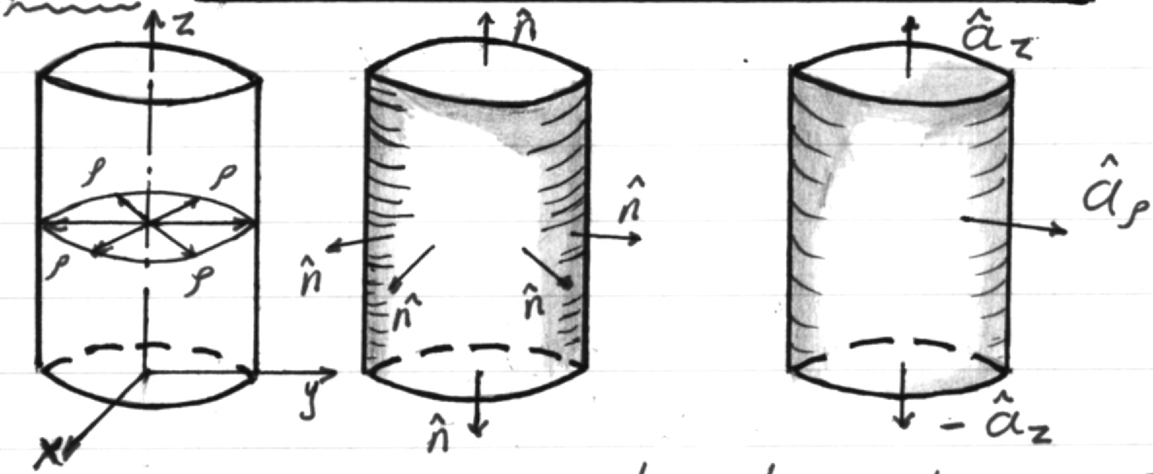
نظرية المجالات
هندسة اتصالات الحاسبات
المرحلة الثالثة

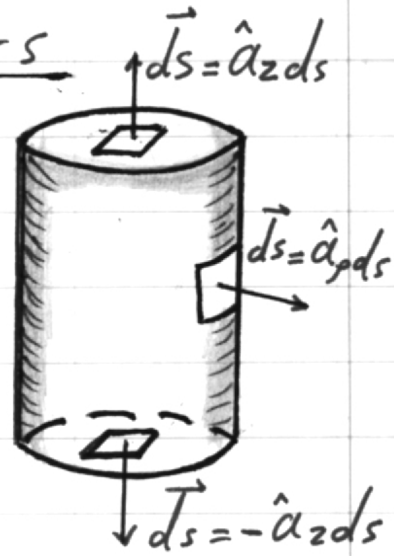
EM5

1

Note: Vectors normal to closed surfaces



Normal unit vectors

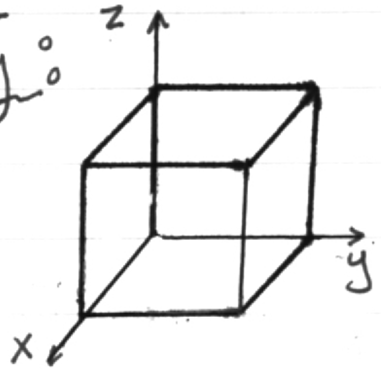


Top: $\hat{n} \equiv \hat{a}_z \Rightarrow \vec{ds} = \hat{a}_z\, ds$

Side: $\hat{n} \equiv \hat{a}_\rho \Rightarrow \vec{ds} = \hat{a}_\rho\, ds$

Bottom: $\hat{n} \equiv -\hat{a}_z \Rightarrow \vec{ds} = -\hat{a}_z\, ds$

Try:



Try to write the forms of ($\vec{ds}$) for all of the six faces.

Preliminary Examples

Ex. a: A closed cylindrical surface is charged uniformly with a density of $0.5\ C/m^2$.
If the radius of the cylinder is $1m$ and it's length is $3\ m$, determine:

(1): the charge of the side surface (lateral surface) of that cylinder.

(2): The total charge on the cylinder.

Sol.:

(1): $Q = \rho_s\ S_{cylinder}$

Let the radius is (ρ) and the length is (L).

$$Q_{sd.} = \rho_s\ (2\pi\rho L)$$

$$Q_{sd.} = 0.5\ (2 \times 3.14 \times 1 \times 3)$$

$$Q_{sd} = 9.42\ C.$$

(2): $Q_T = Q_{sd} + 2\ (\pi\rho^2)\ \rho_s$

$$= 9.42 + 2\ (3.14 \times 1^2) \times 0.5$$

$$Q_T = 12.56\ C$$

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*- In General: For a closed cylindrical surface:

$$Q_T = \underbrace{\rho_s (2\pi\rho L)}_{\text{Side}} + \underbrace{\rho_s\ 2\pi\rho^2}_{\text{Top \& Bottom}}$$

Ex. (b): The surface of a sphere is charged uniformly with a density of $2\ C/m^2$. If the radius of the sphere is $1/\sqrt{2}$ m, find the charge on that sphere.

Sol.: We have: $Q = \rho_s\ S_{sphere}$

Let the radius of the sphere is (r).

$$Q = \rho_s\ (4\pi r^2)$$

$$= 2\ (4 \times 3.14 \times \tfrac{1}{2})$$

$$Q = 12.56\ C.$$

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*- Similarly, for a uniformly line charge we have: $Q_T = \rho_\ell\ L$, where (L) is the length of the line charge.

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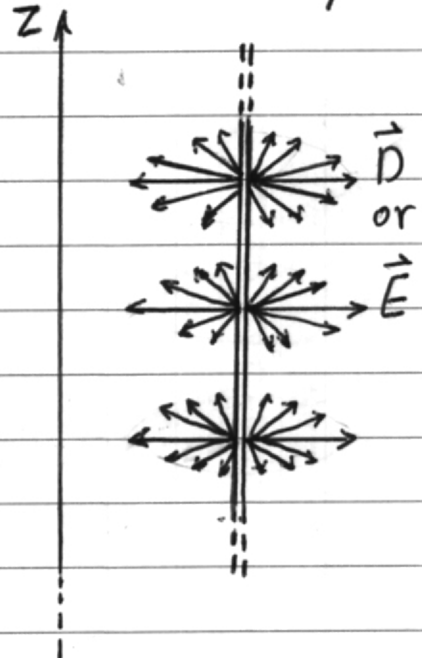
*- The following examples are applications on the Gauss' Law.

<u>Ex. 1</u>: Use Gauss's law to derive the form of the electric flux density due to a line charge distribution (ρ_l) lying along the z-axis and extends from $-\infty$ to ∞.

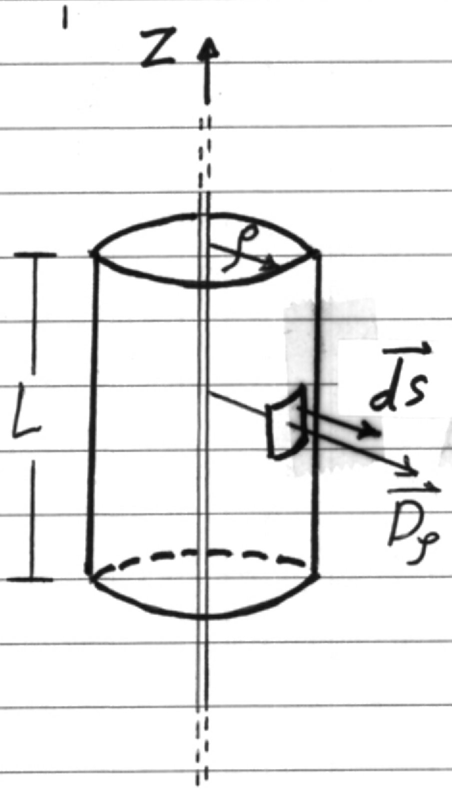
<u>Sol</u>: *- For a line charge, only the radial component of $\vec{D}$ (or $\vec{E}$) is present.



*- Using cylindrical coordinates

$$\vec{D} = \vec{D}_\rho = D_\rho \, \hat{a}_\rho$$

*- <u>Gauss's surface</u>: A cylindrical surface of radius (ρ) and length (L) {extending from $z=0$ to $z=L$}.



*- Applying Gauss's law

$$\oint_{cyl} \vec{D}_\rho \cdot \vec{ds} = Q$$

$$\oint_{cyl} = \int_{Side} + \int_{Top} + \int_{Bottom} = Q$$

Side: $\vec{ds} = \hat{a}_\rho \, ds$

$$\int_{side} \vec{D}_\rho \cdot \vec{ds} = \int_{side} D_\rho \, \hat{a}_\rho \cdot \hat{a}_\rho \, ds = \int_{sd} D_\rho \, ds$$

$$\therefore \int_{sd} \vec{D}_\rho \cdot \vec{ds} = D_\rho \int_{sd} ds = D_\rho \, (2\pi \rho L)$$

Top: $\vec{ds} = \hat{a}_z \, ds \Rightarrow \int_{Top} D_\rho \, \hat{a}_\rho \cdot \hat{a}_z \, ds = 0$

Bottom: $\vec{ds} = -\hat{a}_z \, ds \Rightarrow -\int_{Bot} D_\rho \, \hat{a}_\rho \cdot \hat{a}_z \, ds = 0$

$$\therefore \oint_{Cyl} \vec{D}_\rho \cdot \vec{ds} = \int_{sd} \vec{D}_\rho \cdot \vec{ds} = Q$$

$$D_\rho \, (2\pi \rho L) = Q$$

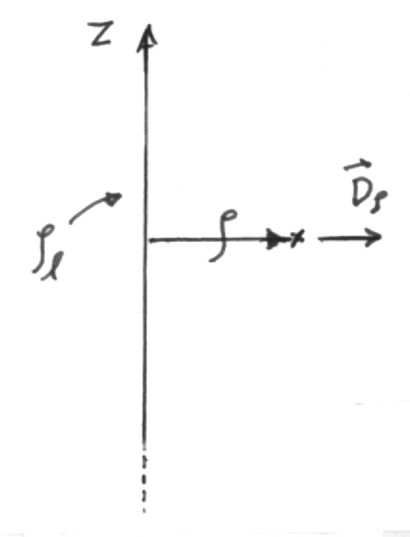
$$D_\rho = \frac{Q}{2\pi \rho L} \Rightarrow \boxed{\vec{D}_\rho = \frac{Q}{2\pi \rho L} \, \hat{a}_\rho}$$

Q: Charge enclosed inside Gauss's surface.

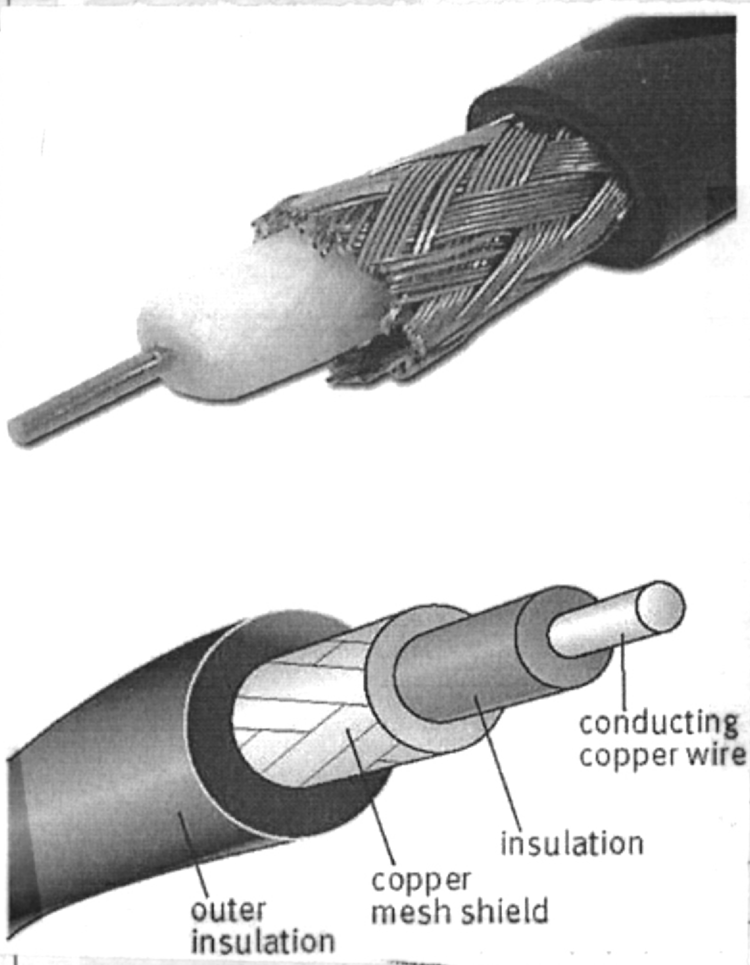
We have that: $\frac{Q}{L} = \rho_L$

$$\therefore \boxed{\vec{D}_\rho = \frac{\rho_L}{2\pi \rho} \, \hat{a}_\rho}$$

$$\text{or,} \quad \boxed{\vec{D}_\rho = \frac{\rho_L}{2\pi \rho^2} \, \vec{\rho}}$$



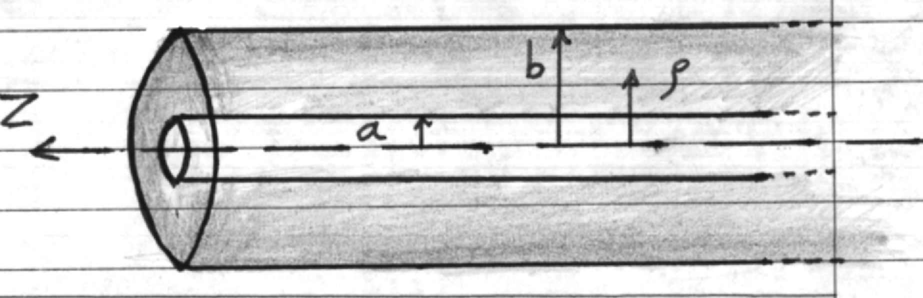
Coaxial Cables — الأسلاك المحورية



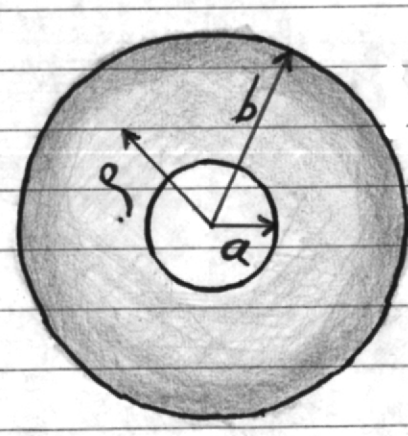
*- Two coaxial cylindrical conductors separated by an insulator.

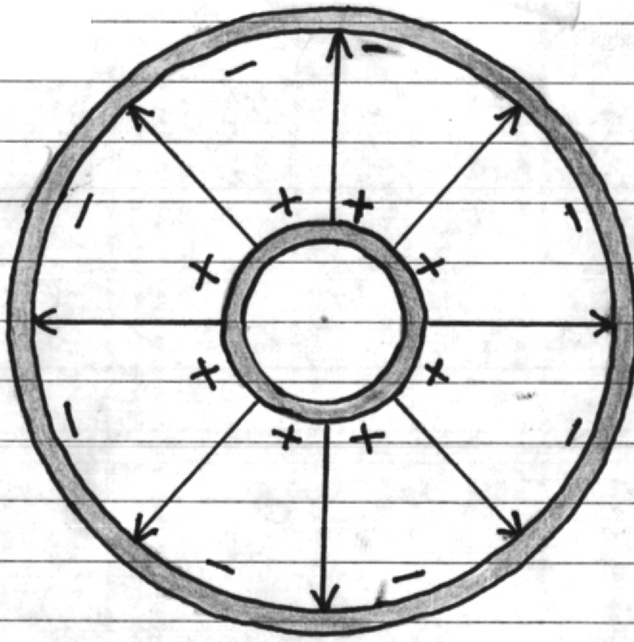
*- عبارة عن موصلين اسطوانيين متحدي المحور (coaxial) تفصلهما ماده عازلة (insulator) أو (dielectric).

Side View



Front View

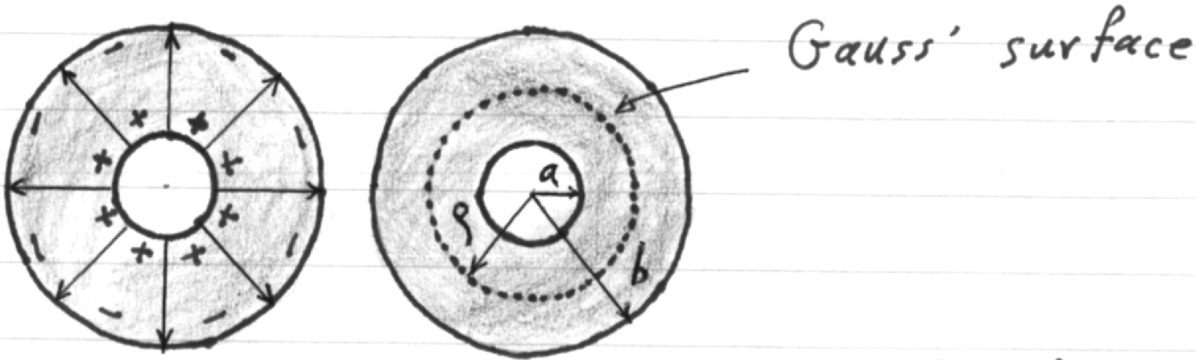


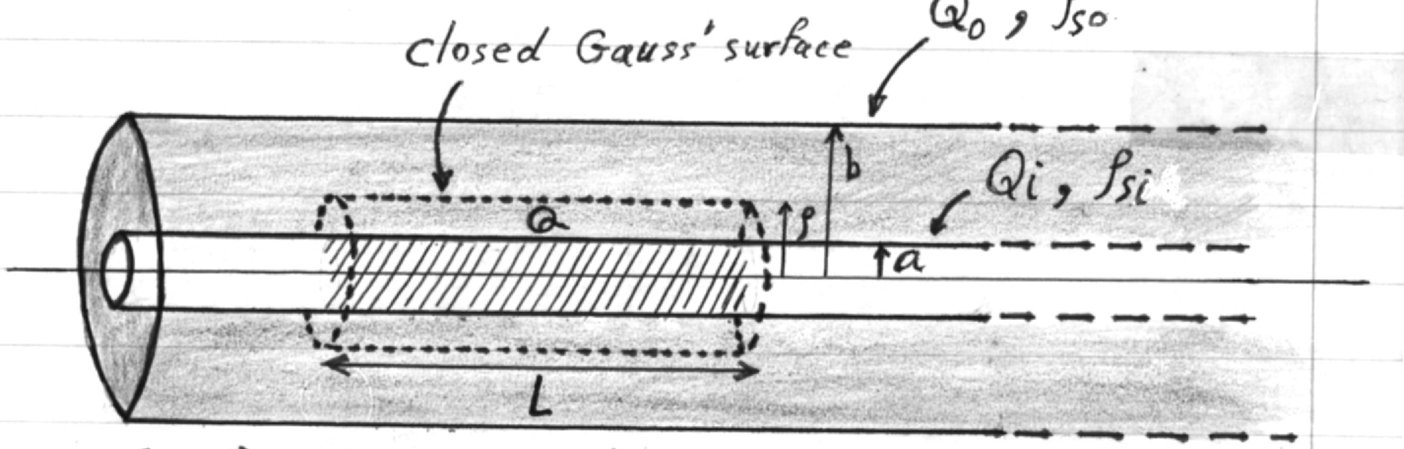


*- عند وضع شحنة موجبة على الموصل الداخلي وأخرى سالبة على الموصل الخارجي ، فأن الشحنات تتوزع على الموصلين بهيئة كثافات سطحية (أو طولية) بحيث نحصل على كثافة شحنات موجبة على الموصل الداخلي وكثافة شحنات سالبه على الموصل الخارجي.

*- يتولد مجال كهربائي شعاعي بالاتجاه النصف قطري ويكون عمودياً على الموصلين، وينبع من الموصل الداخلي (الشحنه الموجبه) وينتهي عند الموصل الخارجي (الشحنه السالبة).

Ex. 2 : Derive the expressions for the electric flux density ($\vec{D}$) and field intensity ($\vec{E}$) between the conductors of a coaxial cable, with a dielectric of permitivity (ϵ).





*- The $\vec{D}$-field (or $\vec{E}$-field) is radial :

$$\vec{D} \equiv \vec{D}_\rho \equiv D_\rho \, \hat{a}_\rho$$

Gauss' Surface : A closed cylinder of length (L) and radius (ρ) $\{ a < \rho < b \}$.

Gauss' Law : $\oint_{cyl.} \vec{D} \cdot d\vec{s} = Q$

where (Q) is the charge enclosed inside the Gauss' surface.

*- $\oint_{cyl} \vec{D_\rho} \cdot \vec{ds} = \int_{side} \vec{D_\rho} \cdot \vec{ds} + \int_{Top} \vec{D_\rho} \cdot \vec{ds} + \int_{Bottom} \vec{D_\rho} \cdot \vec{ds}$

<u>Side</u>: $\int \vec{D_\rho} \cdot \vec{ds} = \int D_\rho \hat{a}_\rho \cdot \hat{a}_\rho ds = \int D_\rho ds$

<u>Top</u>: $\int \vec{D_\rho} \cdot \vec{ds} = \int D_\rho \hat{a}_\rho \cdot \hat{a}_z ds = 0$

<u>Bottom</u>: $\int \vec{D_\rho} \cdot \vec{ds} = - \int D_\rho \hat{a}_\rho \cdot \hat{a}_z ds = 0$

$\therefore$ Gauss' law becomes : $\int_{side} D_\rho ds = Q$

*- Since D_ρ is constant at the side surface then:

$$D_\rho \int ds = Q \implies D_\rho S = Q$$

$$S = 2\pi \rho L \quad \text{and} \quad Q = \rho_{si} (2\pi a L)$$

*- Substituting in Gauss' law :

$$D_\rho (2\pi \rho L) = \rho_{si} (2\pi a L)$$

$$D_\rho = \frac{a}{\rho} \rho_{si}$$

*- Putting direction:

$$\boxed{\vec{D_\rho} = \frac{a}{\rho} \rho_{si} \hat{a}_\rho} \quad , \quad [a < \rho < b]$$

*- We have : $\vec{D}_\rho = \epsilon \vec{E}_\rho$

or, $\vec{E}_\rho = \dfrac{\vec{D}_\rho}{\epsilon}$

$$\therefore \boxed{\vec{E}_\rho = \frac{a}{\rho} \frac{\rho_{si}}{\epsilon} \hat{a}_\rho} , \quad a < \rho < b$$

For the previous example

Note 1: The results may be written in terms of the line charge density ρ_L {charge per unit length}

*- $\rho_L = \dfrac{Q}{L} = \dfrac{\rho_{si}(2\pi a L)}{L} \Rightarrow \rho_L = \rho_{si}(2\pi a)$

or, $\rho_{si} = \dfrac{\rho_L}{2\pi a}$

*- Substituting in D_ρ form :

$$\vec{D}_\rho = \frac{a \rho_{si}}{\rho} \hat{a}_\rho = \frac{a \, \rho_L}{2\pi a \rho} \hat{a}_\rho = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho$$

$$\therefore \boxed{\vec{D}_\rho = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho} , \text{ and } \boxed{\vec{E}_\rho = \frac{\rho_L}{2\pi\epsilon\rho} \hat{a}_\rho}$$

*- These are the same results obtained for a line charge distribution.

Note ②:

*- The charges on outer and inner cylinders are equal and opposite in sign :

$$Q_{\text{outer cyl.}} = -Q_{\text{inner cyl.}}$$

or, $Q_o = -Q_i$

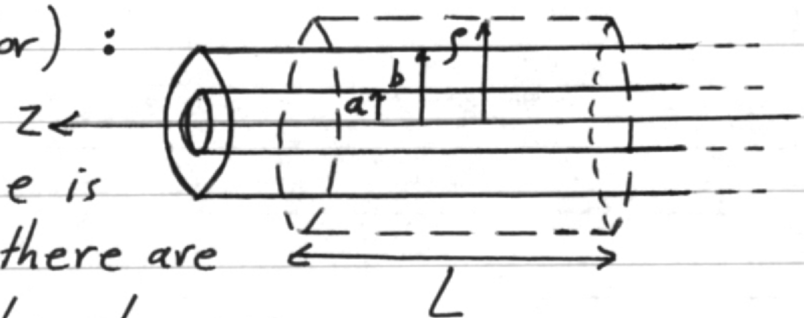
$$(2\pi b L)\,\rho_{so} = -(2\pi a L)\,\rho_{si}$$

$$\therefore \quad \boxed{\rho_{si} = -\frac{b}{a}\,\rho_{so}}$$

Note ③

*- For $\rho > b$ (Gauss surface is outside the outer conductor) :

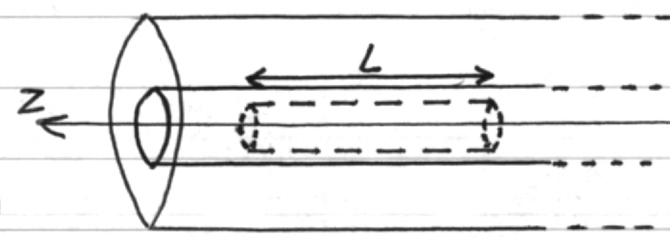
*- The total enclosed charge is zero because there are equal and opposite charges on both cylinders.



$$*- \therefore \oint_{cyl.} \vec{D}\cdot\vec{ds} = Q = 0 \Rightarrow \oint_{cyl.} \vec{D}\cdot\vec{ds} = 0$$

$$\Rightarrow \boxed{D = 0 \quad \text{for } \rho > b.}$$

*- For $\rho < a$ (Gauss surface is inside the inner conductor):



*- Charge enclosed by Gauss surface is zero ($Q = 0$) because there is no electrical charges inside a conductor, hence:

$$\oint_{cyl.} \vec{D} \cdot \vec{ds} = 0 \Rightarrow \boxed{D = 0 \quad \text{for } \rho < a}.$$

Ex.3: A coaxial cable of 50 cm length, the inner radius is 1 mm, the outer is 4 mm. The dielectric material (insulator) between the conductors is Polyethylene ($\epsilon \simeq 20 \times 10^{-12}$ F/m). The total charge on the inner conductor is 30 nC. Find the surface charge density on each conductor, and the $\vec{E}$ and $\vec{D}$-fields at a distance 2.5 mm from the cable axis.

Sol.:

*- Charge density of inner conductor:

$$\rho_{si} = \frac{Q_i}{2\pi a L} = \frac{30 \times 10^{-9}}{2 \times 3.14 \times 10^{-3} \times 50 \times 10^{-2}}$$

$$\rho_{si} = 0.0955 \times 10^{-4} \ C/m^2$$

*- Charge density on the outer electrode :

$$\rho_{so} = -\frac{a}{b}\,\rho_{si}$$

$$\rho_{so} = -\frac{1}{4} \times 0.0955 \times 10^{-4} = -2.3875 \times 10^{-6}\ C/m^2$$

*- From Gauss' law we have :

$$\vec{D}_\rho = \frac{a}{\rho}\,\rho_{si}\,\hat{a}_\rho = \frac{0.0955 \times 10^{-4}}{2.5}\,\hat{a}_\rho$$

$$\vec{D}_\rho = 0.0382 \times 10^{-4}\,\hat{a}_\rho\ C/m^2 .$$

* We have : $\vec{D} = \epsilon \vec{E}$

or, $\vec{E} = \vec{D} / \epsilon$

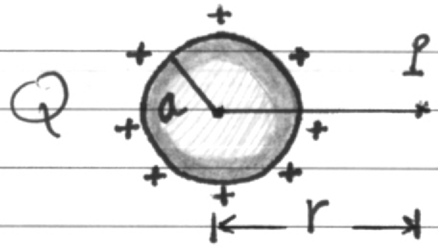
$\therefore\ \vec{E}_\rho = \vec{D}_\rho / \epsilon$

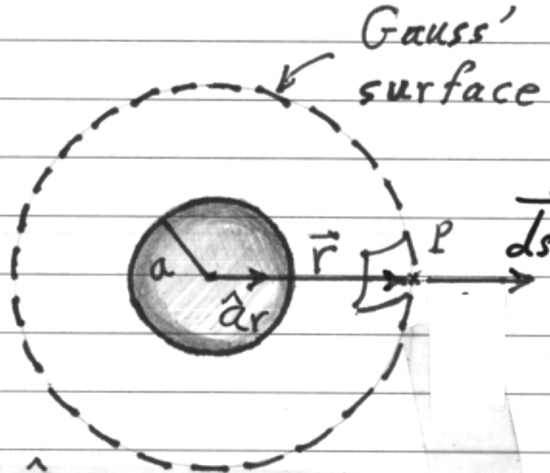
$$\vec{E}_\rho = \frac{0.0382 \times 10^{-4}}{20 \times 10^{-12}}\,\hat{a}_\rho = 0.0019 \times 10^{8}\,\hat{a}_\rho\,\frac{N}{C}$$

or, $\vec{E}_\rho = 190\,\hat{a}_\rho\ kN/C$.

<u>Ex. 4:</u> Use Gauss' law to derive the electric field intensity ($\vec{E}$) at point (P) located at a distance (r) from the centre of a charged sphere of radius (a) and charge (Q).

<u>Sol.:</u>





*- We have: $\vec{E} \equiv \vec{E_r} \equiv E_r \hat{a}_r$

*- <u>Gauss' law:</u> $\oint_s \vec{E} \cdot \vec{ds} = Q/\epsilon$

$$\oint_s \vec{E_r} \cdot \vec{ds} = Q/\epsilon$$

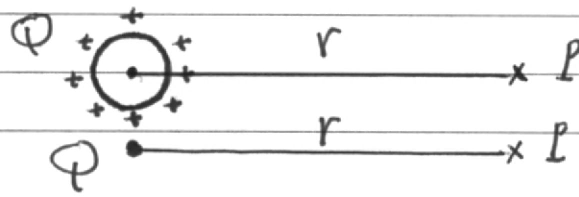
$$\oint_s E_r \hat{a}_r \cdot \hat{a}_r \, ds = Q/\epsilon \Rightarrow \oint_s E_r \, ds = Q/\epsilon$$

$$E_r \oint_s ds = Q/\epsilon \Rightarrow E_r (4\pi r^2) = Q/\epsilon$$

$$\therefore E_r \equiv E = \frac{Q}{4\pi\epsilon r^2}$$

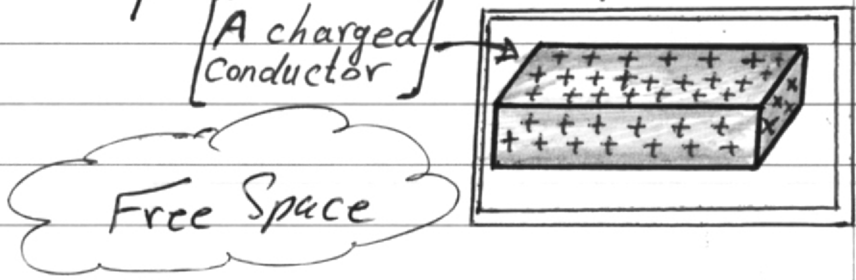
*- <u>Putting direction:</u> $\vec{E} = \dfrac{Q}{4\pi\epsilon r^2} \hat{a}_r$

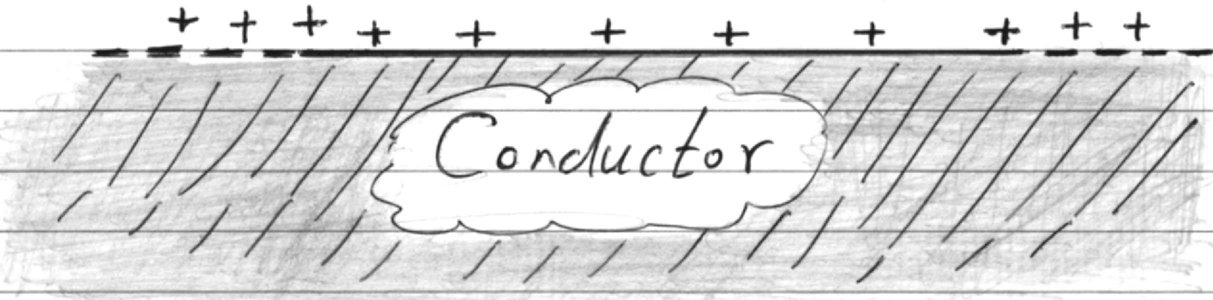
*- <u>This is the same form for a point charge case.</u>



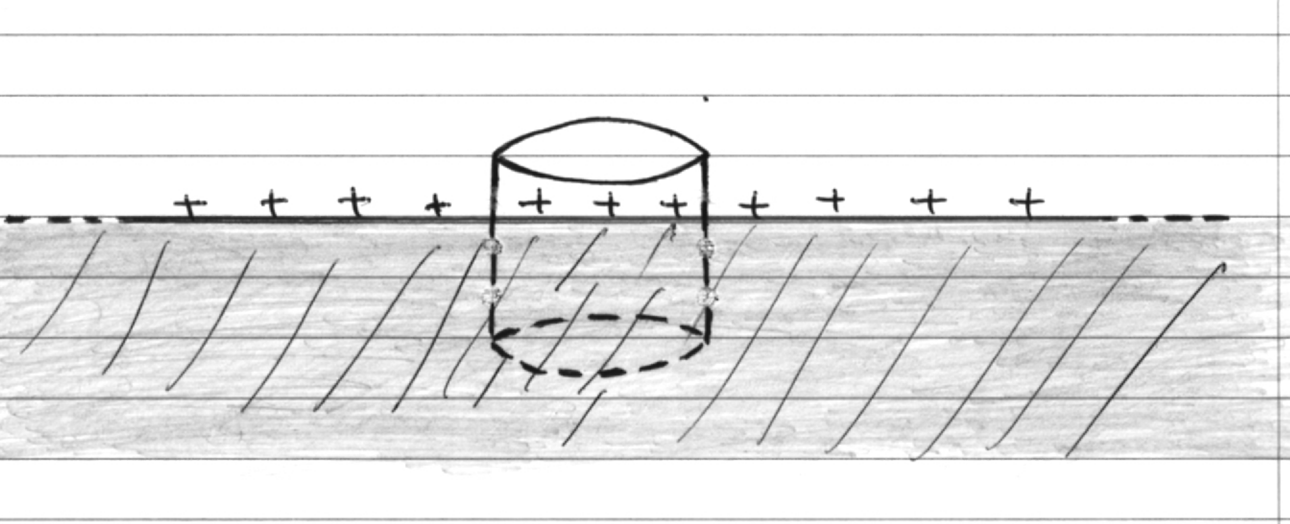
<u>Ex. 5</u>: Find an expression for the electric flux density ($\vec{D}$) and field intensity ($\vec{E}$) just outside a charged conductor surface placed in free space.

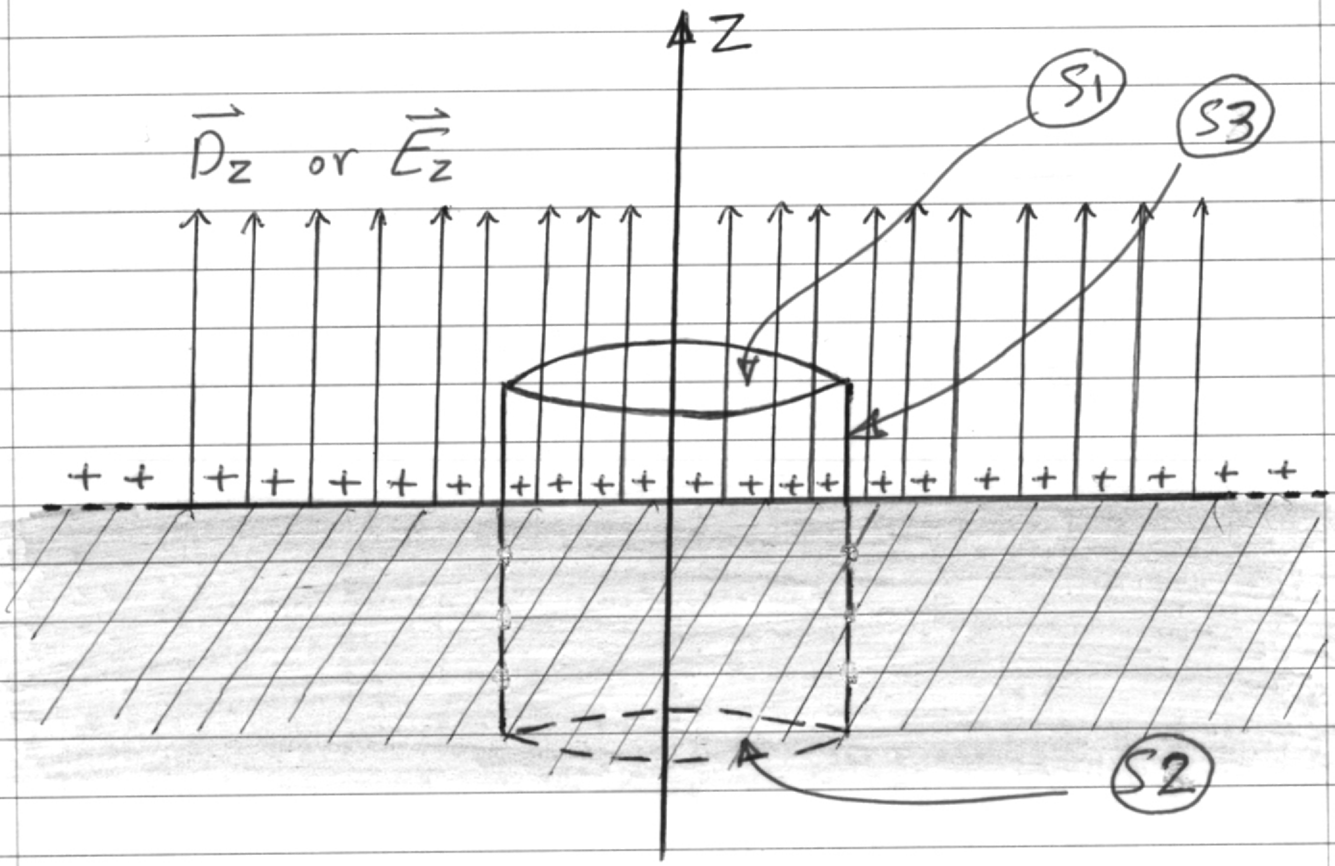
<u>Sol.</u>:





*- <u>Gauss's surface</u>: A small cylinder with it's top outside the conductor, and it's bottom inside the conductor's material.





*- The $\vec{D}$ component in this case is :

$$\vec{D}_z = \hat{a}_z D_z$$

*- Gauss's law : $\oint_{cyl.} \vec{D}_z \cdot \vec{ds} = \int_{S1} + \int_{S2} + \int_{S3} = Q$

S1 : $\int_{S1} \vec{D}_z \cdot \vec{ds} = \int_{S1} D_z \hat{a}_z \cdot \hat{a}_z \, ds = \int_{S1} D_z \, ds$

S2 : $\int_{S2} \vec{D}_z \cdot \vec{ds} = 0$; There is no $\vec{E}$ or $\vec{D}$ fields inside a conductor .

S3 : $\int_{S3} \vec{D}_z \cdot \vec{ds} = \int D_z \hat{a}_z \cdot \hat{a}_r \, ds = 0$

$$\therefore \oint_{cyl} \vec{D_z} \cdot \vec{ds} = \int_{s1} D_z\, ds = D_z \int_{s1} ds = D_z\, S1$$

$$D_z\, S1 = Q \quad \{\text{by Gauss's law}\}$$

$$D_z = \frac{Q}{S1}$$

Q: Surface charge enclosed inside Gauss's surface of cross sectional area = S1.

$$\therefore \boxed{D_z = \rho_s}$$ ← Near the surface of a charged conductor.

$$\vec{D_z} = \rho_s\, \hat{a}_z \quad , \quad \vec{E_z} = \vec{D_z} / \epsilon_0$$

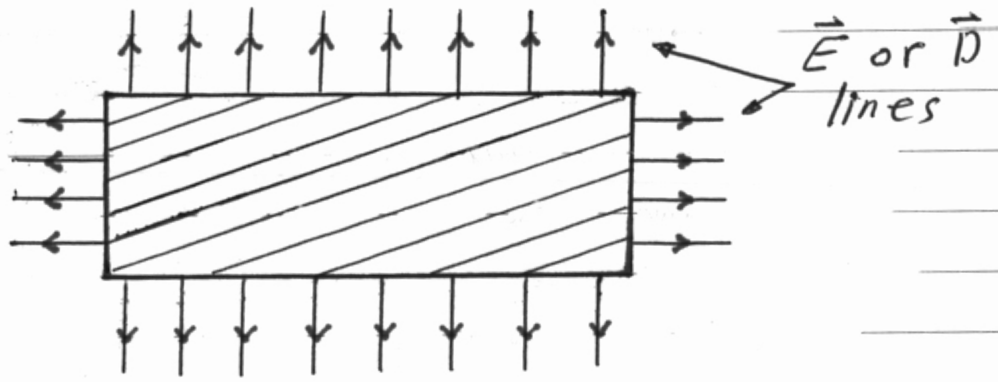
$$\vec{E_z} = \frac{\rho_s}{\epsilon_0}\, \hat{a}_z$$

*- In general: The electric flux density just outside (near) a charged conductor surface is equal in magnitude to the surface charge density on that surface.

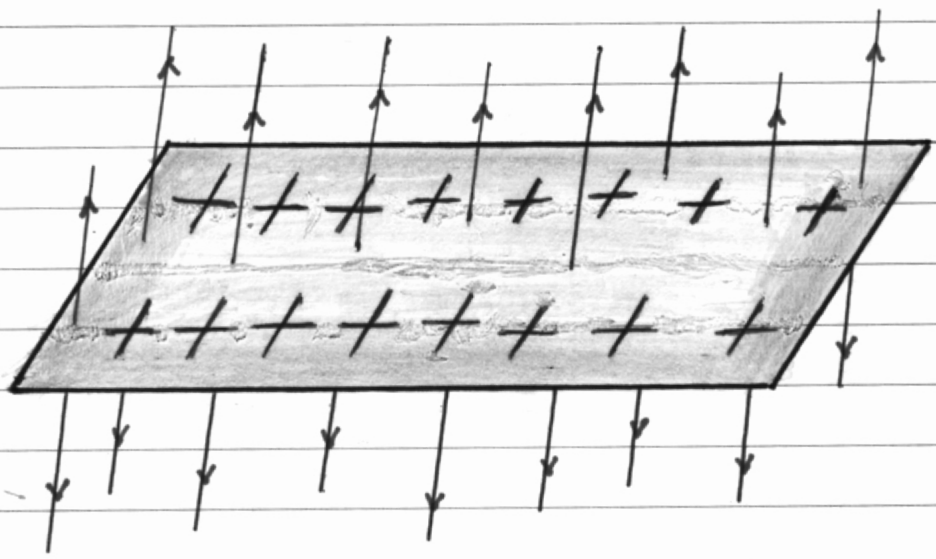
Notes:

*- For charged objects (Ex. 5), there is no electric flux leaving the back surfaces towards the interior of the object.

*- Hence D in front of each face is ρ_s :



*- Surface charge (of charge density ρ_s) :



*- The electric flux leavs the charged surface normally in both directions.

*- The magnitude of D in front of each face is $(\rho_s / 2)$.

ملاحظة حول الصيغة التفاضلية لقانون كاوس ($\vec{\nabla}\cdot\vec{D} = \rho_v$):

*- في المسائل التي لا تحوي على أي شكل من أشكال التناظر، يكون من المستحيل الحصول على سطح كاوس بسيط بحيث تكون $\vec{D}$ خلاله عمودية وثابتة أو صفر.

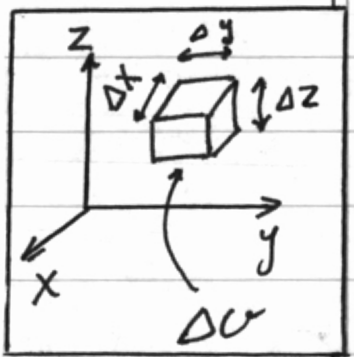
*- بدون سطح كاوس لا يمكن اجراء التكامل السطحي.

*- هنالك طريقة واحدة لتجاوز هذه المسألة، وهي بأتخاذ حجم مغلق صغير جداً (Δv) بحيث تكون $\vec{D}$ ثابتة تقريباً خلاله {التغير في $\vec{D}$ بسيط جداً خلال هذا الحجم الصغير}.

*- يتم استخدام الصيغة التفاضلية لقانون كاوس ($\vec{\nabla}\cdot\vec{D} = \rho_v$) لايجاد الكثافة الحجمية للشحنة الكهربائية (ρ_v).

*- مشتقة $\vec{D}$ في ($\vec{\nabla}\cdot\vec{D}$) تعني اننا نأخذ الغاية (limit) لهذا الحجم بحيث $\Delta v \to 0$:

$$\Delta v = \Delta x \, \Delta y \, \Delta z \quad (\text{Cartesian Coordinates})$$



$$\vec{\nabla}\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v \quad (\text{at a point})$$

$$\vec{\nabla}\cdot\vec{D} = \lim_{\Delta x \to 0} \frac{\Delta D_x}{\Delta x} + \lim_{\Delta y \to 0} \frac{\Delta D_y}{\Delta y} + \lim_{\Delta z \to 0} \frac{\Delta D_z}{\Delta z}$$

*- عندما نأخذ الغاية $\Delta v \to 0$ {$\Delta x \to 0$ ، $\Delta y \to 0$ ، $\Delta z \to 0$} نحصل على الكثافة الحجمية في نقطة لأن الحجم Δv عندما يتقلص الى الصفر ($\Delta v \to 0$) فأنه يتحول الى نقطة.

Ex.6: Find the value of the total charge enclosed inside a volume of $1.5 \times 10^{-9}\ m^3$ (very small), located at the origin if:

$$\vec{D} = [e^{-x} \sin(y)]\hat{i} - [e^{-x} \cos(y)]\hat{j} + 2z\hat{k} \quad C/m^2.$$

Sol: Differential form of Gauss' law: $\vec{\nabla} \cdot \vec{D} = \rho_v$

We have: $\vec{D} = \hat{i} D_x + \hat{j} D_y + \hat{k} D_z$

$\therefore D_x = e^{-x} \sin(y)$, $D_y = -e^{-x} \cos(y)$, $D_z = 2Z$

$$\vec{\nabla} \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \frac{\partial}{\partial x}\left(e^{-x} \sin(y)\right) + \frac{\partial}{\partial y}\left(-e^{-x} \cos(y)\right) + \frac{\partial}{\partial z}(2Z)$$

$$\vec{\nabla} \cdot \vec{D} = -e^{-x} \sin(y) + e^{-x} \sin(y) + 2$$

At the origin: $x = y = Z = 0$

$\therefore \vec{\nabla} \cdot \vec{D} = 0 + 0 + 2 = 2$

$\because \vec{\nabla} \cdot \vec{D} = \rho_v \Longrightarrow \therefore \rho_v = 2\ C/m^3$

Total charge enclosed in this volume:

$$q = \rho_v V = 2 \times 1.5 \times 10^{-9}$$
$$= 3 \times 10^{-9}\ C$$

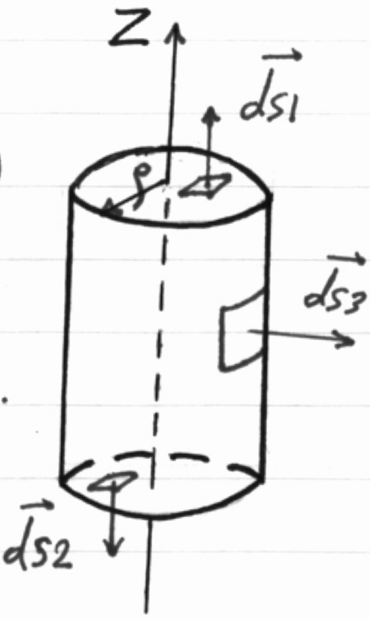
or, $\boxed{q = 3\ nC.}$

Reminder:

For a cylindrical coordinate system:

ds_1, ds_2 : const. Z (Top or Bottom)

ds_3: const. ρ (Side).



$\vec{ds_1}$: the unit vector is $\hat{k}$ (or $\hat{a}_z$).

$\vec{ds_2}$: the unit vector is $-\hat{k}$ (or $-\hat{a}_z$).

$\vec{ds_3}$: the unit vector is $\hat{a}_\rho$.

$\vec{ds_1} = ds_1\,\hat{a}_z$, $\vec{ds_2} = ds_2(-\hat{a}_z) = -ds_2\,\hat{a}_z$,

$\vec{ds_3} = ds_3\,\hat{a}_\rho$.

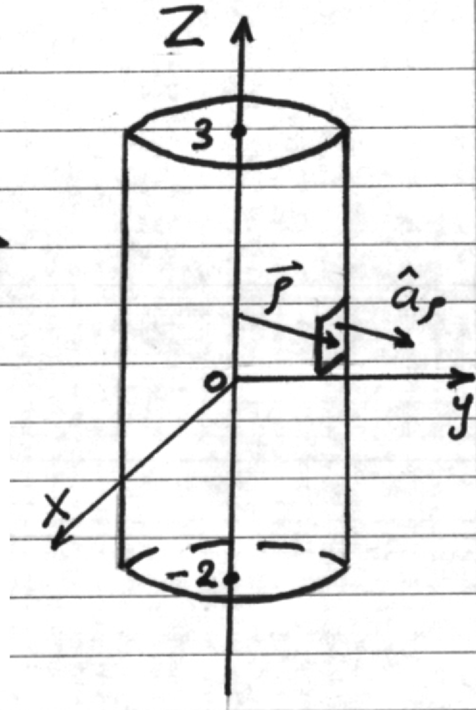
$\vec{ds_1} = \rho\, d\rho\, d\Phi\,\hat{a}_z$, $\vec{ds_2} = -\rho\, d\rho\, d\Phi\,\hat{a}_z$

$\vec{ds_3} = \rho\, d\Phi\, dz\,\hat{a}_\rho$.

Ex. 7: A given charge distribution inside a cylindrical volume produces an electric flux density given as:

$\vec{D} = \frac{k_1}{\rho}\,\hat{a}_\rho + k_2 Z\,\hat{a}_z$. Determine the amount of charge enclosed inside this cylinder which extends from $Z = -2\,m$ to $3\,m$, and it's radius is $2\,m$.

Given : $k_1 = 6\ C/m$ and $k_2 = 2\ C/m^3$.



Sol:

$$\oint_{cyl} \vec{D}\cdot\vec{ds} = \int_{Top} \vec{D}\cdot\vec{ds} + \int_{Bot.} \vec{D}\cdot\vec{ds} + \int_{side} \vec{D}\cdot\vec{ds}$$

① **Top:** $\int \vec{D}\cdot\vec{ds} = \int \left(\frac{k_1}{\rho}\hat{a}_\rho + k_2 Z\hat{a}_z\right)\cdot(\rho\, d\rho\, d\phi\ \hat{a}_z)$

$$\int \vec{D}\cdot\vec{ds} = \int_0^{2\pi}\int_0^{2} k_2 Z \rho\, d\rho\, d\phi \quad , \quad Z = 3\ m$$

$$= 3\,k_2 \int_0^2 \rho\, d\rho \int_0^{2\pi} d\phi = 3k_2 \left\{ \frac{\rho^2}{2}\Big|_0^2 \ \phi\Big|_0^{2\pi} \right\}$$

$$= 3k_2 \{2 \times 2\pi\} = 12\pi k_2$$

$$\therefore \boxed{\int_{Top} \vec{D}\cdot\vec{ds} = 12\pi k_2} \ .$$

② **Bottom:**

$$\int \vec{D}\cdot\vec{ds} = -\int \left(\frac{k_1}{\rho}\hat{a}_\rho + k_2 Z\hat{a}_z\right)\cdot(\rho\, d\rho\, d\phi\ \hat{a}_z)$$

$$= -\iint k_2 Z \rho\, d\rho\, d\phi \quad , \quad Z = -2\ m$$

$$= 2\,k_2 \int_0^{2\pi}\int_0^{2} \rho\, d\rho\, d\phi$$

$$\therefore \boxed{\int_{Bot.} \vec{D}\cdot\vec{ds} = 8\pi k_2}$$

③ Side :

$$\int \vec{D}\cdot\vec{ds} = \int\left(\frac{k_1}{\rho}\hat{a}_\rho + k_2 Z\hat{a}_z\right)\cdot\left(\rho\, d\phi\, dz\, \hat{a}_\rho\right)$$

$$= \int_{-2}^{3}\int_{0}^{2\pi} \frac{k_1}{\rho}\,\rho\, d\phi\, dz \quad , \quad \rho = 2\ m.$$

$$= k_1 \int_0^{2\pi} d\phi \int_{-2}^{3} dz \quad = k_1\, \phi\Big|_0^{2\pi}\ Z\Big|_{-2}^{3}$$

$$= k_1\ (2\pi)\,[3-(-2)] \quad = k_1 \times 2\pi \times 5$$

$$= 10\ \pi\ k_1$$

$$\therefore \boxed{\int_{\text{Side}} \vec{D}\cdot\vec{ds} = 10\ \pi\ k_1}$$

$$\oint_{cyl} \vec{D}\cdot\vec{ds} = 12\pi k_2 + 8\pi k_2 + 10\pi k_1$$

$$= 20\pi k_2 + 10\pi k_1 = 10\pi(2k_2 + k_1)$$

$$= 10 \times 3.14\ (2\times 2 + 6) = 314$$

*- From Gauss' law : $\oint_{cyl} \vec{D}\cdot\vec{ds} = Q$

$$\therefore\ Q = 314\ C\ .$$

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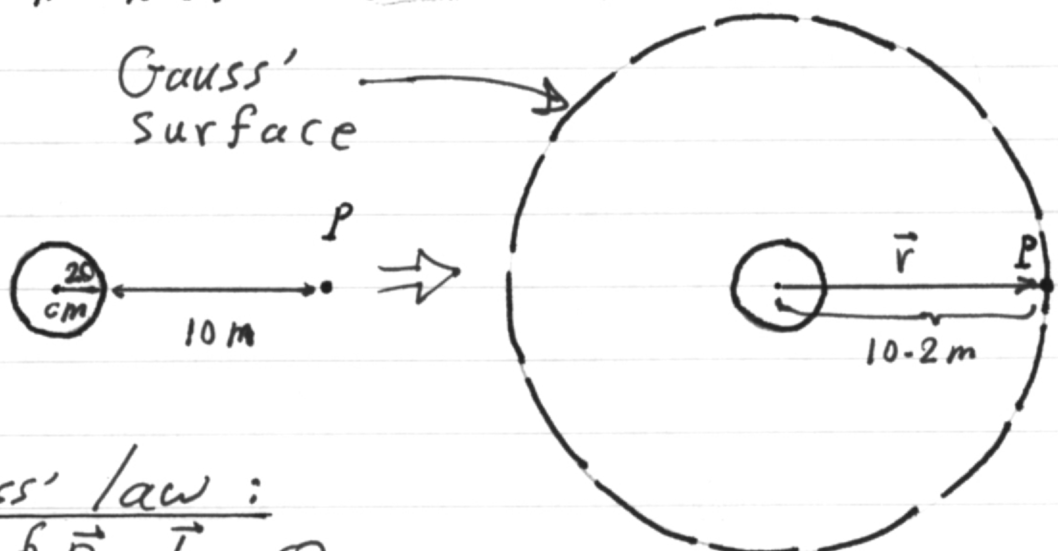
*- Important : Try to solve Hw. 7 on page 27.

Hw.1: Use Gauss' law to find the electric flux density ($\vec{D}$) and electric field intensity ($\vec{E}$) for a point (P) placed at a radial distance of 10 m from the surface of a spherical conductor of radius $a = 20\,cm$, charged by 300 nC, for:

(a): Free-space medium [$\epsilon_o = 8.854 \times 10^{-12}$ F/m].

(b): The medium is ice [$\epsilon = 37.1868 \times 10^{-12}$ F/m].

Hints: *- Define Gauss' surface.



*- Gauss' law:

$$\oint_s \vec{D} \cdot \vec{ds} = Q$$

*- $\vec{D} = D_r \hat{a}_r$, $\vec{ds} = \hat{a}_r \, ds$

*- Ans.: (a) $\vec{D} = 0.2295 \times 10^{-9} \, \hat{a}_r \ C/m^2$

$\vec{E} = 25.9 \, \hat{a}_r \ N/C$

(b) $\vec{D} = 0.2295 \times 10^{-9} \, \hat{a}_r \ C/m^2$

$\vec{E} = 6.1 \, \hat{a}_r \ N/C$.

Hw.2: A coaxial cable of $a = 2$ mm and $b = 6$ mm. The surface charge density on the outer conductor is (-3 nC/m²). Find the electric flux density ($\vec{D}$) and electric field intensity ($\vec{E}$) at a distance (4 mm) from the cable axis. Assume the space between conductors to be filled with Nylon of permitivity $\epsilon \simeq 31 \times 10^{-12}$ F/m.

Hints:

*- $\vec{D} = \vec{D}_\rho$

*- $\rho_{si} = -\frac{b}{a} \rho_{so}$

*- $\boxed{\vec{D}_\rho = \frac{a \rho_{si}}{\rho} \hat{a}_\rho} \Leftarrow \{\rho = 4 \text{ mm}\}$

*- $\vec{E}_\rho = \vec{D}_\rho / \epsilon$.

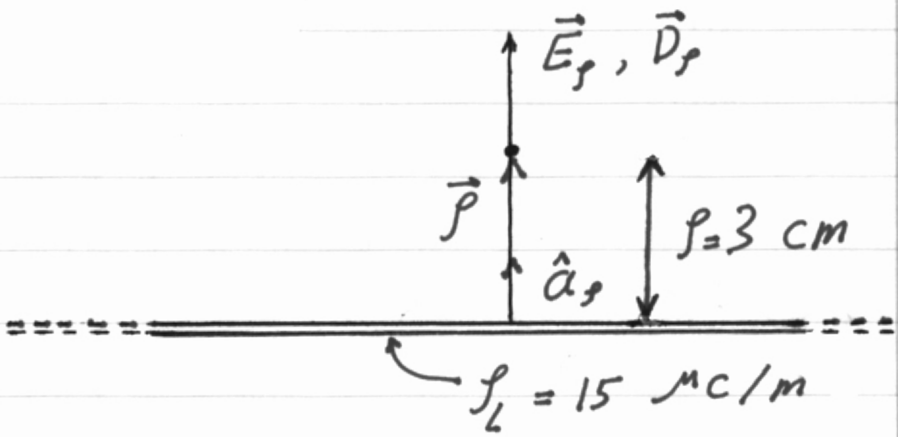
*- Ans.:

$$\vec{D}_\rho = 4.5 \times 10^{-9} \hat{a}_\rho \quad C/m^2$$

$$\vec{E}_\rho = 0.1451 \times 10^{3} \hat{a}_\rho \quad N/C$$

or, $\vec{E}_\rho = 145.1 \hat{a}_\rho$ N/C

Hw. 3 : Determine the electric flux density ($\vec{D}$) and electric field intensity ($\vec{E}$) at a point placed at a normal distance (3 cm) from an infinitly long line charge of a charge density 15 $\mu C/m$ if the surrounding medium is ice of electric permitivity $\epsilon \simeq 37 \times 10^{-12}$ F/m.



Hint : Use the forms of $\vec{D}$ and $\vec{E}$ for the line charge {Page 3}

Ans.

$$\vec{D} \quad (\text{or } \vec{D}_\rho) = 0.7961 \times 10^{-4} \hat{a}_\rho \frac{C}{m^2}$$

$$\vec{E} \quad (\text{or } \vec{E}_\rho) = 0.0215 \times 10^{8} \hat{a}_\rho \frac{N}{C}$$

Optional :

$$\vec{D} = 79.61 \times 10^{-6} \hat{a}_\rho \frac{C}{m^2} = 79.61 \hat{a}_\rho \frac{\mu C}{m^2}$$

$$\vec{E} = 2.15 \times 10^{6} \hat{a}_\rho N/C = 2.15 \hat{a}_\rho M N/C$$

Hw. 4: Find the electric flux density $\vec{D}$ and electric field intensity ($\vec{E}$) in the region about a uniform line charge of 8 nC/m lying along the z-axis in free space. What is the total flux leaving a 5 m length of this line charge?

Ans.: *- $\vec{D} = \frac{1.2738}{\rho} \times 10^{-9} \, \hat{a}_\rho$, C/m^2

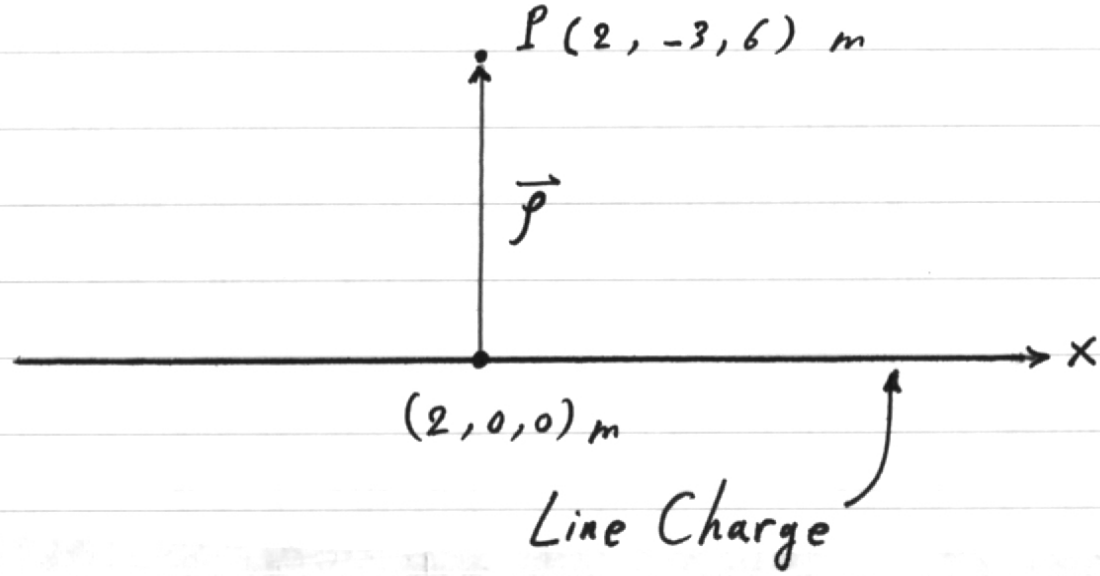
or, $\vec{D} = \frac{1.2738}{\rho} \, \hat{a}_\rho$, nC/m^2

*- $\Psi = 40 \; nC$.

Ex. 8: Calculate $\vec{D}$ in rectangular coordinates at point $P(2,-3,6)$ m produced by: (a) a point charge $Q = 55$ mC at $(-2,3,-6)$ m; (b) a uniform line charge $\rho_l = 20$ mC/m on the x-axis; (c): a uniform charge density $\rho_s = 120 \, \mu C/m^2$ on the plane $z = -5$ m.

S.l.: (a): Hw.
Ans: $\vec{D} = 6.38\,\hat{i} - 9.57\,\hat{j} + 19.14\,\hat{k}$ $\mu C/m^2$.

(b):



We have: $\vec{D}_\rho = \frac{\rho_l}{2\pi\rho}\hat{a}_\rho$

or: $\vec{D}_\rho = \frac{\rho_l}{2\pi\rho^2}\vec{\rho}$

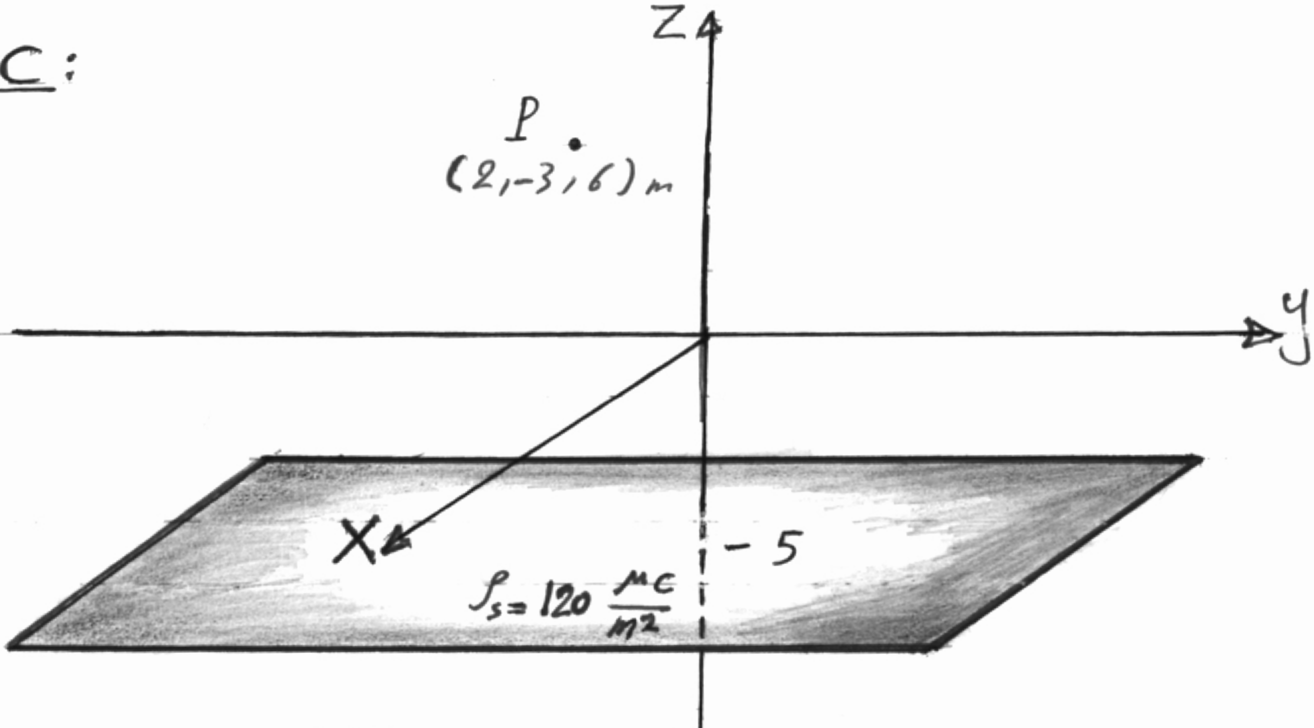
$\vec{\rho} = -3\hat{j} + 6\hat{k} \Rightarrow \rho = \sqrt{9+36} = \sqrt{45}$ m.

$$\vec{D}_\rho = \frac{20\times10^{-3}}{2\times3.14\times45}(-3\hat{j}+6\hat{k})$$

$\vec{D}_\rho = -0.000212\,\hat{j} + 0.000424\,\hat{k}$ C/m^2

or, $\vec{D}_\rho = -212\,\hat{j} + 424\,\hat{k}$ $\mu C/m^2$.

C:



Hw.. Ans. $\boxed{\vec{D} = \quad}$

H.W.5: Given the electric flux density:

$$\vec{D} = \frac{r}{\sin\theta}\,\hat{a}_r + \frac{6\sin\phi}{r}\,\hat{a}_\theta + r\sin\theta\,\hat{a}_\phi, \left(\frac{C}{m^2}\right) \text{ in}$$

free space.

(a): Find $\vec{D}$ at point $P(r = 2\,m,\ \theta = 30^\circ,\ \phi = 90^\circ)$.

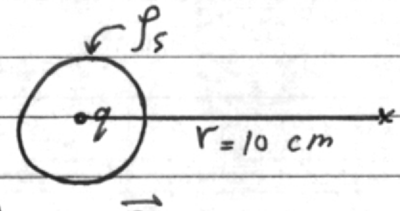
(b): If that flux is produced by a charge inside a sphere of radius 3m, find the amount of charge in the sphere.

Ans.:

(a): $\vec{D}_P = 4\,\hat{a}_r + 3\,\hat{a}_\theta + \hat{a}_\phi, \left(\frac{C}{m^2}\right)$

(b): $Q = 532.4184$ C.

Hw. 6: A point charge $q = 3.72\ \mu C$ placed at the centre of a sphere of radius $a = 0.5$ cm. The surface of the sphere is charged with $\rho_s = 2\times10^{-6}\ C/m^2$. Determine the electric flux density $(\vec{D})$ at a point (P) located at a distance of 10 cm ($r = 10$ cm) from the centre of the sphere.



Hint:

* Determine $\vec{D}$ at (P) due to the point charge (q) : $\vec{D}_1$
* Determine $\vec{D}$ at (P) due to the charge distribution (ρ_s) : $\vec{D}_2$.
* Find the total flux density at (P): $\vec{D}_P = \vec{D}_1 + \vec{D}_2$

Ans.: $\vec{D}_P = 2961.5\times10^{-8}\ \hat{a}_r\ C/m^2$.
or, $\vec{D}_P = 29.615\ \mu C/m^2$.

Hw. 7: Determine the amount of charge enclosed inside a cylindrical volume of radius (4 m) and length (10 m) if the expression of the resulted electric flux density is given as:

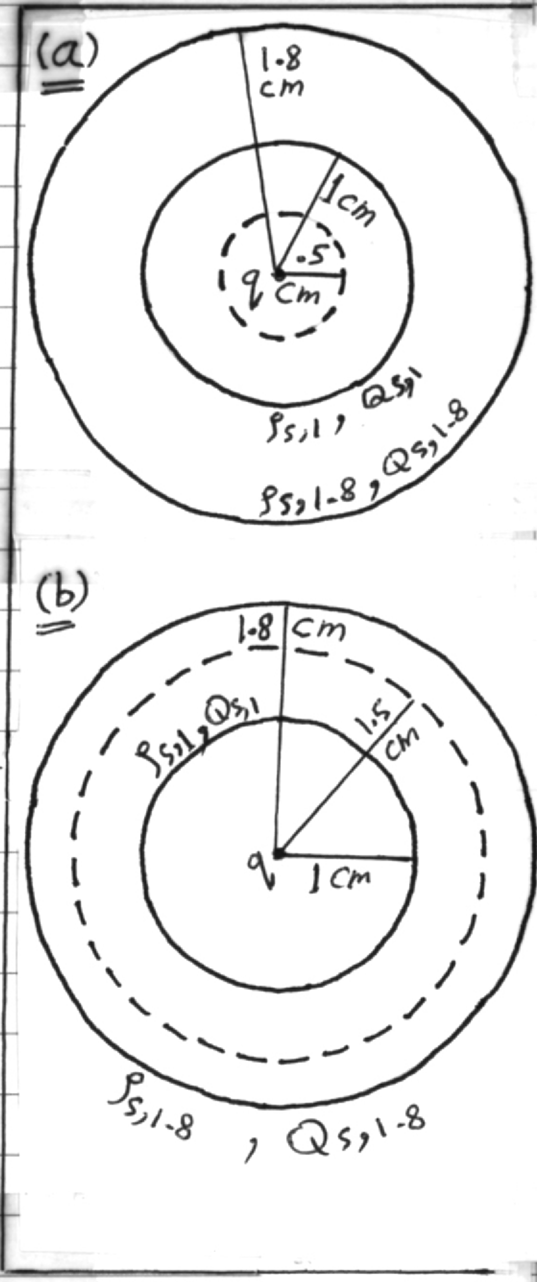
$$\vec{D} = \left(2\rho z - 2.19\, e^{0.7\phi}\right)\hat{a}_\rho + \frac{20}{z+100}\,\rho\, e^{0.25\phi}\,\hat{a}_z,$$

C/m^2.

Ans. $Q = 15.0017$ C.

Ex. 9:

A point charge of 0.25 μC is located at $r=0$, and uniform surface charge densities are located as follows: $2\,mC/m^2$ at $r=1$ cm, and $-0.6\,mC/m^2$ at $r=1.8$ cm.
Calculate $\vec{D}$ at: (a) $r=0.5$ cm; (b) $r=1.5$ cm; (c) $r=2.5$ cm. (d) What uniform surface charge density should be established at $r=3$ cm to cause $\vec{D}=0$ at $r=3.5$ cm?



Sol.:

(a): $\vec{D}_{.5} = \dfrac{q}{4\pi r^2}\,\hat{a}_r$

$= \dfrac{0.25\times10^{-6}}{4\times3.14\times(0.5)^2\times10^{-4}}\,\hat{a}_r$

$= \dfrac{0.25\times10^{-2}}{3.14}\,\hat{a}_r$

$\vec{D}_{.5} = 0.07961\times10^{-2}\,\hat{a}_r\,C/m^2$

or, $\boxed{\vec{D}_{.5} = 796.1\,\hat{a}_r\,\mu C/m^2}$

(b): $\vec{D}_{1.5} = \dfrac{Q}{4\pi r^2}\,\hat{a}_r$

$Q = q + Q_{s,1}$

$Q_{s,1} = \rho_{s,1}\times S$

$Q_{s,1} = 2\times10^{-3}\times[4\times3.14\times1^2\times10^{-4}]$

$Q_{s,1} = 25.12\times10^{-7}\,C$

$$Q = 0.25\times10^{-6} + 25.12\times10^{-7}$$
$$= 0.25\times10^{-6} + 2.512\times10^{-6} = 2.762\times10^{-6}\ C.$$

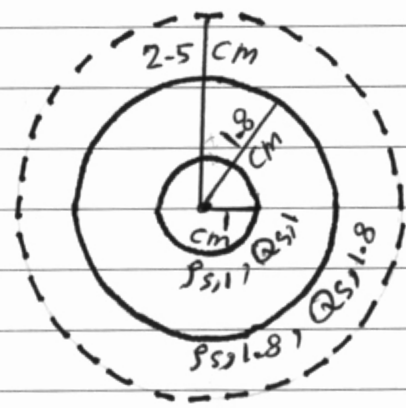
$$\therefore \vec{D}_{1.5} = \frac{2.762\times10^{-6}}{4\times3.14\times(1.5)^2\times10^{-4}}\ \hat{a}_r$$

$$= \frac{2.762\times10^{-2}}{28.26}\ \hat{a}_r = 0.0977\times10^{-2}\ \hat{a}_r\ C/m^2$$

$$\vec{D}_{1.5} = 0.0977\times10^{-2}\ \hat{a}_r\ C/m^2$$

or, $\boxed{\vec{D}_{1.5} = 977\ \hat{a}_r\ \mu C/m^2}$.

(c): $\vec{D}_{2.5} = \dfrac{Q}{4\pi r^2}\ \hat{a}_r$



$$Q = q + Q_{s,1} + Q_{s,1.8}$$

We have that:

$$q + Q_{s,1} = 2.762\times10^{-6}\ C.$$

$$Q_{s,1.8} = \rho_{s,1.8}\times S = -0.6\times10^{-3}\times4\times3.14\times(1.8)^2\times10^{-4}$$

$$Q_{s,1.8} = -24.4166\times10^{-7}\ C$$

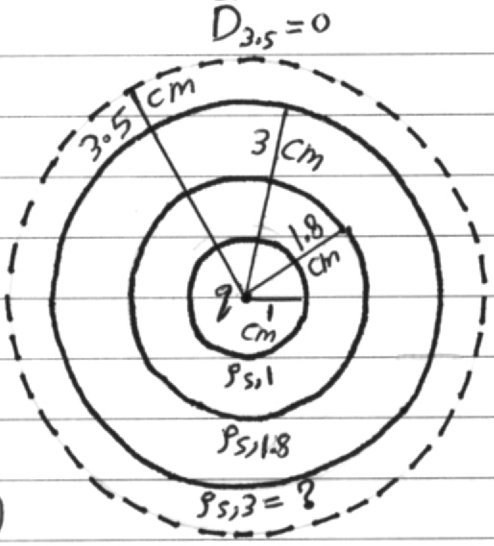
$$\therefore Q = 2.762\times10^{-6} - 2.4416\times10^{-6} = 0.3204\times10^{-6}\ C.$$

$$\vec{D}_{2.5} = \frac{0.3204\times10^{-6}\ \hat{a}_r}{4\times3.14\times(2.5)^2\times10^{-4}} = 0.00408\times10^{-2}\ \hat{a}_r\ \frac{C}{m^2}$$

or, $\boxed{\vec{D}_{2.5} = 40.8\ \hat{a}_r\ \mu C/m^2}$

(d):

*- To make $\vec{D}$ at the surface of $r = 3.5$ cm equals zero, the net charge enclosed by that surface must be zero. Hence, we must put:



$$Q_{s,3} = -(q + Q_{s,1} + Q_{s,1.8})$$

$$Q_{s,3} = -0.3204 \times 10^{-6} \text{ C.} \quad \{\text{from part (c)}\}.$$

$$\rho_{s,3} = \frac{Q_{s,3}}{S_{r=3}} = \frac{-0.3204 \times 10^{-6}}{4 \times 3.14 \times 3^2 \times 10^{-4}} = -0.002834 \times 10^{-2} \frac{C}{m^2}$$

$$\rho_{s,3} = -0.002834 \times 10^{-2} \text{ C/m}^2 .$$

or, $\boxed{\rho_{s,3} = -28.34 \ \mu\text{C/m}^2}$.

*- Comment on part (d):

To make $\vec{D} = 0$ on a closed surface enclosing electric charges, the net enclosed charge must be zero, according to Gauss's law:

$$\oint \vec{D} \cdot \vec{ds} = Q_{enclosed}$$

when $\oint \vec{D} \cdot \vec{ds} = 0$, Then $\vec{D} = 0$.