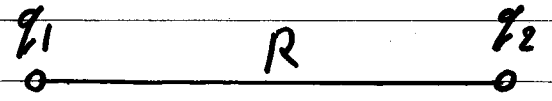
EM.2

Electrostatics

*- Coulomb's law for point charges :



$$\boxed{F = k\,\frac{q_1 q_2}{R^2}}$$

<u>F</u> : The magnitude of the mutual force between the point charges q_1 and q_2.

<u>k</u> : A constant that depends on the medium material.

$$k = \frac{1}{4\pi\epsilon} \quad , \quad \epsilon : \text{Electric permitivity of the medium.}$$

*- <u>For free space :</u> $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

$$k = \frac{1}{4\pi\epsilon_0} \simeq 9 \times 10^{9} \ N.m^2/C^2$$

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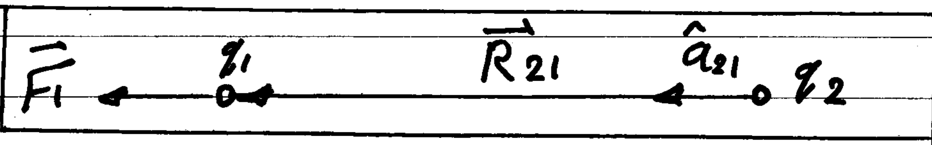
*- Directional forces

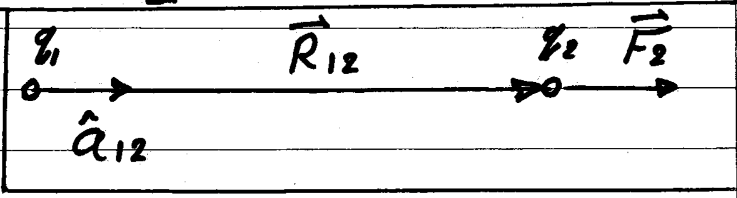
*- <u>To find $\vec{F_1}$:</u>



$$\vec{F_1} = k\,\frac{q_1 q_2}{R_{21}^2}\,\hat{a}_{21} \quad , \text{or: } \vec{F_1} = k\,\frac{q_1 q_2}{R_{21}^3}\,\vec{R}_{21}$$

Class

*- To find $\vec{F_2}$:



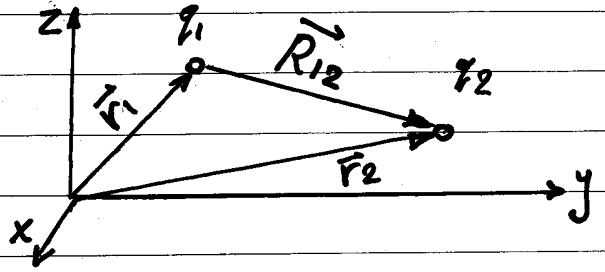
$$\vec{F_2} = k \frac{q_1 q_2}{R_{12}^2} \hat{a}_{12} \text{ , or, } \vec{F_2} = k \frac{q_1 q_2}{R_{12}^3} \vec{R_{12}}$$

*- Finding the directional distance ($\vec{R_{12}}$ or $\vec{R_{21}}$):

Taking $\vec{R_{12}}$ for example :

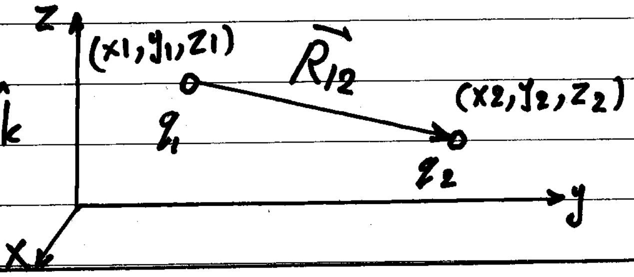
Case (1)

$$\vec{R_{12}} = \vec{r_2} - \vec{r_1}$$



Case (2)

$$\vec{R_{12}} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

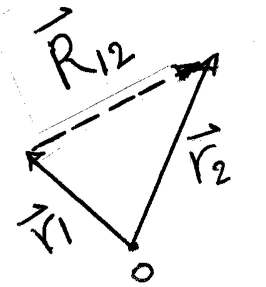


Ex.: Find $\vec{R_{12}}$ from:

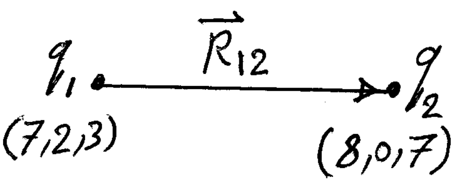
ⓐ $\vec{r_1} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{r_2} = 10\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{R_{12}} = \vec{r_2} - \vec{r_1} = 10\hat{i} - 2\hat{j} - 2\hat{k} - (4\hat{i} - \hat{j} + 3\hat{k})$

$\vec{R_{12}} = 6\hat{i} - \hat{j} - 5\hat{k}$



ⓑ $q_1 (7, 2, 3)_m$, $q_2 (8, 0, 7)\, m$



$\vec{R_{12}} = (8-7)\hat{i} + (0-2)\hat{j} + (7-3)\hat{k}$

$\vec{R_{12}} = \hat{i} - 2\hat{j} + 4\hat{k}$, (m)

The Electric Field

*- In electrostatics, any electric charge generates an electric field among the space around that charge.

The Electric Field Intensity

*- We may determine the electric field intensity at any point around a point charge.

*- Illustration: The magnitude of the electric field intensity (E) at point (P) located at a distance (r) from a point charge (q) is determined as:

$$E = k\,\frac{q}{r^2}\,, \quad k = \frac{1}{4\pi\epsilon}$$

*- The electric field intensity is defined as the force per unit charge: $E = F/q$ [or the force exerted on one Coulomb of charge].

*- $\boxed{\text{Units of } E : N/C}$.

*- Illustration: Determine the amount of force exerted on a point charge of ($0.2\ C$) placed in an electric field of ($8\times10^{-4}\ N/C$).

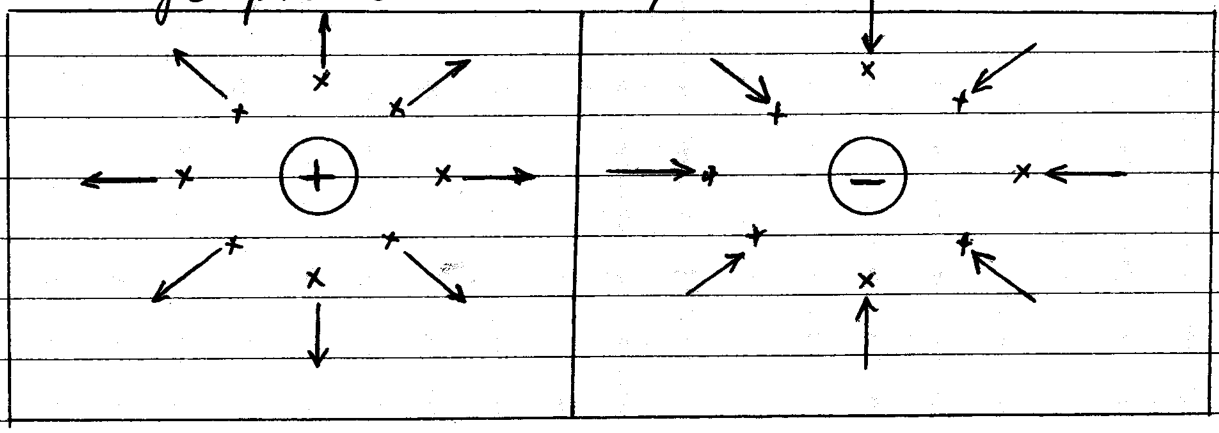
Sol. $q = 0.2\ C$, $E = 8\times10^{-4}\ N/C$

$$E = \frac{F}{q} \Longrightarrow F = qE = 0.2\times8\times10^{-4}$$

$$F = 1.6\times10^{-4}\ N.$$

*- The electric field intensity is a vector quantity ($\vec{E}$).

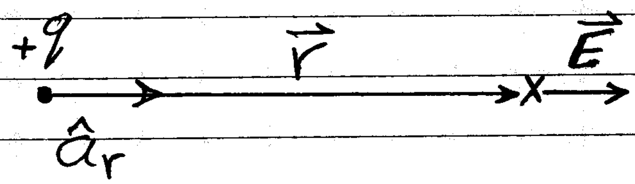
*- The direction of ($\vec{E}$) at any point is the same direction of the force exerted on a positive test charge placed at that point.



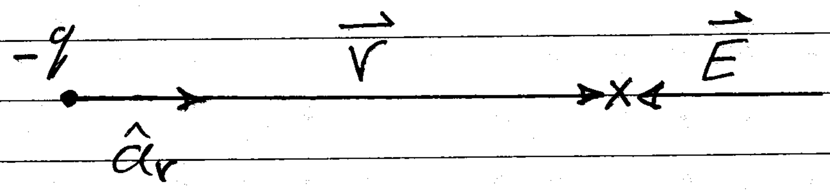
*- Determining the Directional Electric Field

$$\vec{E} = k \frac{q}{r^2} \hat{a}_r$$

$$\text{or, } \vec{E} = k \frac{q}{r^3} \vec{r}$$



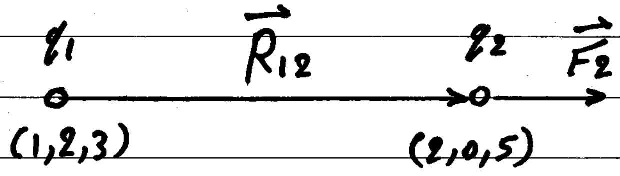
Note: For a (-ve) point charge



Examples and Homeworks

Ex. 1: Given the two point charges: $q_1 = 3\times10^{-4}$ C at (1, 2, 3) m and $q_2 = 10^{-4}$ C at (2, 0, 5) m in free space. Find the force exerted on q_2 by q_1. { for free space: $k = 9\times10^{9}\ N.m^2/C^2$ }

Sol.



$$\vec{F}_2 = k\,\frac{q_1 q_2}{R_{12}^3}\,\vec{R}_{12}$$

$$\vec{R}_{12} = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$R_{12} = \sqrt{1+4+4} = \sqrt{9} = 3\ m$$

$$\vec{F}_2 = 9\times10^{9}\ \frac{3\times10^{-4}\times10^{-4}}{3^3}\ (\hat{i} - 2\hat{j} + 2\hat{k})$$

$$= 10\,(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\therefore\ \vec{F}_2 = 10\,\hat{i} - 20\,\hat{j} + 20\,\hat{k}\ ,\ (N)$$

Try: *- Find the unit vector along $\vec{F}_2$ (the direction of $\vec{F}_2$).

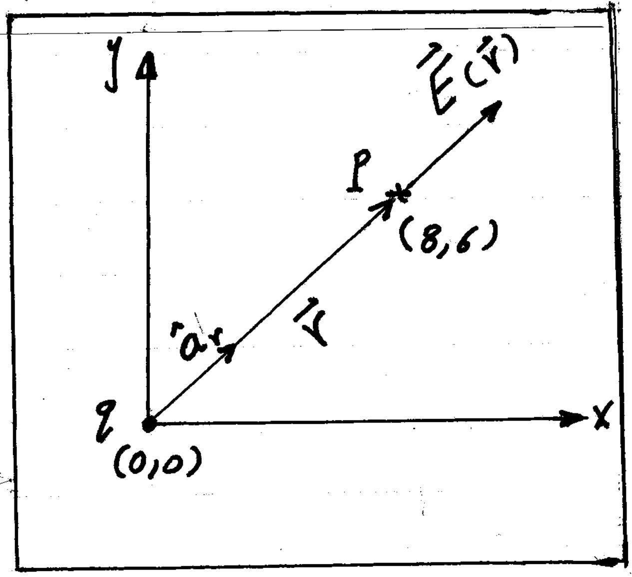
Ans.: $\hat{a} = \frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$

*- What is the magnitude of $\vec{F}_2$?

Ans. 30 N

Class

Ex. 1: Find the electric field intensity $\vec{E}$ at point $P(8,6)$ m resulted by a point charge $q = 20$ nC placed at the origin. (The medium is free space).



Sol.:

$$\vec{E} = k \frac{q}{r^2} \hat{a}_r$$

$$\vec{r} = 8\hat{i} + 6\hat{j}$$

$$|\vec{r}| = \sqrt{8^2 + 6^2} = \sqrt{64+36}$$

$$|\vec{r}| = \sqrt{100} = 10 \text{ m}.$$

$$\hat{a}_r = \frac{\vec{r}}{|\vec{r}|} = \frac{1}{10}(8\hat{i} + 6\hat{j})$$

$$\hat{a}_r = 0.8\hat{i} + 0.6\hat{j}$$

$$\vec{E} = \frac{9\times10^{9} \times 20\times10^{-9}}{10^2}(0.8\hat{i} + 0.6\hat{j})$$

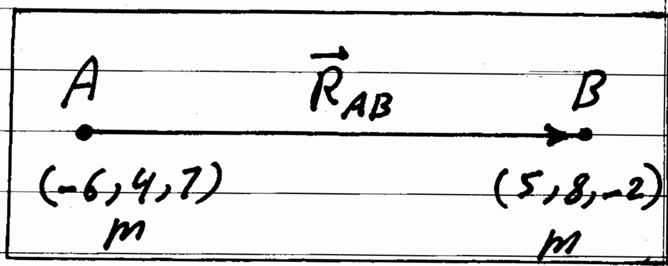
$$= \frac{180}{100}(0.8\hat{i} + 0.6\hat{j})$$

$$\vec{E} = 1.8(0.8\hat{i} + 0.6\hat{j})$$

$$\vec{E} = 1.44\hat{i} + 1.08\hat{j} \quad , (N/C).$$

Class

Ex. 2 : A charge $Q_A = -20\ \mu C$ is located at $A(-6, 4, 7)\,m$, and a charge $Q_B = 50\ \mu C$ is at $B(5, 8, -2)\,m$ in free space.
Find: (a) $\vec{R}_{AB}$; (b) R_{AB} ; (c) the vector force on Q_A by Q_B.



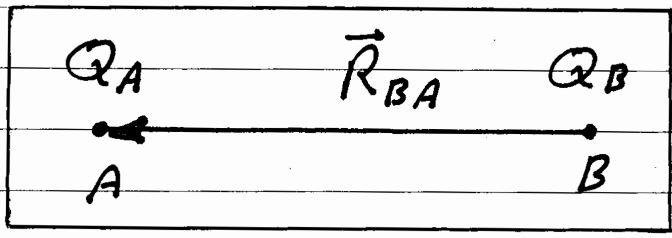
Sol. :

(a):

$$\vec{R}_{AB} = 11\,\hat{i} + 4\,\hat{j} - 9\,\hat{k} \quad (m)$$

(b): $R_{AB} = \sqrt{11^2 + 4^2 + (-9)^2} = 14.7648\ m.$

(c):



$$\vec{F}_A = k\,\frac{Q_A\,Q_B}{R_{BA}^3}\,\vec{R}_{BA}$$

$$\vec{R}_{BA} = -\vec{R}_{AB} \Rightarrow \vec{R}_{BA} = -11\,\hat{i} - 4\,\hat{j} + 9\,\hat{k} \quad (m)$$

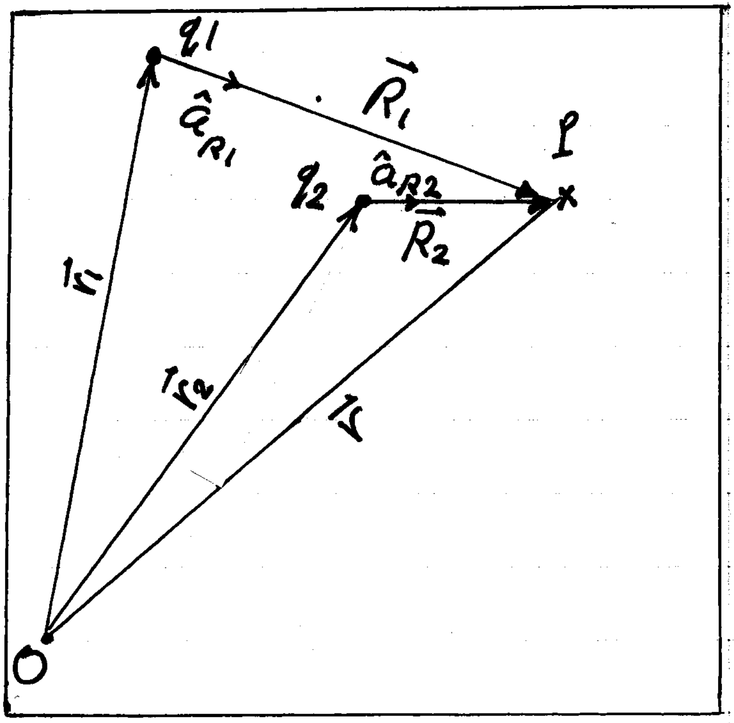
$$|\vec{R}_{BA}| = |\vec{R}_{AB}| \Rightarrow R_{BA} = 14.7648 \quad (m)$$

$$\vec{F}_A = 9\times10^{9}\ \frac{-20\times10^{-6}\times 50\times10^{-6}}{14.7648^3}\left(-11\,\hat{i} - 4\,\hat{j} + 9\,\hat{k}\right)$$

$$\vec{F}_A = \frac{-9000\times10^{-3}}{3218.7163}\left(-11\,\hat{i} - 4\,\hat{j} + 9\,\hat{k}\right)$$

$$\vec{F}_A = 0.0307\,\hat{i} + 0.0111\,\hat{j} - 0.0251\,\hat{k}\ , \quad (N).$$

Ex.3 : Find the electric field intensity ($\vec{E}$) at point P of a position vector $\vec{r} = \hat{i} + 10\hat{j} + 2k$ (m) due to charges $q_1 = 20$ nC at $\vec{r}_1 = -5\hat{i} + 4\hat{j} + 5\hat{k}$ (m) and $q_2 = 10$ nC at $\vec{r}_2 = 5\hat{i} + 7\hat{j} + 2\hat{k}$ (m).



Sol.:

$\vec{E}_1$: E- field at P due to q_1.

$\vec{E}_2$: E- field at P due to q_2.

$$\vec{E}_1 = \frac{k q_1}{R_1^2} \hat{a}_{R1}$$

or, $$\vec{E}_1 = \frac{k q_1}{R_1^3} \vec{R}_1$$

$$\vec{R}_1 = \vec{r} - \vec{r}_1 = (\hat{i} + 10\hat{j} + 2k) - (-5\hat{i} + 4\hat{j} + 5\hat{k})$$

$$\vec{R}_1 = 6\hat{i} + 6\hat{j} - 3\hat{k} \quad , (m)$$

$$|\vec{R}_1| = \sqrt{36 + 36 + 9} = \sqrt{81} = 9 \text{ m}.$$

$$\vec{E}_1 = \frac{9 \times 10^9 \times 20 \times 10^{-9}}{9^3} (6\hat{i} + 6\hat{j} - 3\hat{k})$$

$$\vec{E}_1 = \frac{180}{729} (6\hat{i} + 6\hat{j} - 3\hat{k})$$

$$= 0.2469 (6\hat{i} + 6\hat{j} - 3\hat{k})$$

$$\vec{E}_1 = 1.4814\,\hat{i} + 1.4814\,\hat{j} - 0.7407\,\hat{k} \quad , (N/C).$$

To find $\vec{E}_2$:

$$\vec{E}_2 = \frac{k q_2}{R_2^3}\,\vec{R}_2$$

$$\vec{R}_2 = \vec{r} - \vec{r}_2 = -4\hat{i} + 3\hat{j} \quad , (m)$$

$$|\vec{R}_2| = \sqrt{16+9} = \sqrt{25} = 5 \; m$$

$$\vec{E}_2 = \frac{9\times10^9 \times 10\times10^{-9}}{5^3}\,(-4\hat{i} + 3\hat{j})$$

$$\vec{E}_2 = \frac{90}{125}\,(-4\hat{i} + 3\hat{j}) = 0.72\,(-4\hat{i} + 3\hat{j})$$

$$\vec{E}_2 = -2.88\,\hat{i} + 2.16\,\hat{j} \quad , (N/C)$$

Net electric field $(\vec{E})$:

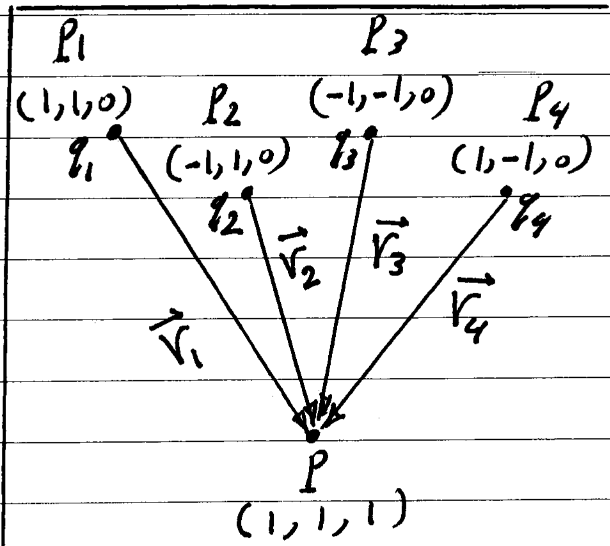
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = (1.4814\hat{i} + 1.4814\hat{j} - 0.7407\hat{k}) + (-2.88\hat{i} + 2.16\hat{j})$$

$$\vec{E} = 1.4814\hat{i} + 1.4814\hat{j} - 0.7407\hat{k} - 2.88\hat{i} + 2.16\hat{j}$$

$$\vec{E} = -1.3986\,\hat{i} + 3.6414\,\hat{j} - 0.7407\,\hat{k} \quad , (N/C).$$

Ex. 4: Find $\vec{E}$ at $P(1,1,1)$ m caused by four identical 3 nC charges located at $P_1(1,1,0)$ m, $P_2(-1,1,0)$ m, $P_3(-1,-1,0)$ m, and $P_4(1,-1,0)$ m, where the medium is free space.



Note: Positions in the figure are arbitrary.

Sol. $q_1 = q_2 = q_3 = q_4 = q$

$$\vec{E} = \sum_{i=1}^{4} k \frac{q}{r_i^3} \vec{r}_i$$

$$\vec{E} = kq \sum_{i=1}^{4} \frac{\vec{r}_i}{r_i^3}$$

$\vec{r}_1 = \hat{k}$, $|\vec{r}_1| = 1$ m

$\vec{r}_2 = 2\hat{i} + \hat{k}$, $|\vec{r}_2| = \sqrt{5}$ m

$\vec{r}_3 = 2\hat{i} + 2\hat{j} + \hat{k}$, $|\vec{r}_3| = \sqrt{9} = 3$ m

$\vec{r}_4 = 2\hat{j} + \hat{k}$, $|\vec{r}_4| = \sqrt{5}$ m

$$\vec{E} = kq \left[\frac{\vec{r}_1}{r_1^3} + \frac{\vec{r}_2}{r_2^3} + \frac{\vec{r}_3}{r_3^3} + \frac{\vec{r}_4}{r_4^3} \right]$$

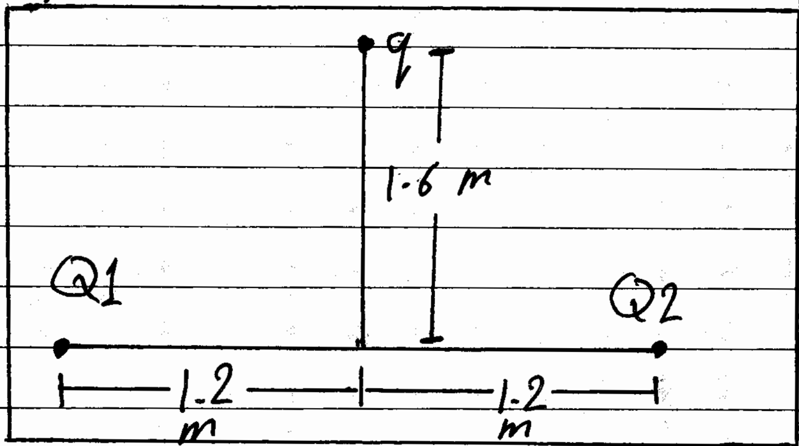
$$\vec{E} = 9\times10^{9} \times 3\times10^{-9} \left[\hat{k} + \frac{2\hat{i}+\hat{k}}{\sqrt{5}^{\,3}} + \frac{2\hat{i}+2\hat{j}+\hat{k}}{27} + \frac{2\hat{j}+\hat{k}}{\sqrt{5}^{\,3}} \right]$$

$$\vec{E} = 27\left[0.2528\,\hat{i} + 0.2528\,\hat{j} + 1.2158\,\hat{k}\right]$$

$$\vec{E} = 6.8256\,\hat{i} + 6.8256\,\hat{j} + 32.8266\,\hat{k} \;,\; (N/C)$$

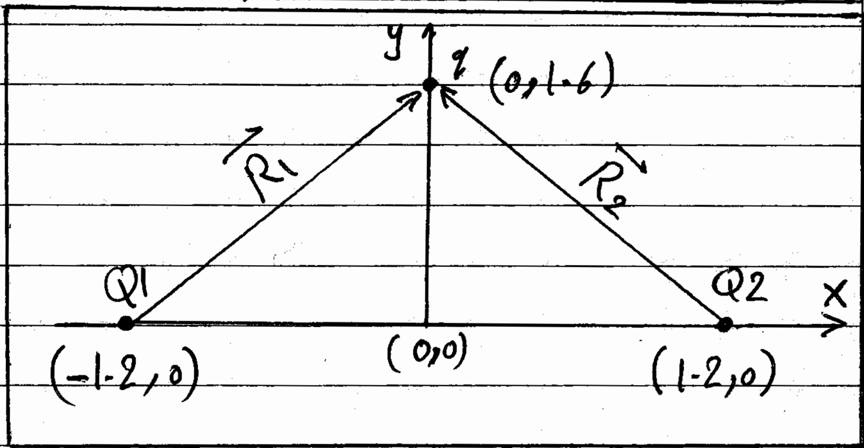
Class

Ex. 5: Calculate the net electrostatic force on charge $q = 4\mu C$ resulted by charges $Q1 = 6\mu C$ and $Q2 = -6\mu C$ placed as shown. {The medium is free space}.



Sol.:

*- First we introduce the x-y coordinates and complete the figure.



* $\vec{F1} = k \dfrac{q\,Q1}{R_1^3} \vec{R_1}$

* $\vec{F2} = k \dfrac{q\,Q2}{R_2^3} \vec{R_2}$

* $\vec{R_1} = 1.2\,\hat{i} + 1.6\,\hat{j}$

* $\vec{R_2} = -1.2\,\hat{i} + 1.6\,\hat{j}$

* $|\vec{R_1}| = \sqrt{1.2^2 + 1.6^2} = \sqrt{1.44 + 2.56} = \sqrt{4} = 2\ m.$

* $|\vec{R_2}| = \sqrt{(-1.2)^2 + 1.6^2} = \sqrt{1.44 + 2.56} = \sqrt{4} = 2\ m.$

* $\vec{F} = \vec{F_1} + \vec{F_2}$

* $\vec{F} = k \dfrac{q\,Q1}{R_1^3} \vec{R_1} + k \dfrac{q\,Q2}{R_2^3} \vec{R_2}$

* We have: $R_1 = R_2 = 2m$, and $Q2 = -Q1$

$$\therefore \vec{F} = k\frac{qQ_1}{2^3}\vec{R_1} - k\frac{qQ_2}{2^3}\vec{R_2}$$

$$\vec{F} = \frac{kqQ1}{2^3}(\vec{R_1} - \vec{R_2})$$

$$= \frac{9\times10^{9}\times4\times10^{-6}\times6\times10^{-6}}{8}\left[1.2\hat{i} + 1.6\hat{j} - (-1.2\hat{i} + 1.6\hat{j})\right]$$

$$= 27\times10^{-3}\,(1.2\hat{i} + 1.6\hat{j} + 1.2\hat{i} - 1.6\hat{j})$$

$$= 27\times10^{-3}\times 2.4\,\hat{i}$$

$$\vec{F} = 64.8\times10^{-3}\,\hat{i}\ \ N$$

or, $\vec{F} = 64.8\,\hat{i}\ mN$.

Hw. 1: Two charges: $q_1 = 500\,\mu C$ and $q_2 = 100\,\mu C$ located on the x-y plane at positions $\vec{r_1} = 3\hat{j}$ (m) and $\vec{r_2} = 4\hat{i}$ (m) respectively in free space.

Calculate the electric force exerted on q_2.

Ans.: $\vec{F_2} = 14.4\hat{i} - 10.8\hat{j}$, (N).

Hw-2: A charge of $-0.3\,\mu C$ is located at $A(25, -30, 15)$ cm, and a second charge of $0.5\,\mu C$ is at $B(-10, 8, 12)$ cm.

Find $\vec{E}$ at: (a) The origin.

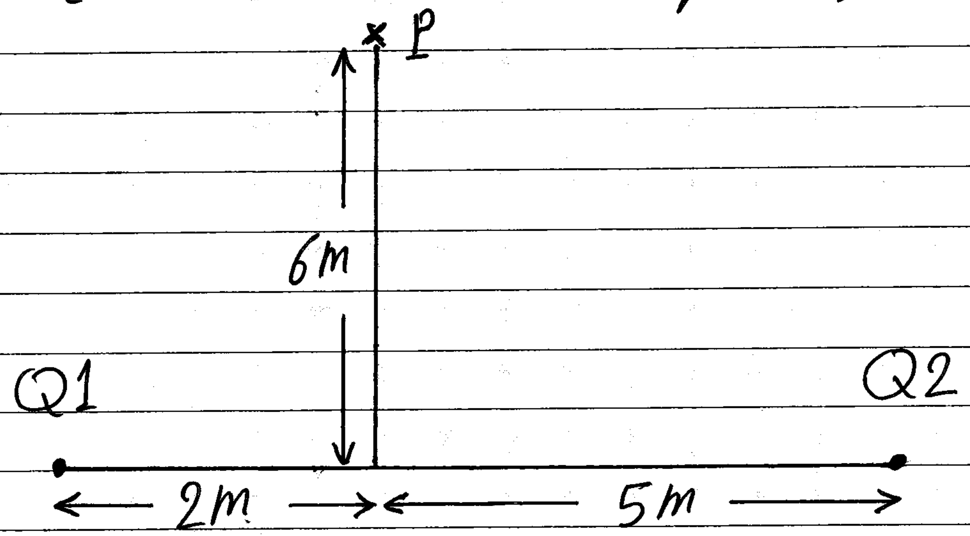
(b) Point $P(15, 20, 50)$ cm.

{The medium is free space}.

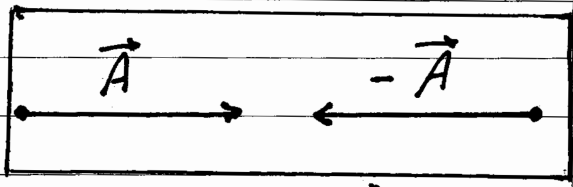
Ans.: (a) $\vec{E} = (92.6141\,\hat{i} - 77.7801\,\hat{j} - 94.5369\,\hat{k}) \times 10^{3}$, (N/C).

(b) $\vec{E} = (11.9498\,\hat{i} - 0.5206\,\hat{j} + 12.4324\,\hat{k}) \times 10^{3}$, (N/C).

Hw-3: Determine the net electric field intensity at point (P) resulted by two point charges: $Q1 = 8$ nC and $Q2 = 6$ nC as shown in the figure. {The medium is free space}.



Notes:



△ $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
$-\vec{A} = -A_x\hat{i} - A_y\hat{j} - A_z\hat{k}$; $|\vec{A}| = |-\vec{A}|$

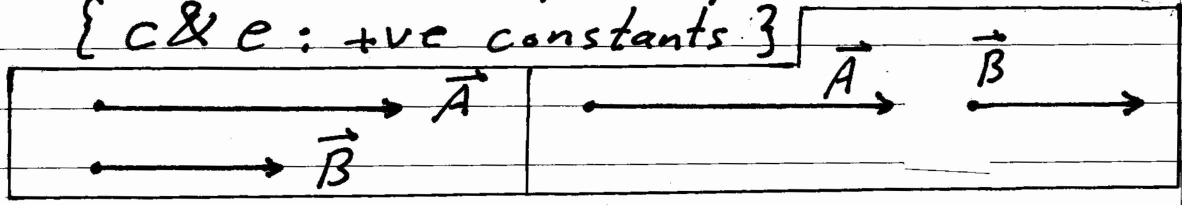
△ $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$; $|\vec{A}| \neq |\vec{B}|$

- Two special cases:

① $\vec{A}$ & $\vec{B}$ are of the same direction (parallel) if:

$$\vec{A} = c\,\vec{B}$$
$$\text{or, } \vec{B} = e\,\vec{A}$$

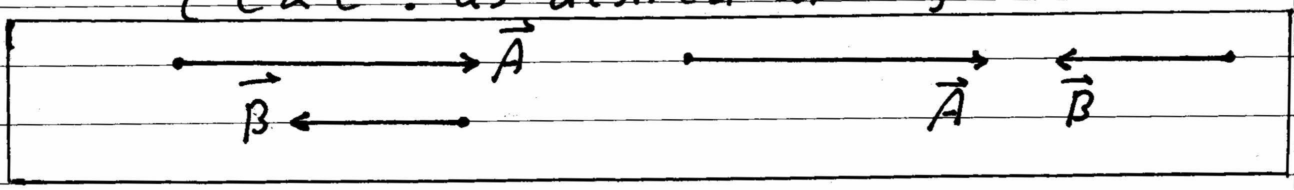
{ c & e : +ve constants }



② $\vec{A}$ & $\vec{B}$ are of opposite directions (parallel) if:

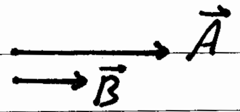
$$\vec{A} = -c\vec{B}, \quad \text{or} \quad \vec{B} = -e\vec{A}$$

{ c & e : as defined above }



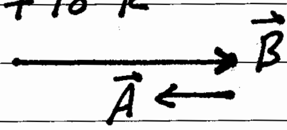
Examples:

$\vec{A} = 4\hat{i} - 8\hat{j} + 2\hat{k}$
$\vec{B} = 2\hat{i} - 4\hat{j} + \hat{k}$
$\therefore \vec{A} = 2\vec{B}$



$\vec{A}$ & $\vec{B}$ are in the same direction.

$\vec{A} = 3\hat{i} + 2\hat{j} - 6\hat{k}$
$\vec{B} = -9\hat{i} - 6\hat{j} + 18\hat{k}$
$\therefore \vec{B} = -3\vec{A}$



$\vec{A}$ & $\vec{B}$ are of opposite directions.

Class

Ex. 6: A point charge $Q1 = 300\ \mu C$, located at $(1,-1,-3)$ m, experiences a force:

$\vec{F_1} = 8\hat{i} - 8\hat{j} + 4\hat{k}$ (N) due to a point charge $Q2$ at $(3,-3,-2)$ m. Determine $Q2$.

Sol.: We have:

$$\vec{F_1} = 8\hat{i} - 8\hat{j} + 4\hat{k}$$

$$\vec{R_{21}} = -2\hat{i} + 2\hat{j} - \hat{k}$$

or, $\vec{F_1} = -4\,\vec{R_{21}}$

$\therefore$ $\vec{F_1}$ and $\vec{R_{21}}$ are of opposite directions $\Rightarrow$ The forces between $Q1$ and $Q2$ are attractive.
$\Rightarrow$ $Q1$ & $Q2$ are of opposite signs.

$\because$ $Q1$ is +ve $\Rightarrow$ $\therefore$ $Q2$ is −ve.

*- The magnitude force (F_1):

$$F_1 = k\,\frac{Q1\,(-Q2)}{R_{21}^2} \Rightarrow Q2 = -\frac{F_1 \times R_{21}^2}{k\,Q1}$$

$$F1 = \sqrt{8^2 + (-8)^2 + 4^2} = \sqrt{144} = 12\ N.$$

$$R_{21} = \sqrt{(-2)^2 + 2^2 + (-1)^2} = \sqrt{9} = 3\ m$$

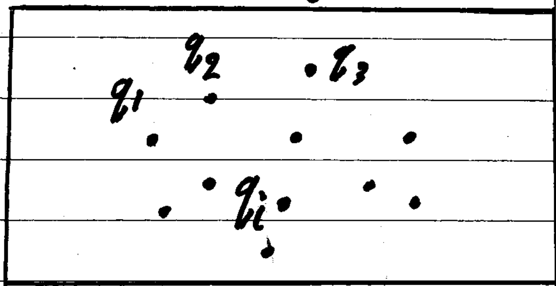
$$Q2 = \frac{-12 \times 9}{9\times10^9 \times 300\times10^{-6}} = -0.04\times10^{-3}\ C$$

$Q2 = -0.04$ mC, or $Q_2 = -40\ \mu C$

Electric Charge Configurations

1- Collection (System) of Point Charges

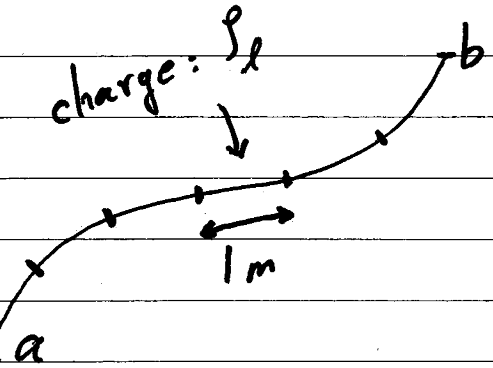
$$Q = \sum_{i=1}^{n} q_i$$



n-Point Charges

2- Distributions

a- Line Distribution



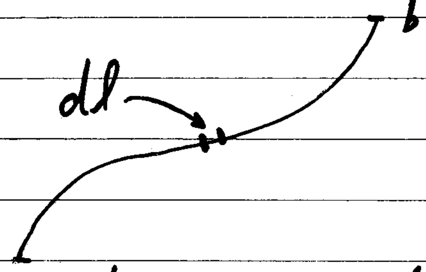
*- ρ_l : Line Charge Density.

ρ_l : Amount of charge per one meter length (unit length).

*- Units: ρ_l : C/m.

*- Total charge (Q) along the whole length (L) {from a to b}:

$$Q = \int_a^b \rho_l \, dl = \int_L \rho_l \, dl$$

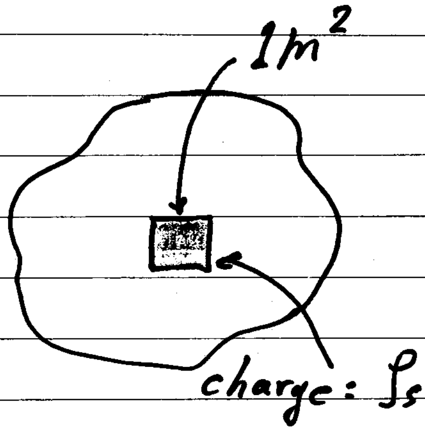


*- If ρ_l is homogeneous (uniform) along the whole length (ρ_l : const.) then:

$$Q = \rho_l \int_a^b dl = \rho_l L$$

L: Total length of the line charge.

b- Surface Distribution



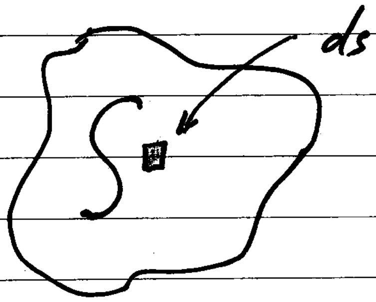
*- ρ_s: Surface Charge Density.

ρ_s: Amount of charge per one meter square (unit area).

*- Units: ρ_s: C/m^2.

*- Total charge among the whole area (S):

$$Q = \int_S \rho_s \, ds$$

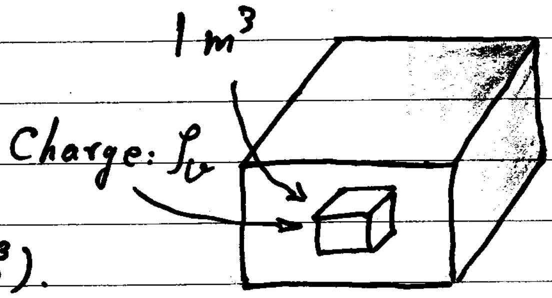


*- If ρ_s is uniform among the whole area (ρ_s: const.) then:

$$Q = \rho_s \int_S ds = \rho_s S.$$

c- Volume Distribution

*- ρ_v: Volume Charge Density

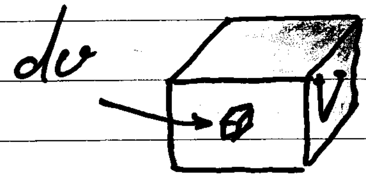


ρ_v: Amount of charge per unit volume ($1m^3$).

*- Units: ρ_v: C/m^3

*- Total charge among the whole volume (V):

$$Q = \int_V \rho_v \, dv$$



Class

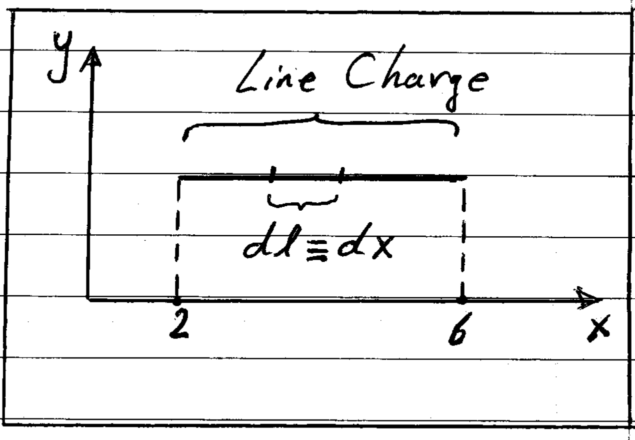
*- If ρ_v is homogeneous (ρ_v : const.), then:

$$Q = \rho_v \int_v dv = \rho_v V$$

Charge Distribution	Uniform		non-Uniform	
	Charge Density	Total Charge	Charge Density	Total Charge
Linear	$\rho_l = \frac{Q}{L}$	$Q = \rho_l L$	$\rho_l = \frac{dq}{dl}$	$dq = \rho_l \, dl$ $Q = \int dq$ $Q = \int_L \rho_l \, dl$
Surface	$\rho_s = \frac{Q}{S}$	$Q = \rho_s S$	$\rho_s = \frac{dq}{ds}$	$dq = \rho_s \, ds$ $Q = \int dq$ $Q = \int_s \rho_s \, ds$
Volume	$\rho_v = \frac{Q}{V}$	$Q = \rho_v V$	$\rho_v = \frac{dq}{dv}$	$dq = \rho_v \, dv$ $Q = \int dq$ $Q = \int_v \rho_v \, dv$

Examples of non-uniform charge distribution

Ex. 7: A line charge of charge density $\rho_\ell = 4x^3 - x + 3$ mC/m lying along the x-axis. Determine the total charge if the line charge extends from $x = 2$ m to $x = 6$ m.



Sol.: $Q = \int \rho_\ell \, d\ell$

Here, $d\ell \equiv dx$

$$Q = \int_2^6 (4x^3 - x + 3)\, dx$$

$$Q = 4\int_2^6 x^3 dx - \int_2^6 x\, dx + 3\int_2^6 dx$$

$$Q = 4\frac{x^4}{4}\Big|_2^6 - \frac{x^2}{2}\Big|_2^6 + 3x\Big|_2^6$$

$$= (6^4 - 2^4) - \frac{1}{2}(6^2 - 2^2) + 3(6-2)$$

$$Q = 1276 \text{ mC}$$

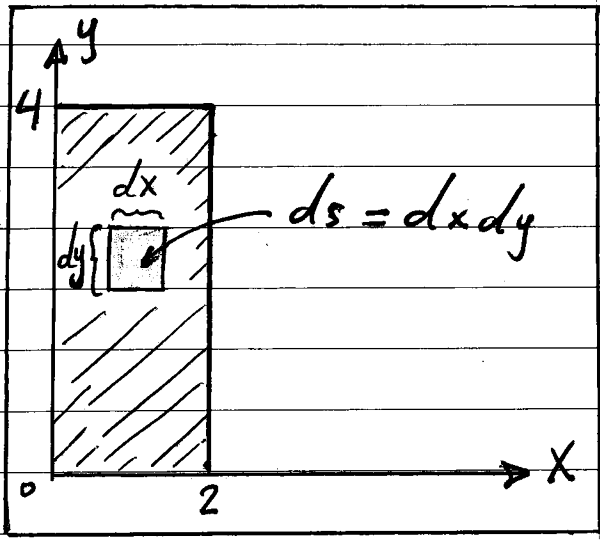
$$Q = 1.276 \text{ C}.$$

Class

*- Surface Distribution

Ex. 8 : A rectangular area with $x = 0 \rightarrow 2\,m$ and $y = 0 \rightarrow 4\,m$ has a charge density given as: $12x^3y^2\ mC/m^2$.

Determine the total charge on that area.



Sol. : $Q = \int_s \rho_s \, ds$

$ds = dx\,dy$, $\rho_s = 12x^3y^2\ mC/m^2$.

$Q = 12 \int_s x^3y^2 \, dx\,dy$

$= 12 \int_0^2 x^3 dx \int_0^4 y^2 dy$

$= 12 \left[\frac{x^4}{4}\Big|_0^2 \ \frac{y^3}{3}\Big|_0^4 \right]$

$= 12 \left[\frac{1}{12} (2^4 - 0)(4^3 - 0) \right] = 2^4 \times 4^3 = 16 \times 64$

$Q = 1024\ mC$

Or, $Q = 1.024\ C$.

Class

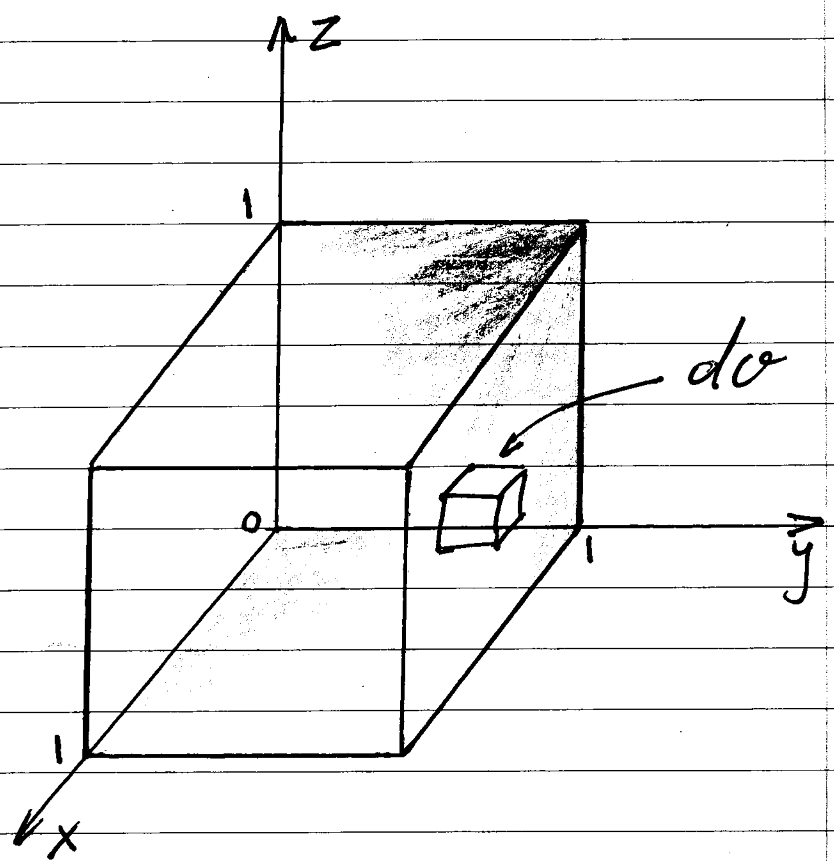
*- Volume Distribution

Hw. 4: A cubic volume with an edge of 1m, has a charge density given as: $xyz\ C/m^3$. Determine the charge in that cube.

Hints: $\rho_v = xyz$

$$Q = \int_v \rho_v \, dv$$

$$dv = dx\, dy\, dz$$



Ans.:

$Q = 0.125\ C.$

Hw. 5: Try to solve Hw. 4 with:

$$\rho_v = x^2 e^{2y} + 3z$$

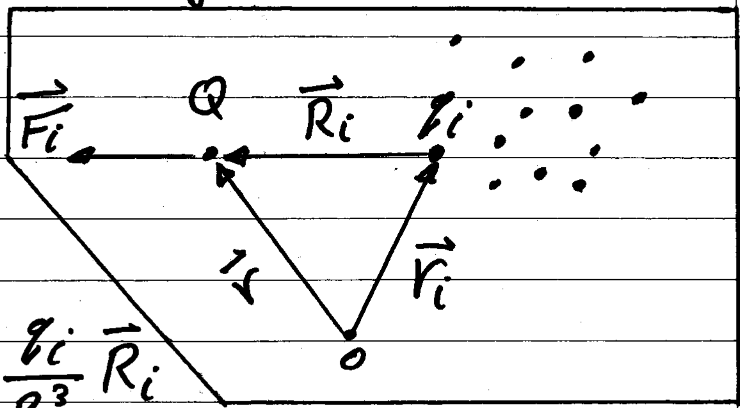
Ans.: $Q = 2.5648\ C.$

Class

Force and Electric Field Intensity for Charge Configurations

1- System of point charges

a- Force

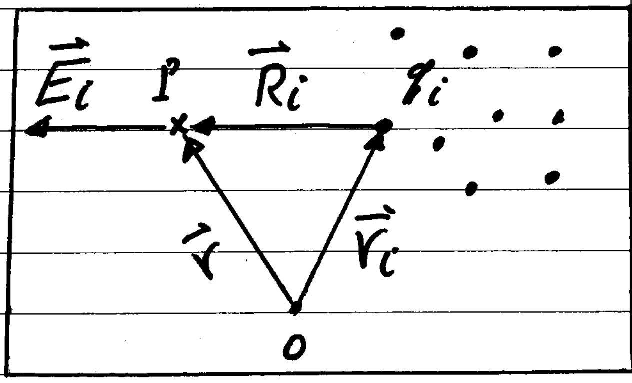


$$\vec{F_i} = k \frac{Q q_i}{R_i^3} \vec{R_i}$$

$$\vec{F} = \sum_{i=1}^{n} \vec{F_i} = kQ \sum_{i=1}^{n} \frac{q_i}{R_i^3} \vec{R_i}$$

where $\vec{R_i} = \vec{r} - \vec{r_i}$

b- Electric Field



$$\vec{E_i} = k \frac{q_i}{R_i^3} \vec{R_i}$$

$$\vec{E_p} = \sum_{i=1}^{n} \vec{E_i}$$

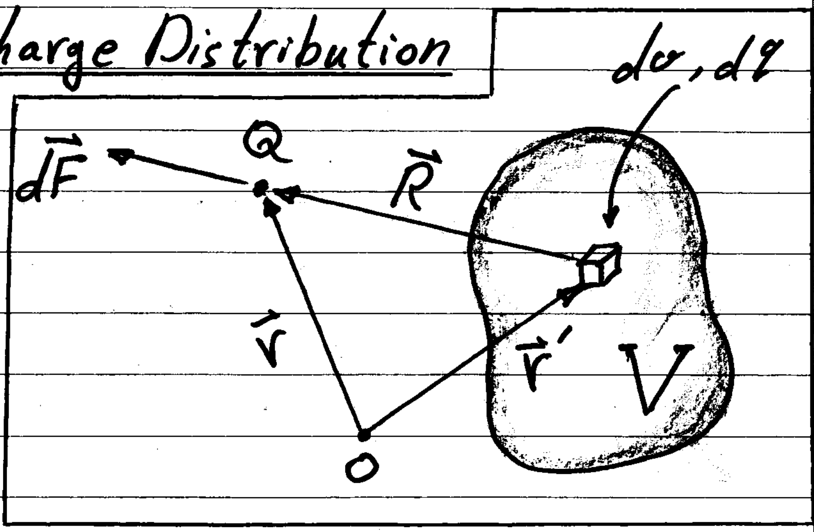
$$\vec{E_p} = k \sum_{i=1}^{n} \frac{q_i}{R_i^3} \vec{R_i}$$

$$\vec{R_i} = \vec{r} - \vec{r_i}$$

Class

2- Charge Distributions

*- Volume Charge Distribution



a- Force

$$\vec{dF} = k\frac{Q\,dq}{R^3}\vec{R}$$

$$\vec{F} = \int \vec{dF}$$

$$\vec{F} = kQ\int \frac{\vec{R}}{R^3}\,dq \quad , \quad dq = \rho_v\,dv$$

$$\therefore\ \vec{F} = kQ\int_v \frac{\rho_v\vec{R}}{R^3}\,dv \quad , \quad \vec{R} = \vec{r} - \vec{r}'$$

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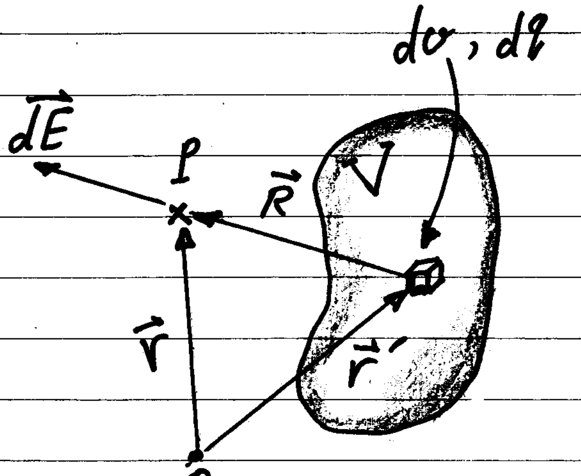
b- Electric Field



$$\vec{dE} = k\frac{dq}{R^3}\vec{R}$$

$$\vec{E}_p = \int \vec{dE}$$

$$\vec{E}_p = k\int \frac{\vec{R}}{R^3}\,dq$$

$$dq = \rho_v\,dv$$

$$\vec{E}_p = k\int_v \frac{\rho_v\vec{R}}{R^3}\,dv \quad , \quad \vec{R} = \vec{r} - \vec{r}'$$

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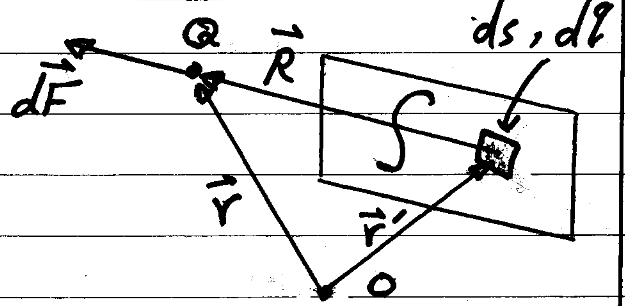
*- Surface Charge Distribution

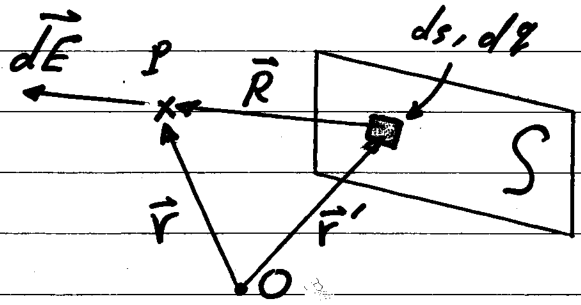
a- Force



$$\vec{F} = kQ \int_s \frac{\rho_s \vec{R}}{R^3} \, ds$$

$$\vec{R} = \vec{r} - \vec{r}\,'$$

b- Electric Field

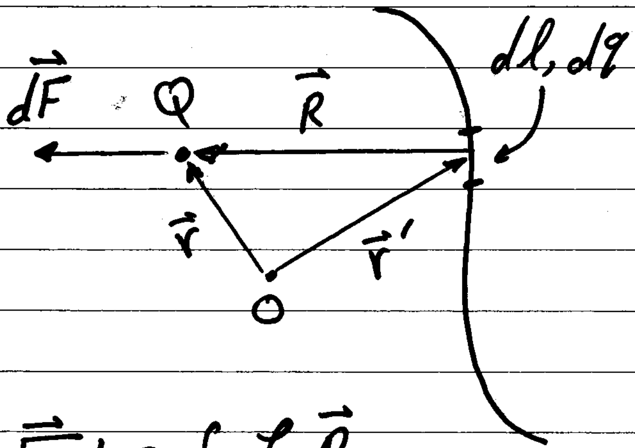


$$\vec{E}_p = k \int_s \frac{\rho_s \vec{R}}{R^3} \, ds$$

$$\vec{R} = \vec{r} - \vec{r}\,'$$

*- Line Charge Distribution

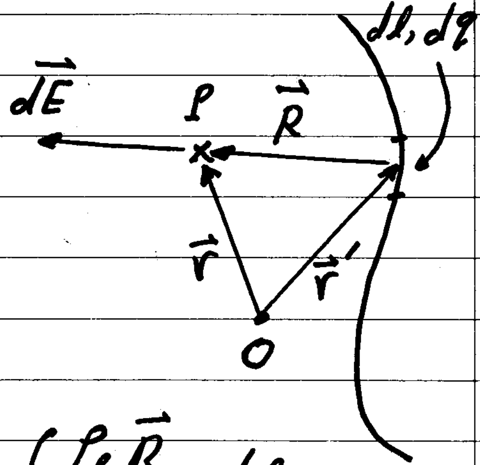
a- Force



$$\vec{F} = kQ \int_L \frac{\rho_l \vec{R}}{R^3} \, dl$$

$$\vec{R} = \vec{r} - \vec{r}\,'$$

b- Electric Field



$$\vec{E} = k \int_L \frac{\rho_l \vec{R}}{R^3} \, dl$$

$$\vec{R} = \vec{r} - \vec{r}\,'$$

*- Achieve the steps for the surface and line distributions yourself.

Class