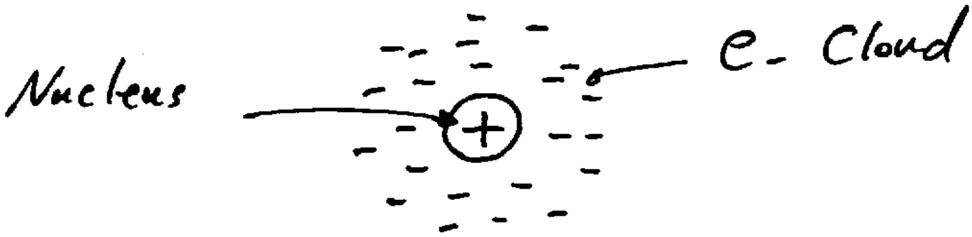
_EM. 8_

-1-

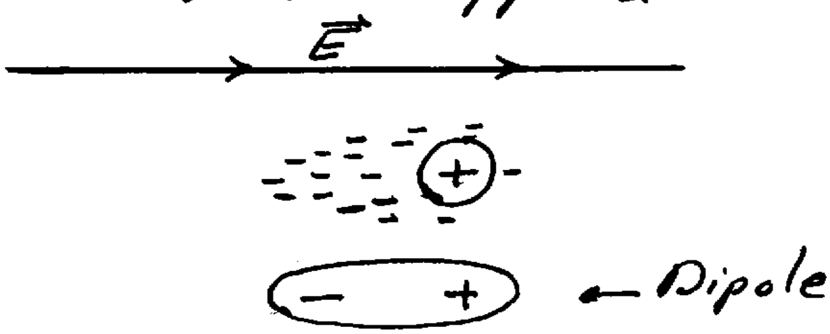
Dielectric Materials (Insulators)

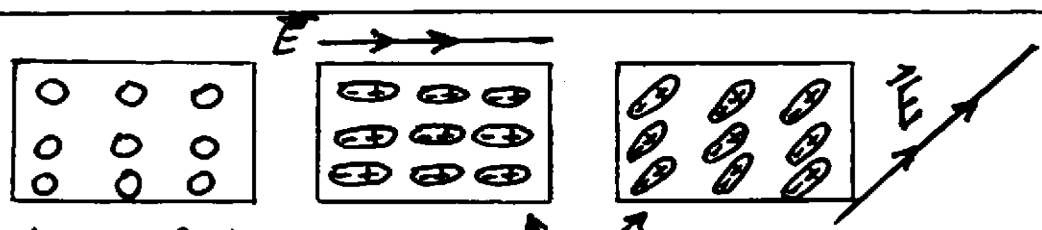
*- In dielectrics there are very few of free e_s (approximately no free e_s).

*- The atom when no E-field applied can be viewed as:



*- When E-field is applied $\Longrightarrow$ Polarization:





No E-field
No polarization

Polarization

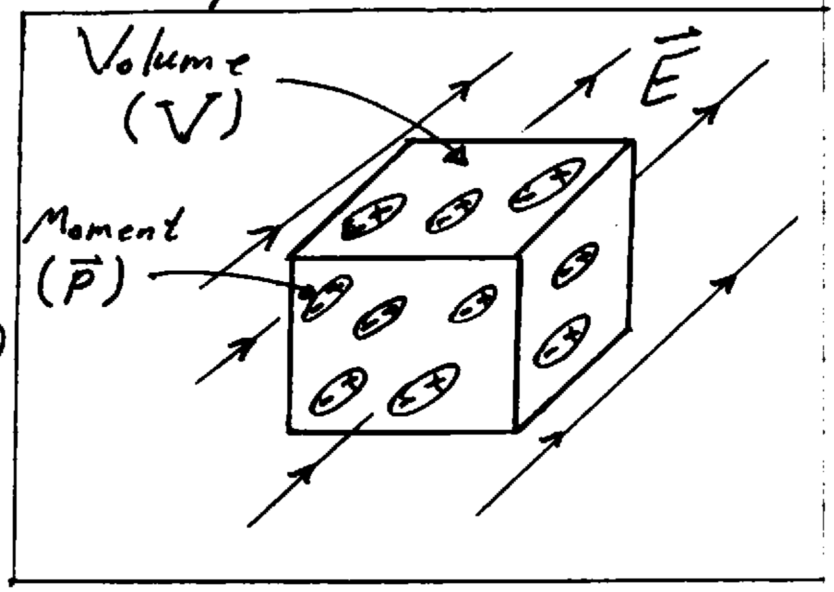
*- Each polarized atom has a moment:

$\vec{p} = Q\,\vec{d}$ ← This is the moment of a single atom or molecule.

*- Polarization ($\vec{P}$): The net dipole moment per unit volume.

*- No. of dipoles in volume (V) is n.

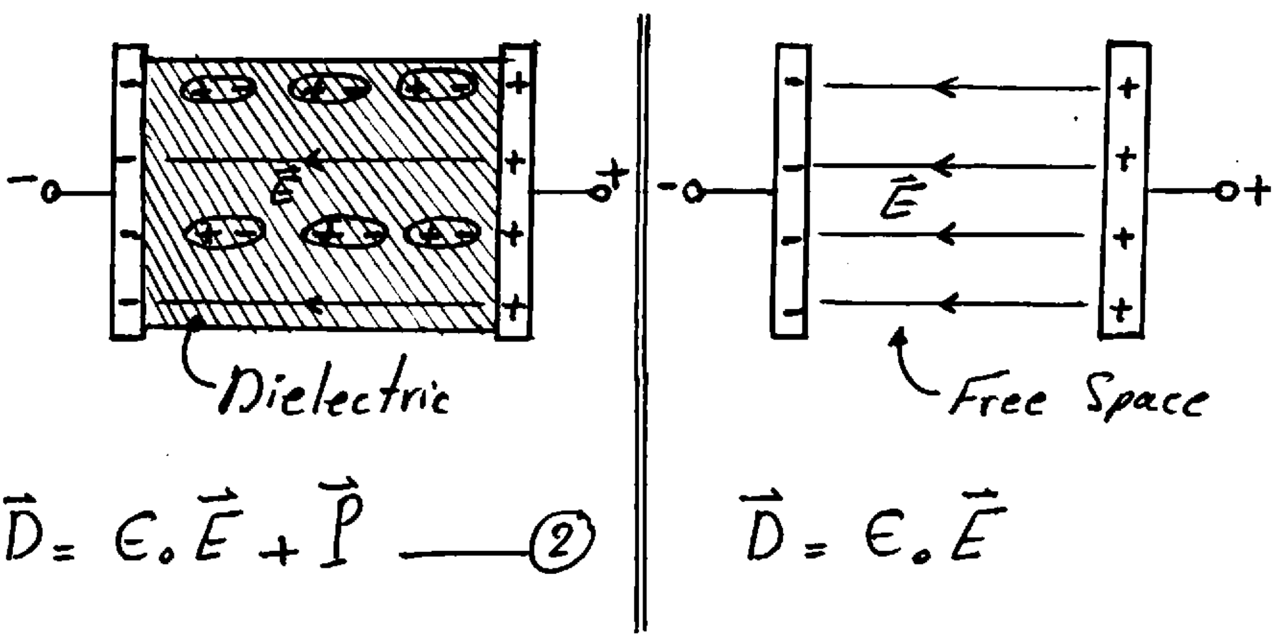
*- $\vec{P} = \dfrac{\sum_{i=1}^{n} \vec{p}_i}{V}$ ——— (1)



*- Units of $\vec{P}$:
(C/m²). *- More precisely, eq.(1) is written as:
$\vec{P} = \lim_{\Delta V \to 0} \left(\sum_{i=1}^{n} \vec{p}_i / \Delta V \right)$.

*- In polarized dielectric materials we have: (Bound Charges).

*- In general, we may have free charges and bound charges, then the total charge is:
$Q_T = Q_f + Q_b$



$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ ——— (2)

$\vec{D} = \epsilon_0 \vec{E}$

*- The polarization ($\vec{P}$) is proportional to the electric field ($\vec{E}$) and it depends on the dielectric material :

$$\vec{P} = \chi_e \epsilon_\circ \vec{E} \quad \text{(3)}$$

χ_e: Electric susceptibility of the dielectric (dimensionless).

*- Substituting (3) in (2) :

$$\vec{D} = \epsilon_\circ (1 + \chi_e) \vec{E}$$

or, $\vec{D} = \epsilon_\circ \epsilon_R \vec{E}$

where, $\epsilon_R = \chi_e + 1$ —— (4)

ϵ_R : Relative permitivity (or dielectric constant) of the medium (dimensionless).

*- ($\vec{D}$) may also be written as:

$$\vec{D} = \epsilon \vec{E} \quad \text{(5)}$$

where, $\epsilon = \epsilon_\circ \epsilon_R$ —— (6)

or, $\epsilon_R = \dfrac{\epsilon}{\epsilon_\circ}$

$\underline{\epsilon:}$ Permitivity of the dielectric material.

H.W.: A slab of dielectric material has a relative dielectric constant of 3.8 and contains a uniform electric flux density of $8\ nC/m^2$. Find (a) E, (b) P, (c) the average No. of dipoles per cubic meter if the average dipole moment is 10^{-29} C.m. Given $\epsilon_o = 8.854 \times 10^{-12}$ F/m.

(a): Hint: Use scaler form of eq. 5 (and eq. 6 \ optional).

Ans. $E \simeq 237.7$ V/m.

(b): Hint: Use 4 to find χ_e, and 3 (scaler form) to find P.

Ans. $P = 5892.8682 \times 10^{-12}$ C/m^2.

or, $P = 5.8928$ nC/m^2.

(c): Hint: Av. No. of dipoles per unit volume {cubic meter} is $\dfrac{\text{Dipole moment per unit volume}}{\text{Av. Value of moment}}$

→ We have that P is the dipole moment per unit volume

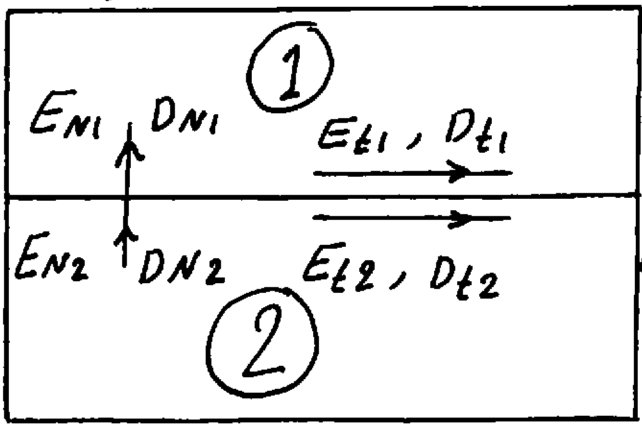
→ ∴ Av. No. of dipoles per unit volume is $\dfrac{P}{\text{Av. moment}}$

Ans. $5.8928 \times 10^{20}\ m^{-3}$

{Note: m^{-3} : $\dfrac{\text{Dipoles}}{m^3}$}

Boundary Conditions for Perfect Dielectric Materials

* - نتناول المركبات العمودية (Normal) والمماسية (tangential) للمجالين $\vec{E}$ و $\vec{D}$ عند السطح الفاصل (السطح الحدودي) (boundary surface) الذي يفصل بين مادتين عازلتين : المادة ①: [ϵ_1] والمادة ②: [ϵ_2]. {أو مادة عازلة ومادة أخرى}



* - عند السطح الفاصل :

← المركبات العمودية: E_N, D_N

← المركبات المماسية: E_t , D_t

* - يقال عن المركبات التي تتساوى عند الحد الفاصل بأنها مستمرة في القيمة (continuous) ، وعن غير المتساوية بأنها غير مستمرة في القيمة (discontinuous) .

*- Boundary Conditions :

$\vec{E}$ - field

$$E_{t1} = E_{t2} \quad \text{——} (7)$$

[continuous]

$$\frac{E_{N1}}{E_{N2}} = \frac{\epsilon_2}{\epsilon_1} \quad \text{——} (8)$$

[discontinuous]

$$\left\{ \begin{array}{l} E_{N1} \neq E_{N2} \\ \text{at the boundary} \end{array} \right\}$$

$\vec{D}$ - field

$$D_{N1} = D_{N2} \quad \text{——} (9)$$

[continuous]

$$\frac{D_{t1}}{D_{t2}} = \frac{\epsilon_1}{\epsilon_2} \quad \text{——} (10)$$

[discontinuous]

$$\left\{ \begin{array}{l} D_{t1} \neq D_{t2} \\ \text{at the boundary} \end{array} \right\}$$

Ex.1:

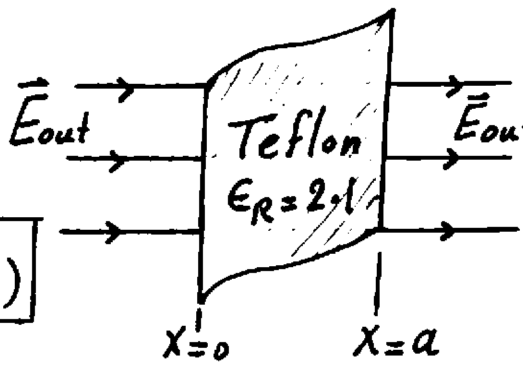
A slab of Teflon [$\epsilon_R = 2.1$] is located in the region $0 \leq x \leq a$, assume free space where $x < 0$ and $x > a$. Outside the Teflon there is a uniform field $\vec{E}_{out} = E_o\, \hat{a}_x$ (V/m).

(a) Find $\vec{D}$, $\vec{E}$, and $\vec{P}$ everywhere (outside and inside the Teflon slab).

(b) What would be the values of $\vec{D}$, $\vec{E}$, and $\vec{P}$ in part (a) if $E_o = 10$ (V/m) $\{\vec{E}_{out} = 10\, \hat{a}_x\}$?

Sol.: (a):

Outside the slab



We have $\boxed{\vec{E}_{out} = E_o\, \hat{a}_x \text{ (V/m)}}$

from $\vec{D} = \epsilon_o \vec{E} \Rightarrow \vec{D}_{out} = \epsilon_o \vec{E}_{out}$

$$\therefore \boxed{\vec{D}_{out} = \epsilon_o E_o\, \hat{a}_x \quad (C/m^2)}$$

$\boxed{\vec{P}_{out} = 0}$ {Because there is no dielectric outside the slab}.

Inside the slab

*- $\vec{E}$-field is normal to slab surface:

$\vec{E}_{out} \equiv \vec{E}_N \Longrightarrow$ Hence: $\vec{D}_{out} \equiv \vec{D}_N$.

{Note: Slab surface is the boundary between free space and Teflon}.

*- To find $\vec{D}_{in}$:

[eq. 9] : $\vec{D}_{in} = \vec{D}_{out} = \epsilon_o E_o \hat{a}_x$

$\therefore \boxed{\vec{D}_{in} = \epsilon_o E_o \hat{a}_x \quad (C/m^2)}$

*- To find $\vec{E}_{in}$:

$\rightarrow \quad \vec{D}_{in} = \epsilon \vec{E}_{in} \Rightarrow \vec{E}_{in} = \vec{D}_{in} / \epsilon$

$$\vec{E}_{in} = \frac{\epsilon_o E_o \hat{a}_x}{\epsilon_o \epsilon_R} \Rightarrow \vec{E}_{in} = \frac{E_o}{\epsilon_R} \hat{a}_x$$

or, $\boxed{\vec{E}_{in} = 0.4761 \, E_o \hat{a}_x \quad (V/m)}$

⚠ Try to find $\vec{E}_{in}$ using 8

*- To find $\vec{P}_{in}$:

[eq. 2]: $\vec{D} = \epsilon_o \vec{E} + \vec{P}$

$\vec{P} = \vec{D} - \epsilon_o \vec{E}$

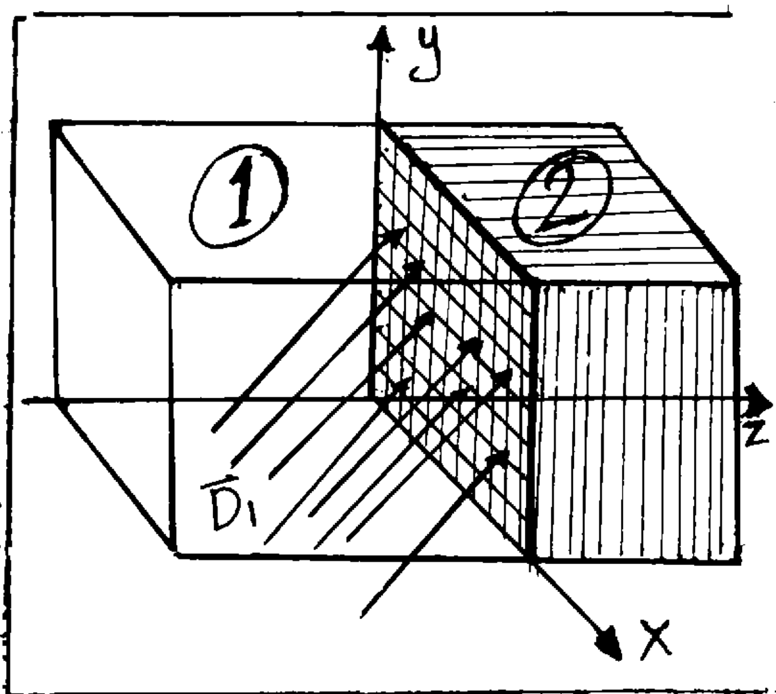
$$\begin{aligned} \vec{P}_{in} &= \vec{D}_{in} - \epsilon_o \vec{E}_{in} \\ &= \epsilon_o E_o \hat{a}_x - \epsilon_o \times 0.4761 E_o \hat{a}_x \\ &= \epsilon_o E_o \hat{a}_x (1 - 0.4761) \end{aligned}$$

$\therefore \boxed{\vec{P}_{in} = 0.5239 \, \epsilon_o E_o \hat{a}_x \quad (C/m^2)}$

⚠ Try to find $\vec{P}_{in}$ using 3 & 4

(b): Calculate yourself.

Ex. 2: Let the region $z<0$ be composed of a uniform dielectric material for which $\epsilon_R = 3.2$, while the region $z>0$ is characterized by $\epsilon_R = 2$. Let $\vec{D}_1 = -30\hat{i} + 50\hat{j} + 70\hat{k}$ nC/m^2,
Find: (a) D_{N1}, (b) $\vec{D}_{t1}$, (c) D_{t1}, (d) $D1$, (e) $\vec{P}_1$, (f) $\vec{D}_{N2}$, (g) $\vec{D}_{t2}$, (h) $\vec{D}_2$, (i) $\vec{P}_2$.



Sol. $\vec{D}_1 = \vec{D}_{x1} + \vec{D}_{y1} + \vec{D}_{z1}$

$\vec{D}_{N1} \equiv \vec{D}_{z1}$

$\vec{D}_{N1} = 70\hat{k}$ nC/m^2

$D_{N1} = 70$ nC/m^2

(b) $\vec{D}_{t1} = \vec{D}_{x1} + \vec{D}_{y1}$

$\vec{D}_{t1} = -30\hat{i} + 50\hat{j}$ $\frac{nC}{m^2}$

(c) $D_{t1} = \sqrt{D_{x1}^2 + D_{y1}^2}$

$= \sqrt{(-30)^2 + 50^2}$

$D_{t1} = 58.3095$ nC/m^2.

(d) $D_1 = \sqrt{D_{x1}^2 + D_{y1}^2 + D_{z1}^2} = \sqrt{(-30)^2 + 50^2 + 70^2}$

$D1 = 91.1043$ nC/m^2.

(e) $\vec{P} = \chi_e \epsilon_o \vec{E} \Rightarrow \vec{P}_1 = \chi_{e1} \epsilon_o \vec{E}_1$

$\vec{D}_1 = \epsilon_1 \vec{E}_1 \Rightarrow \vec{E}_1 = \vec{D}_1 / \epsilon_1 = \frac{\vec{D1}}{\epsilon_{R1}\epsilon_o}$

$\therefore \vec{E}_1 = \frac{\vec{D1}}{\epsilon_{R1}\epsilon_o}$

We have: $\epsilon_R = \chi_e + 1 \Rightarrow \epsilon_{R1} = \chi_{e1} + 1$

or, $\chi_{e1} = \epsilon_{R1} - 1 \quad = 3.2 - 1 \quad = 2.2$

$\chi_{e1} = 2.2$

$$\therefore \vec{P_1} = 2.2\, \epsilon_0 \frac{\vec{D_1}}{\epsilon_0\, \epsilon_{R1}} = \frac{2.2}{3.2} \vec{D}_1$$

$$\vec{P}_1 = \frac{2.2}{3.2}\left(-30\,\hat{i} + 50\,\hat{j} + 70\,\hat{k}\right)$$

$$\vec{P}_1 = -20.625\,\hat{i} + 34.375\,\hat{j} + 48.125\,\hat{k} \quad nC/m^2.$$

(f): $(\vec{D}_{N2})$: Normal component is continuous at the boundary : $\vec{D}_{N2} = \vec{D}_{N1}$
(or, $\vec{D}_{z2} = \vec{D}_{z1}$)

$$\therefore \vec{D}_{N2} = 70\,\hat{k} \quad nC/m^2.$$

(g): $(\vec{D}_{t2})$: We have : $\dfrac{\vec{D}_{t1}}{\vec{D}_{t2}} = \dfrac{\epsilon_1}{\epsilon_2}$ {eq. (10)}

$$\therefore \vec{D}_{t2} = \frac{\epsilon_2}{\epsilon_1} \vec{D}_{t1}$$

$$\vec{D}_{t2} = \frac{\epsilon_0 \epsilon_{R2}}{\epsilon_0 \epsilon_{R1}} \vec{D}_{t1} = \frac{\epsilon_{R2}}{\epsilon_{R1}} \vec{D}_{t1} = \frac{2}{3.2}\left(-30\hat{i} + 50\hat{j}\right)$$

$$\vec{D}_{t2} = -18.75\,\hat{i} + 31.25\,\hat{j} \quad nC/m^2.$$

(h): $(\vec{D}_2)$: $\vec{D}_2 = \vec{D}_{x2} + \vec{D}_{y2} + \vec{D}_{z2}$

$$\vec{D}_2 = -18.75\hat{i} + 31.25\hat{j} + 70\hat{k} \left(\frac{nC}{m^2}\right)$$

(i): ($\vec{P}_2$): Using $\vec{P} = \chi_e \epsilon_o \vec{E}$

$$\vec{P}_2 = \chi_{e2} \epsilon_o \vec{E}_2$$

We have : $\chi_{e2} = \epsilon_{R2} - 1 = 2 - 1 = 1$

$\therefore \chi_{e2} = 1$

From $\vec{D}_2 = \epsilon_2 \vec{E}_2 \Rightarrow \vec{E}_2 = \dfrac{\vec{D}_2}{\epsilon_2}$

$$\vec{E}_2 = \frac{\vec{D}_2}{\epsilon_o \epsilon_{R2}}$$

$$\therefore \vec{P}_2 = (1) \times \epsilon_o \times \frac{\vec{D}_2}{\epsilon_o \epsilon_{R2}} = \frac{\vec{D}_2}{\epsilon_{R2}} = \frac{\vec{D}_2}{2}$$

$$\vec{P}_2 = \frac{1}{2}\left(-18.75\,\hat{i} + 31.25\,\hat{j} + 70\,\hat{k}\right)$$

$$\vec{P}_2 = -9.375\,\hat{i} + 15.625\,\hat{j} + 35\,\hat{k} \ \ nC/m^2.$$

Capacitors :

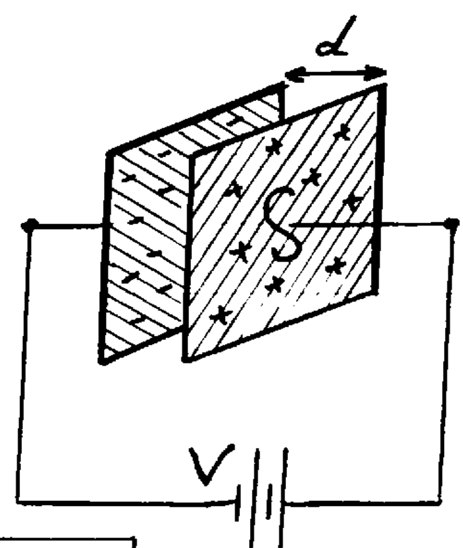
Any two bodies separated by free space or a dielectric material constitute a capacitor with a capacitance [C] :

$C = \frac{Q}{V}$ — for any kind of capacitors.

*- The capacitance does not depend on Q or V. It depends on the geometry of the capacitor and the dielectric material.

<u>Prove</u> : for flat plate capacitor :

We have : $C = \frac{Q}{V}$ —— (11)



$\boxed{Q = \rho_s S}$

$E = \frac{V}{d} \Rightarrow V = E d$

But $D = \epsilon E \Rightarrow E = D / \epsilon$

and $D = \rho_s$

$\therefore E = \rho_s / \epsilon \Rightarrow \boxed{V = \frac{\rho_s d}{\epsilon}}$

Substituting for Q and V in (11) :

$C = \frac{\rho_s S \epsilon}{\rho_s d} \Rightarrow C = \frac{\epsilon S}{d}$

Energy Stored in a Capacitor

*- Energy in a charged capacitor is stored in the electric field ($\vec{E}$) between the conductors.

*- The energy stored in the volume between the conductors is given as:

$$W_E = \frac{1}{2} \int_V \vec{D} \cdot \vec{E} \, dv$$

<u>dv:</u> Volume element in the space between the conductors.

*- We have: $\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E}$

$$\therefore \; W_E = \frac{1}{2} \int_V \epsilon_0 \epsilon_r \vec{E} \cdot \vec{E} \, dv$$

$$\boxed{W_E = \frac{1}{2} \epsilon_0 \epsilon_r \int_V E^2 \, dv}$$

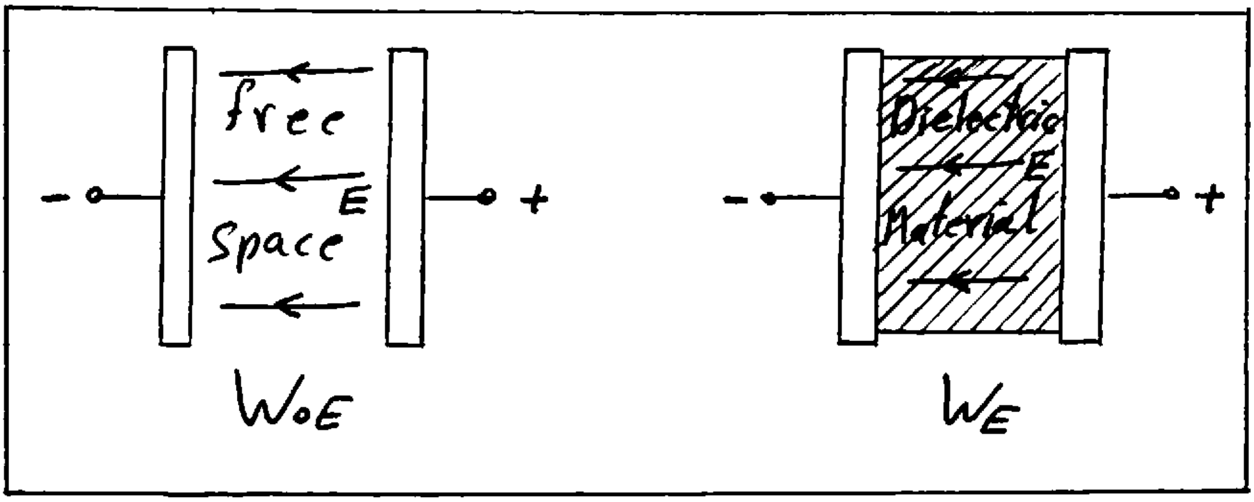
*- For the same E-field as in free space, the presence of a dielectric results in an increase in stored energy by a factor $\epsilon_r > 1$.

→ For free space capacitor: $W_{0E} = \frac{1}{2} \epsilon_0 \int_V E^2 \, dv$
($\epsilon_r = 1$ in free space).

→ If $(\vec{E})$ is the same :

$$\frac{W_E}{W_{oE}} = \frac{\frac{1}{2}\epsilon_o \epsilon_r \int E^2 dv}{\frac{1}{2}\epsilon_o \int E^2 dv} = \epsilon_r$$

$$\therefore \boxed{\frac{W_E}{W_{oE}} = \epsilon_r} \quad \text{or ,} \quad \boxed{W_E = \epsilon_r W_{oE}}$$



*- In terms of C & V :

$$W_E = \frac{1}{2} C V^2$$