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هندسة اتصالات
المرحلة الرابعة



Antenna

2017 – 2018

الهوائيات

P-2

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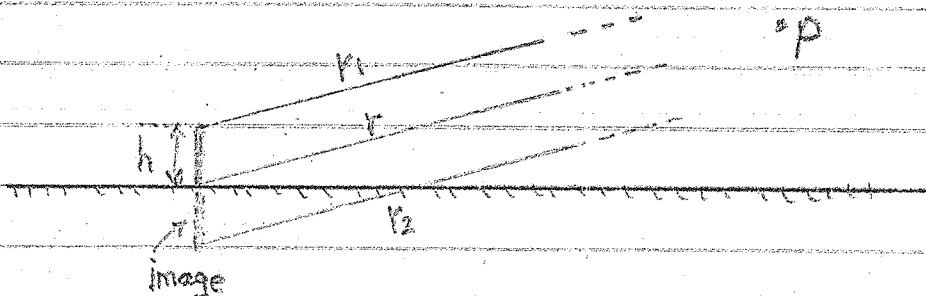
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Vertical dipole over perfectly conducting ground

1) Infinitesimal dipole (Hertzian dipole)



The radiation power decreased because integration from $\theta=0$ to $\pi/2$.

$$P_{\text{rad}} = \iint_S \mathbf{S}_{\text{av}} \cdot d\mathbf{s} = \frac{1}{2} \iint_S \text{Re} \mathbf{E}_\theta \mathbf{H}_\phi^* \cdot d\mathbf{s} = \frac{1}{2} \eta \iint_S |\mathbf{H}_\phi|^2 \cdot d\mathbf{s}$$

$$|\mathbf{H}_\phi| = \frac{\beta I_0 L}{4\pi r} \sin\theta, \quad L = 2h$$

$$\begin{aligned} P_{\text{rad}} &= \frac{1}{2} \eta \int_0^{2\pi} \int_0^{\pi/2} |\mathbf{H}_\phi|^2 r^2 \sin\theta \, d\theta \, d\phi = \frac{1}{2} \eta \int_0^{2\pi} \int_0^{\pi/2} \left(\frac{\beta I_0 L}{4\pi r} \right)^2 r^2 \sin\theta \, d\theta \, d\phi \\ &= \frac{2\pi}{2} \eta \int_0^{\pi/2} \left(\frac{\beta I_0 L}{4\pi r} \right)^2 r^2 \sin^3\theta \, d\theta = \frac{2\pi \beta^2 I_0^2 L^2}{2 \times 16 \times \pi^2} \int_0^{\pi/2} \sin^3\theta \, d\theta \\ &= \frac{\beta^2 L^2 \times 4 I_0^2}{16 \pi \times 6} \eta \end{aligned}$$

$$P_{\text{rad}} = R_{\text{rad}} \frac{I_0^2}{2}$$

$$R_{\text{rad}} = \frac{2 P_{\text{rad}}}{I_0^2} = \frac{2 \times \beta^2 L^2 \times 4 I_0^2}{16 \times \pi \times 6} = \frac{(2\pi)^2 L^2 \times 120\pi}{12 \pi \lambda^2} = \frac{40 \pi^2 L^2}{\lambda^2} = 40 \pi^2 \left(\frac{L}{\lambda} \right)^2$$

R_{rad} decreased by "2"

$$L = 2h$$

The electric field of the infinitesimal dipole at observation point is given by :

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$$E = E_i + E_r = j\eta \frac{\beta I_0 h}{4\pi r_1} \sin\theta + j\eta \frac{\beta I_0 h \sin\theta}{4\pi r_2}$$

$$= j\eta \frac{\beta I_0 h}{2\pi r} \sin\theta = j120\pi \frac{\beta I_0 h}{2\pi r} \sin\theta$$

$$= j \frac{60 \beta I_0 h \sin\theta}{r} \quad r_1 \approx r_2 \approx r$$

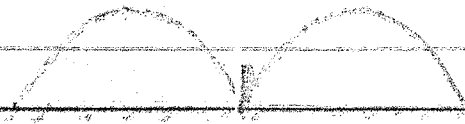
$E = E_i + R_v E_r$
 $R_v = 1$ - reflection coefficient

The Reflected Component E_r can be accounted for by the introduction of the virtual source (image), and it can be written as

$$E_r = j\eta \frac{\beta I_0 h}{4\pi r_2} \sin\theta = j120\pi \frac{\beta I_0 h \sin\theta}{4\pi r_2}$$

$$= j \frac{30 \beta I_0 h \sin\theta}{r_2}$$

The pattern of vertical infinitesimal (Hertzian) dipole shown in figure below (over perfectly conduction ground)



$\sigma = \infty$

ii)

In practice, a wide use has been made of a quarter wavelength monopole ($L = \lambda/4$) mounted above a ground plane.

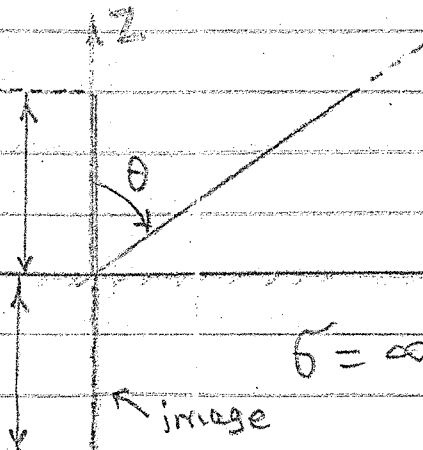
$$R_r (\text{monopole}) = \frac{1}{2} R_r (\text{dipole})$$

$$V_{in} (\text{monopole}) = \frac{1}{2} V_{in} (\text{dipole})$$

$$Z_{in} (\text{monopole}) = \frac{1}{2} Z_{in} (\text{dipole})$$

$$= \frac{1}{2} (73 + j42.5)$$

$$= 36.5 + j21.25 \Omega$$



$$0 < \theta \leq \frac{\pi}{2}$$

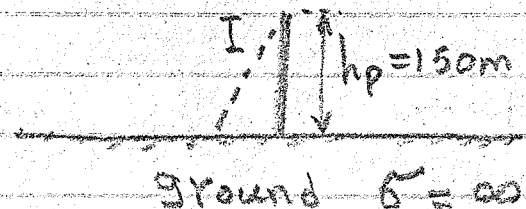
$\sigma = \infty$

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Example

Referring to figure below, a 100 kHz transmitter feeds a 150 m vertical antenna against ground. The loss resistance is 2Ω . Find ~~the effective~~

- The effective height
- The radiation resistance
- The radiation efficiency



Solution

$$a) \lambda = \frac{3 \times 10^8}{f} = \frac{3 \times 10^8}{100 \times 10^3} = 3000 \text{ m}$$

$$h_p = 0.05 \lambda$$

$$h_e = \frac{h_p}{2} = 0.025 \lambda$$

$$\frac{I_0 \cdot h_p}{2} = I_0 \cdot h_e$$

$$b) R_r = 80 \pi^2 \left(\frac{2h_e}{\lambda} \right)^2 / 2 = 40 \pi^2 \left(\frac{2h_e}{\lambda} \right)^2$$

$$= 790 \frac{4 h_e^2}{\lambda^2} / 2 = 395 \times 4 \frac{[0.025 \lambda]^2}{\lambda^2} \approx 1 \Omega$$

- Radiation efficiency

$$P_E = \frac{R_r}{R_l + R_r} = \frac{1}{2 + 1} = \frac{1}{3} = 0.33 \text{ or } 33 \text{ percent}$$

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Example

Calculate the power radiated by $\lambda/16$ dipole in free space if it carries a uniform of ~~100~~ $I = 100 \cos \omega t$ amperes. What is its radiation resistance

Solution

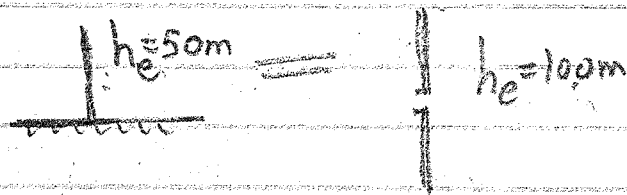
$$P_r = 40 \pi^2 \left(\frac{dL}{\lambda} \right)^2 I_0^2 = 40 \pi^2 \left(\frac{\lambda}{16 \times \lambda} \right)^2 \times 100^2$$

$$= 4 \times 10^5 \times \pi^2 \left(\frac{1}{16} \right)^2 = 15.42 \text{ kW}$$

$$R_r = 80 \pi^2 \left(\frac{\lambda}{16 \lambda} \right)^2 = 3.08 \Omega$$

Example

A short vertical grounded antenna is designed to radiate at 10 MHz. Calculate the radiation resistance if the effective height of antenna is 50 m.

Solution

$$\lambda = \frac{3 \times 10^8}{10 \times 10^6} = 30 \text{ m}$$

$$R_r = 80 \pi^2 \left(\frac{2h_e}{\lambda} \right)^2 / 2 = \frac{80 \pi^2}{2} \frac{100^2}{30^2} = 40 \pi^2 \frac{10^4}{900}$$

$$= 40 \pi^2 \frac{10^2}{9} = 4.386 \text{ k}\Omega$$

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Example

Define radiation resistance. Calculate the radiation resistance and efficiency of a current element (constant current along dipole) whose overall length is $\frac{\lambda}{50}$ and loss resistance is 1.5Ω .

Solution

Radiation resistance is the fictitious resistance which when connected in series with antenna will consume the same amount of power as when actually radiating.

$$R_r = 80 \pi^2 \left(\frac{dL}{\lambda} \right)^2 = 80 \pi^2 \left(\frac{\lambda}{50 \lambda} \right)^2 = 0.3158 \Omega$$

$$\text{efficiency} = \frac{R_r}{R_r + R_L} = \frac{0.3158}{0.3158 + 1.5} = 0.1739 \text{ or } 17.4 \text{ percent}$$

(58)

Impedance matching of Antennas

Impedance matching between a transmission line and antenna may be accomplished in various ways.

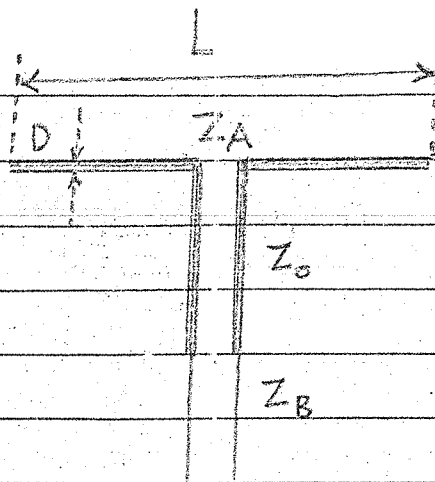
Suppose that the antenna is a cylindrical dipole with a length-diameter ratio of 60 ($L/D = 60$) and that the measured terminal impedance at 5 frequencies are as follows:

Frequency	Antenna Length, L	Terminal impedance, Ω
$1.15 F_0$	$L = 0.53$	$110 + j90$
$1.07 F_0$	$L = 0.49$	$80 + j40$
F_0 - center frequency	$L = 0.46$	$65 + j0$
$0.93 F_0$	$L = 0.43$	$52 - j40$
$0.85 F_0$	$L = 0.39$	$40 - j100$

The center frequency F_0 corresponds to the resonant frequency of the antenna. At this frequency the terminal impedance is $65 + j0 \Omega$.

The dipole antenna may be energized with a 2-wire open type of transmission line. Since the characteristic impedance of convenient sizes of open 2-wire line is in the range of 200 to 600 Ω , an impedance transformer is required between the line and the antenna. Referring to figure below,

(59)



$$Z_B = \frac{Z_A + jZ_0 \tan \beta x}{Z_0 + jZ_A \tan \beta x}$$

Z_A - input impedance of antenna

Z_0 - characteristic impedance of transformer

Z_B - input impedance of transformer = characteristic impedance of transmission line.

When the transformer is $\frac{1}{4}$ Long ($\beta x = \frac{2\pi}{\lambda} \frac{\lambda}{4} = 90^\circ$), last equation reduces to

$$Z_B = \frac{Z_0^2}{Z_A}$$

$$Z_0 = \sqrt{Z_B Z_A} \quad , \quad \text{for example } Z_B = 600 \Omega, Z_A = 65 \Omega$$

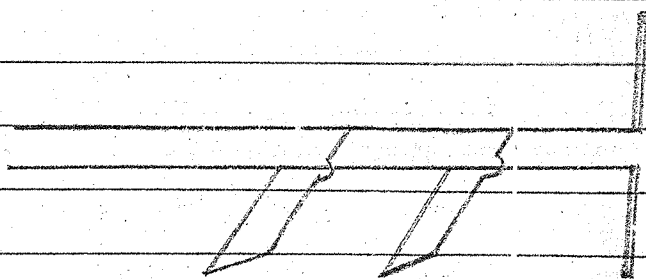
$$Z_0 = \sqrt{600 \times 65} = 197.5 \Omega$$

This type of transformer gives a perfect match at only the center frequency.

Two sections may be connected in series. Each is $\frac{\lambda}{4}$ Long at the center frequency. Also this antenna is perfectly matched with line at only the center frequency. However, this 2-section arrangement is less frequency sensitive than the single section.

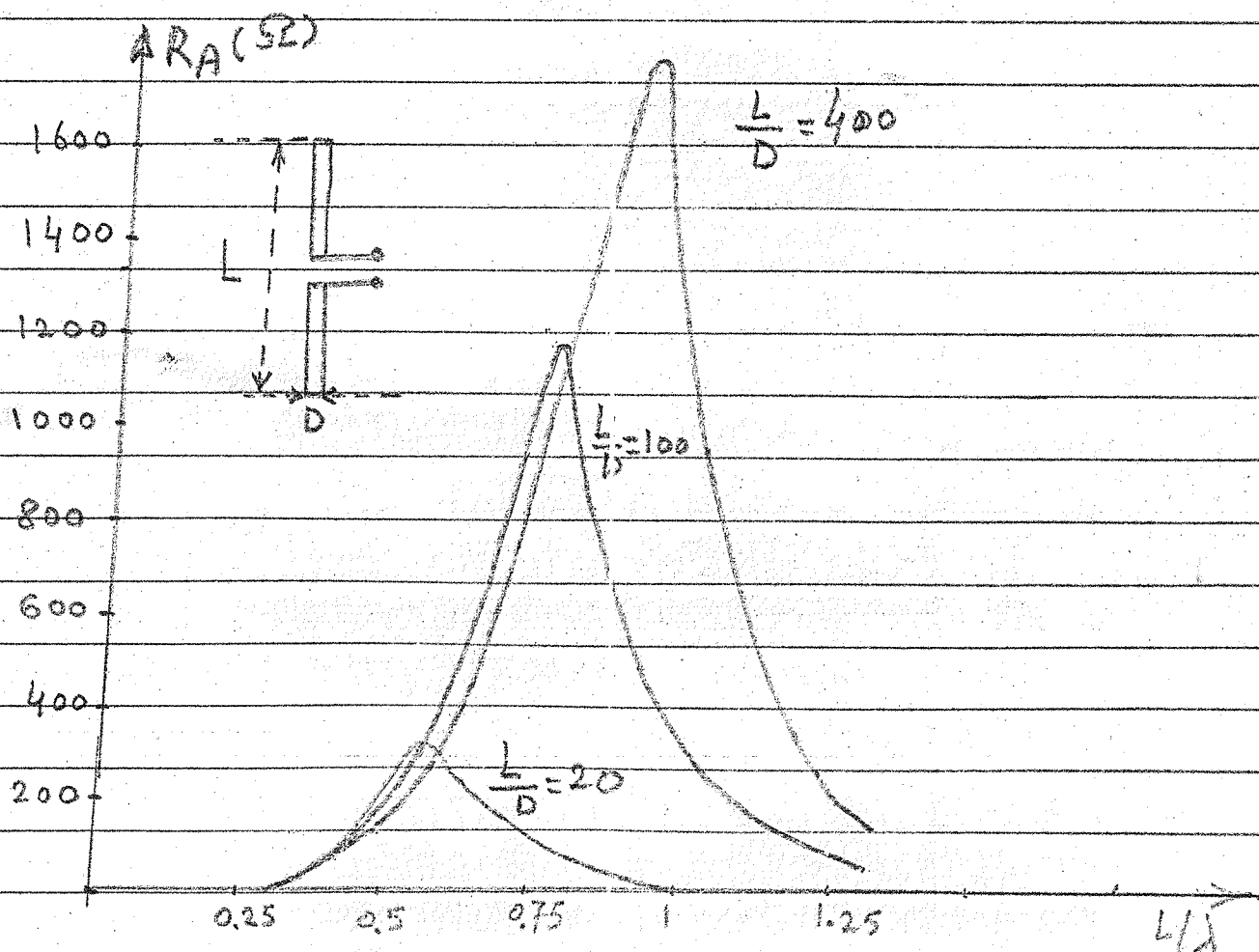
Another more frequency-sensitive method of matching a ~~stub~~ line to a $\frac{\lambda}{2}$ dipole is with stub. [one or two stub]

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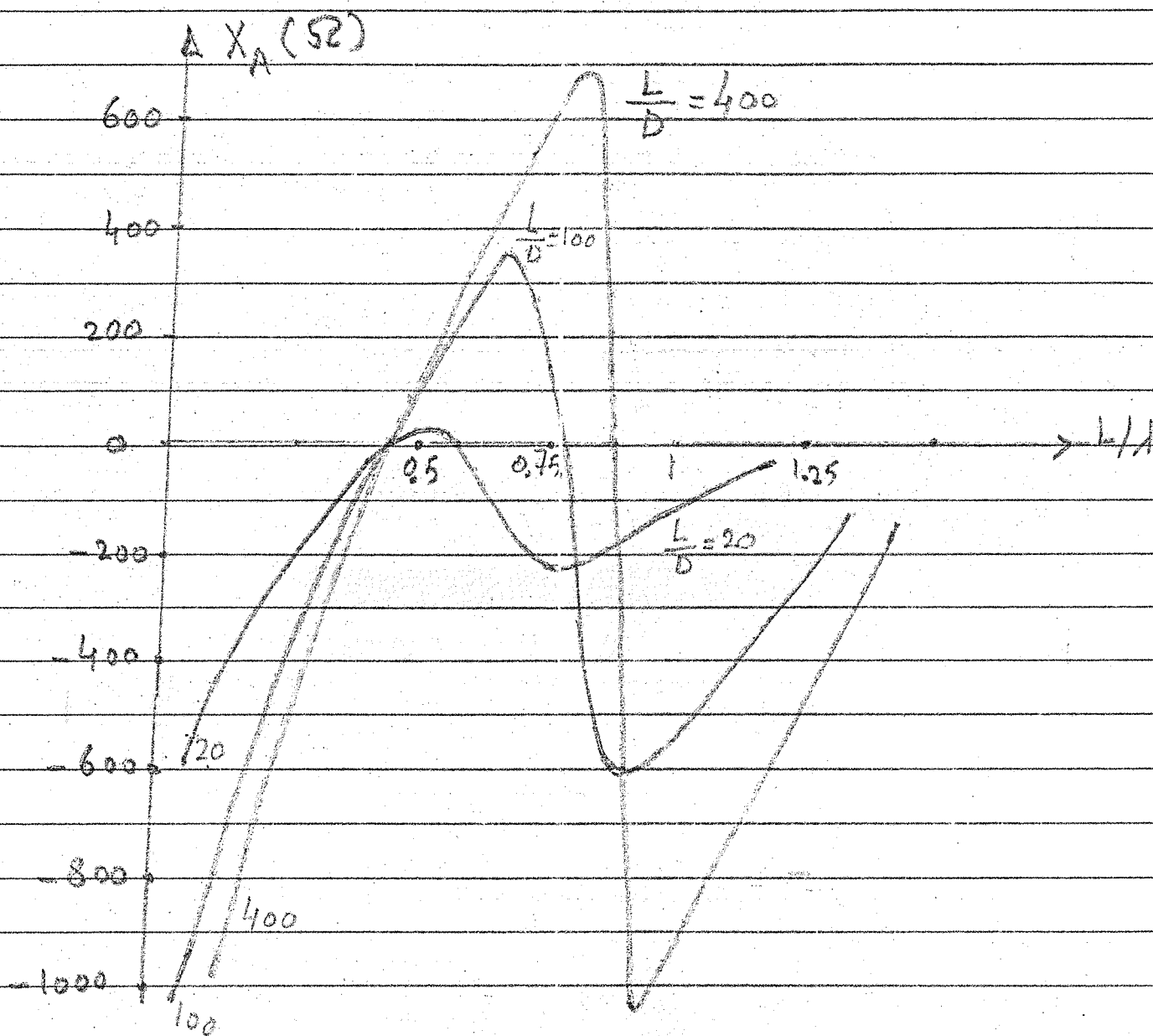


The impedance variation with frequency is inherently less for thick dipoles as compared to thin dipoles. The thick dipole is desirable for very wideband applications.

The figure below shows the measurement & results for the input impedance of a dipole are given.



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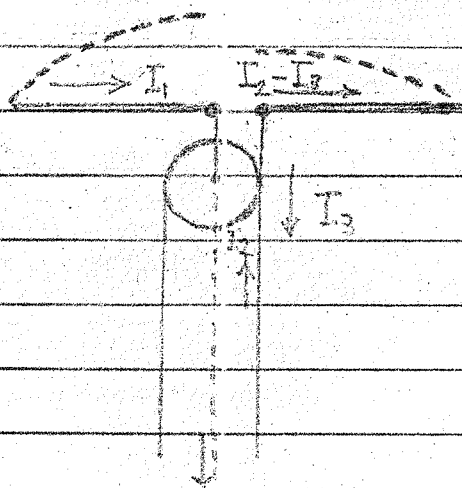


R_A - input resistance of dipole

X_A - input reactance of dipole

Balanced-to-unbalanced feed

If a $\lambda/2$ dipole antenna is fed by a single coaxial line, current can flow back along the outside of the coaxial cable making the current distribution on the dipole unbalanced. In this case the



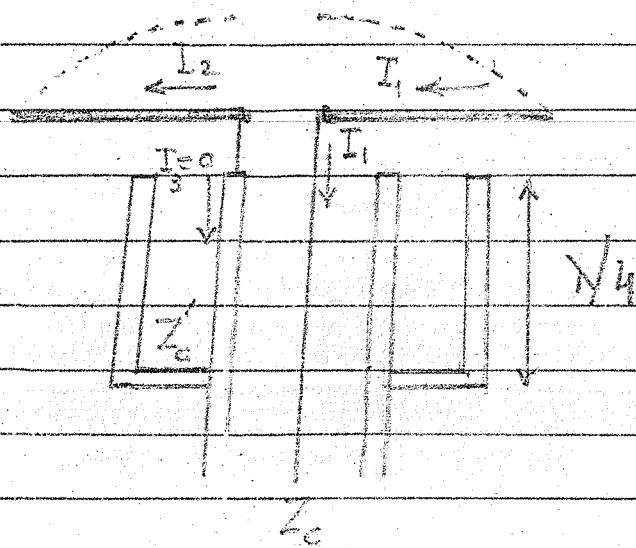
~~Pattern may be deformed and the surface of the coaxial cable screen is radiated. To balance the currents, various devices are used, called baluns (balance-to-unbalanced transformer).~~

pattern may deformed and the surface of the coaxial cable screen is radiated. To balance the currents, various devices are used, called baluns (balance-to-unbalanced transformer).

Sleeve balun 1:1

The sleeve and the outer conductor of the coax form another coax line, which has a characteristic impedance of Z_c' . This line is shorted quarter-wavelength away from the antenna input terminals. $I_3 = 0$ because the input impedance is infinity

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This is a narrowband balun, which does not have impedance transform capability (1:1 balun). It is not very easy to construct.

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Antenna temperature

Every object with a physical temperature above absolute zero ($0\text{ K} = -273^\circ\text{C}$) radiates energy. The amount of energy radiated is usually represented by an equivalent temperature T_B , better known as brightness temperature, and it is defined as

$$T_B(\theta, \varphi) = \epsilon(\theta, \varphi) T_m$$

Where

T_B = brightness temperature (equivalent temperature; K)

ϵ = emissivity (dimensionless)

T_m = molecular (physical) temperature (K)

$$0 \leq \epsilon \leq 1$$

Usually the emissivity is a function of the frequency of operation, polarization of emitted energy, and molecular structure of the object. Some of the better natural emitters of energy at microwave frequencies are (a) the ground with equivalent temperature of about 300 K and (b) the sky, with equivalent temperature of about 5 K when looking toward zenith and about 100–150 K toward the horizon.

The brightness temperature emitted by the different sources is intercepted by antenna, and it appears at their terminals as an antenna temperature.

The antenna temperature (T_A) is

$$T_A = \frac{1}{\Omega_A} \int_0^\pi \int_0^{2\pi} T_B(\theta, \varphi) P_n(\theta, \varphi) d\Omega \quad (\text{K}) \quad (*)$$

$$= \frac{D_0}{4\pi} \int_0^\pi \int_0^{2\pi} T_B(\theta, \varphi) P_n(\theta, \varphi) d\Omega \quad (\text{K})$$

Where

Ω_A - antenna beam solid angle, sr

$P_n(\theta, \phi)$ - normalized antenna power pattern, dimensionless

$d\Omega = \sin\theta d\theta d\phi$ - infinitesimal element of solid angle, sr

$T_B(\theta, \phi)$ - brightness temperature of source or sources as a function of angle, K

D_0 - directivity in maximum direction of maximum pattern.

Assuming no losses or other contributions between the antenna and the receiver, the noise power transferred to the receiver is given by

$$P_r = k T_A \Delta f,$$

where

P_r = antenna noise power (W)

k = Boltzmann's constant ($1.38 \times 10^{-23} \frac{W}{Hz \cdot K}$)

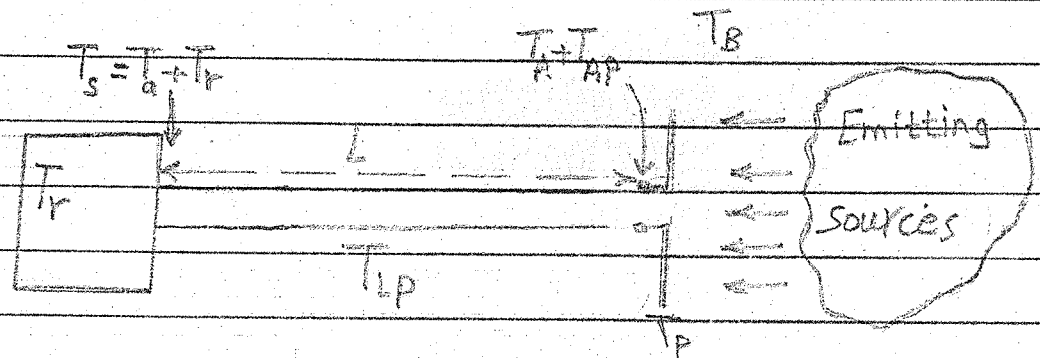
T_A = antenna temperature (K)

Δf = bandwidth (Hz)

If the antenna and transmission line are maintained at certain physical temperatures, and the transmission line between the antenna and receiver is lossy, the antenna temperature T_A must be modified to include the other contributions and the line losses. If the antenna itself is maintained at certain physical temperature T_p and a transmission line of length L ,

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Constant physical temperature T_0 throughout its length and uniform attenuation of α (NP/unit length) is used to connect an antenna to a receiver, as shown in figure:



The effective antenna temperature at the receiver terminals is given by

$$T_a = T_A e^{-2\alpha L} + T_{AP} e^{-2\alpha L} + T_0 \left[\frac{1}{e_L} - 1 \right] e^{-2\alpha L}$$

where

$$T_{AP} = \left[\frac{1}{e_A} - 1 \right] T_P$$

~~$$e_L = \frac{1}{e^{-2\alpha L}}$$~~

$$e_L = \frac{1}{e^{-2\alpha L}}$$

$$T_a = T_A e^{-2\alpha L} + T_{AP} e^{-2\alpha L} + T_0 (1 - e^{-2\alpha L})$$

T_a = antenna temperature at the receiver terminals (K)

T_A = antenna temperature at the antenna terminals given by formula (*) (K)

T_{AP} = antenna temperature at the antenna terminals due to Physical temperature (K)

T_P = antenna physical temperature (K)

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gular waveguide of 10m (Loss of waveguide = 0.13 dB/m)
and at a temperature of 300K, Find the effective
antenna temperature at the receiver terminals.

Solution : $T_p = 300\text{K}$, $T_o = 300\text{K}$

$$\alpha (\text{NP/m}) = \alpha (\text{dB/m}) / 8.68 = \frac{0.13}{8.68} = 0.0149$$

$$T_{Ap} = T_p \left[\frac{1}{e_A} - 1 \right] = 300 \left[\frac{1}{0.99} - 1 \right] = 3.03$$

$$T_a = T_A e^{-2\alpha L} + T_{Ap} e^{-2\alpha L} + T_o (1 - e^{-2\alpha L})$$
$$= 150 e^{-2 \times 10 \times 0.0149} + 3.03 e^{-2 \times 10 \times 0.0149} + 300 [1 - e^{-2 \times 10 \times 0.0149}]$$

$$= 111.345 + 2.249 + 77.31 = 190.904 \text{ K}$$

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Example

An antenna having an effective temperature of 15°K is fed into a microwave amplifier that has an effective noise temperature of 20°K . Calculate the available noise power per unit bandwidth, at the input of microwave amplifier for this particular antenna temperature. Calculate the available noise power for a noise bandwidth of 4 MHz at the input of microwave amplifier.

Solution

$$T_A = 15^\circ\text{K}, T_n = 20^\circ\text{K}, \Delta f = 4 \times 10^6 \text{ Hz}$$

Hence $P_s = k(T_A + T_n) \Delta f$

$$1) \frac{P_s}{\Delta f} = k(T_A + T_n) = 1.38 \times 10^{-23} (15 + 20) = 48.3 \times 10^{-23} \text{ W/Hz}$$

2) The total available noise power for a noise bandwidth of 4 MHz is

$$P_s = 48.3 \times 10^{-23} \times 4 \times 10^6 = 193.2 \times 10^{-17} \text{ W}$$

Antenna Arrays

In many practical applications, it is necessary to design antennas with more energy radiated in some particular directions and less in other directions. This is the ^{desire} tantamount to requiring that the radiation pattern be concentrated in the direction of interest. This hardly achievable with a single antenna element. An antenna array is a group of radiating elements arranged so as to produce some particular radiation characteristics. It is practical and convenient that the array consists of identical elements but this is not fundamentally required. We shall consider the simplest case of a two-element array and extend our results to the more complicated, general case of an N -element array.

Consider an antenna consisting of two Hertzian dipoles placed in free space along the z -axis but oriented parallel to the x -axis as depicted in figure below. We assume that the dipole at $(0, 0, d/2)$ carries current $I_1 = I_0 \angle \psi/2$ and the one at $(0, 0, -d/2)$ carries current $I_2 = I_0 \angle \psi/2$ where ψ is the phase difference between the two currents. By varying the spacing d and phase difference ψ , the fields from the array can be made to interfere constructively (add) in certain directions of interest and interfere destructively (cancel) in other directions. The total electric field at point P is the vector sum of the fields due to the individual elements. If P is in the far field zone, we obtain the total electric field at P as,

$$E = E_1 + E_2 = \frac{j\eta I_0 R d L}{4\pi} \left[\cos\theta_1 \frac{e^{-j\beta r_1}}{r_1} e^{j\psi/2} \hat{a}_{\theta_1} + \cos\theta_2 \frac{e^{-j\beta r_2}}{r_2} e^{j\psi/2} \hat{a}_{\theta_2} \right]$$

Since P is far from the array, $\theta_1 = \theta_2 = \theta$ and $\hat{a}_{\theta_1} = \hat{a}_{\theta_2} = \hat{a}_{\theta}$.

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In the amplitude we can set $r \approx r_1 \approx r_2$ but in phase we use

$$r_1 \approx r - \frac{d}{2} \cos \theta$$

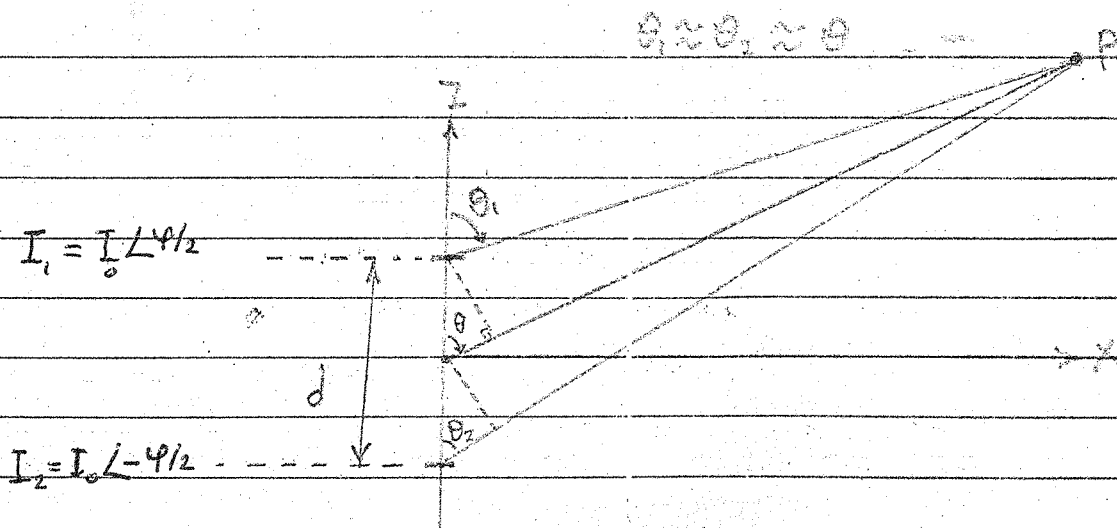
$$r_2 \approx r + \frac{d}{2} \cos \theta$$

$$E = \frac{j\eta \beta I_0 d L}{4\pi r} \cos \theta e^{-j\beta r} \left[e^{j\beta d \cos \theta / 2} e^{j\varphi/2} + e^{-j\beta d \cos \theta / 2} e^{-j\varphi/2} \right] \hat{a}_\theta$$

$$= \frac{j\eta \beta I_0 d L \cos \theta}{4\pi r} e^{-j\beta r} \cdot 2 \cos \left[\frac{1}{2} (\beta d \cos \theta + \varphi) \right] \hat{a}_\theta$$

$$AF = 2 \cos \left[\frac{1}{2} (\beta d \cos \theta + \varphi) \right]$$

AF - array factor



For second case: $I_1 = I_0 \angle \varphi$ and $I_2 = I_0 \angle 0$

$$E = \frac{j\eta \beta I_0 d L}{4\pi r} \cos \theta e^{-j\beta r} e^{j\varphi/2} \left[e^{j\beta d \cos \theta / 2} e^{j\varphi/2} + e^{-j\beta d \cos \theta / 2} e^{-j\varphi/2} \right] \hat{a}_\theta$$

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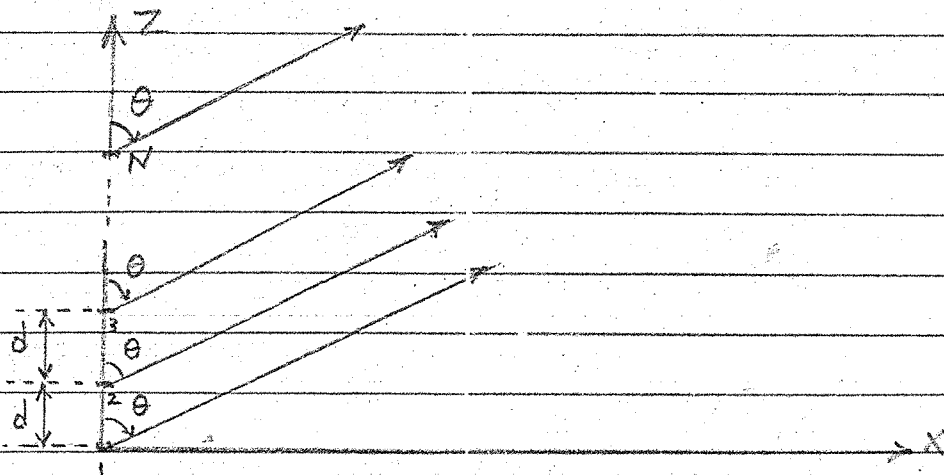
$$= \frac{j\omega B I_0 d L}{4\pi r} \cos\theta e^{-j\beta r} e^{j\phi/2} \times 2 \cos\left[\frac{1}{2}(\beta d \cos\theta + \phi)\right] \hat{a}_\theta$$

$$AF = 2 \cos\left[\frac{1}{2}(\beta d \cos\theta + \phi)\right] e^{j\phi/2}$$

Thus in general, the far field due to two-element array is given by

$$E(\text{total}) = [E \text{ due to single element at origin}] \times [\text{array factor}]$$

Let us now extend the results on the two-element array to the general case of an N -element array shown in figure below. We assume that the array is linear in that the elements are spaced equally along a straight line and lie along the Z -axis. Also we assume that the array is linear in that the elements are spaced equally along a straight line and lie along the Z -axis. Also we assume that the array is uniform so that each element is fed with current of the same magnitude but progressive phase shift ϕ ,



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The array factor is the sum of the contributions by all the elements, Thus,

$$AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{j(N-1)\psi}$$

$$\psi = \beta d \cos \theta + \psi$$

multiplying both sides of AF by $e^{j\psi}$

$$AF e^{j\psi} = e^{j\psi} + e^{j2\psi} + \dots + e^{j(N-1)\psi} + e^{jN\psi}$$

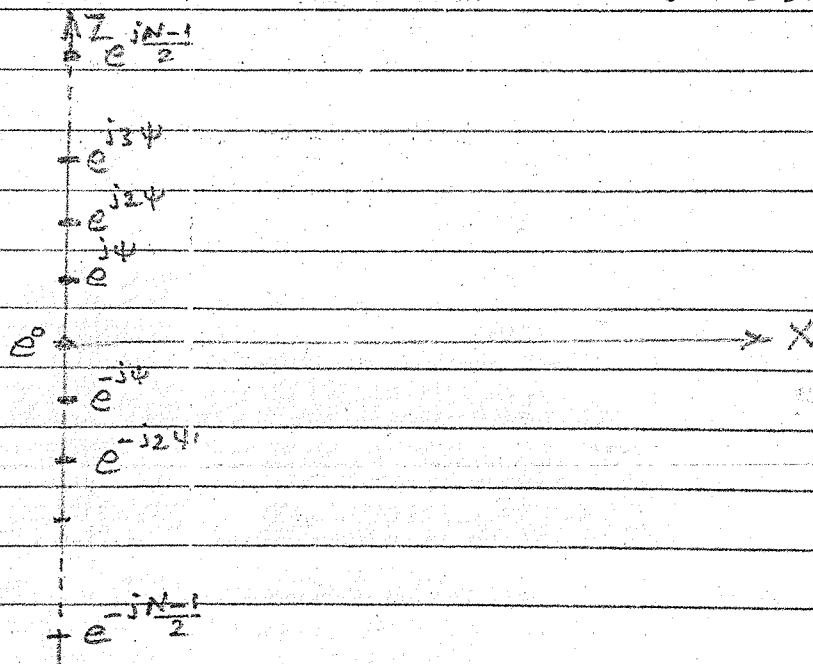
subtraction AF from $AF e^{j\psi}$ reduced to

$$AF(e^{j\psi} - 1) = e^{jN\psi} - 1$$

$$AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1} = \frac{e^{j\frac{N}{2}\psi} \left[\frac{e^{j\frac{N}{2}\psi} - e^{-j\frac{N}{2}\psi}}{e^{j\frac{1}{2}\psi} - e^{-j\frac{1}{2}\psi}} \right]}{e^{j\frac{N-1}{2}\psi} \frac{\sin \frac{N}{2}\psi}{\sin \frac{1}{2}\psi}}$$

The phase factor $e^{j\frac{N-1}{2}\psi}$ would not be present if

the array were centered about the origin as shown in figure.



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$$AF = \frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}}$$

$$\psi = \beta d \cos \theta + \varphi$$

For example $N=21$ and reference point is the physical center of array

$$AF = 1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j10\psi} + e^{-j\psi} + e^{-j2\psi} + \dots + e^{-j10\psi}$$

$$e^{j\psi} AF = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{j11\psi} + 1 + e^{-j\psi} + e^{-j2\psi} + \dots + e^{-j10\psi}$$

$$AF - e^{j\psi} AF = e^{-j10\psi} - e^{j11\psi}$$

$$AF(1 - e^{j\psi}) = e^{-j10\psi} - e^{j11\psi}$$

$$AF = \frac{e^{-j10\psi} - e^{j11\psi}}{1 - e^{j\psi}} = \frac{e^{-j10\psi} - e^{j11\psi}}{e^{j\psi/2} (e^{-j\psi/2} - e^{j\psi/2})}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$$

$$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$AF = \frac{e^{-j10\psi} - e^{j11\psi}}{1 - e^{j\psi}} = \frac{\sin 21\psi/2}{\sin \psi/2}$$

$$= \frac{\cos 10\psi - \cos 11\psi - j[\sin 10\psi + \sin 11\psi]}{\sin^2 \frac{\psi}{2} + \cos^2 \frac{\psi}{2} - \cos^2 \frac{\psi}{2} + \sin^2 \frac{\psi}{2} - j2 \sin \frac{\psi}{2} \cos \frac{\psi}{2}}$$

$$= \frac{-2 \sin \frac{10\psi + 11\psi}{2} \sin \frac{10\psi - 11\psi}{2} - j2 \sin \frac{10\psi + 11\psi}{2} \cos \frac{10\psi - 11\psi}{2}}{2 \sin^2 \frac{\psi}{2} - j2 \sin \frac{\psi}{2} \cos \frac{\psi}{2}}$$

$$= \frac{-2 \sin \frac{10\psi + 11\psi}{2} [\sin \frac{10\psi - 11\psi}{2} + j \cos \frac{10\psi - 11\psi}{2}]}{2 \sin \frac{\psi}{2} [\sin \frac{\psi}{2} - j \cos \frac{\psi}{2}]} = \frac{-2 \sin \frac{21\psi}{2} [-\sin \frac{\psi}{2} + j \cos \frac{\psi}{2}]}{2 \sin \frac{\psi}{2} [\sin \frac{\psi}{2} - j \cos \frac{\psi}{2}]}$$

$$= \frac{2 \sin \frac{21\psi}{2} [\sin \frac{\psi}{2} - j \cos \frac{\psi}{2}]}{2 \sin \frac{\psi}{2} [\sin \frac{\psi}{2} - j \cos \frac{\psi}{2}]} = \frac{2 \sin \frac{21\psi}{2}}{2 \sin \frac{\psi}{2}}$$

$$= \frac{\sin [21 \frac{\psi}{2}]}{\sin \frac{\psi}{2}}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

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Now we discuss the following cases:

* The maximum value of AF equal to N

$$AF_{\max} = \frac{\sin(N \frac{\psi}{2})}{\sin \frac{\psi}{2}} \approx N \quad \left\{ \psi - \text{small value} \right\}$$

* AF has nulls (or zeros) when $AF = 0$, that is

$$\frac{N\psi}{2} = \pm n\pi, \quad n = 1, 2, 3, \dots$$

* A broadside array has its maximum radiation directed normal to the axis of the array, that is, $\psi = 0^\circ$, $\theta = 90^\circ$
so that $\varphi = 0$

* An end-fire array has its maximum radiation directed along the axis of the array, that is, $\psi = 0$

$$\theta = \begin{cases} 0 \\ \pi \end{cases} \quad \text{so that } \varphi = \begin{cases} -\beta d \\ \beta d \end{cases}$$

Example

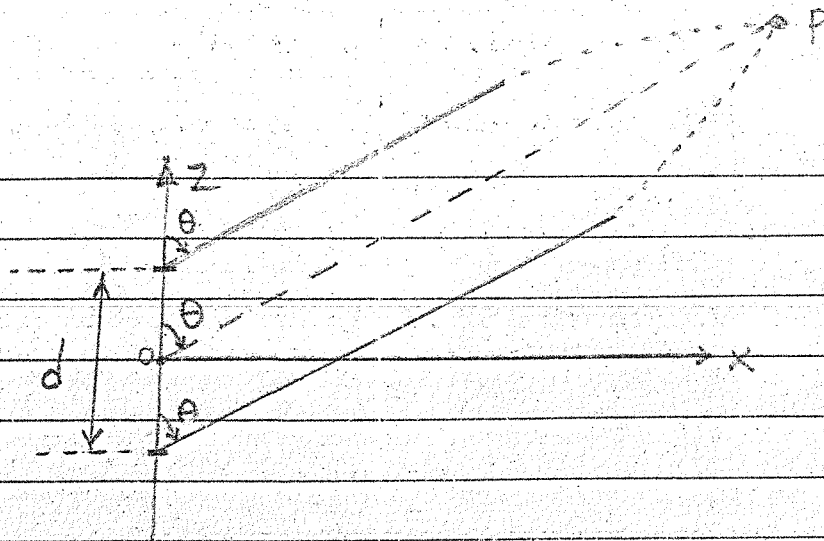
Hertzian.

For the two element antenna of figure below. Sketch the normalized field pattern when the current are:

a) Fed in phase ($\varphi = 0$), $d = \lambda/2$

b) Fed 90° out of phase ($\varphi = 90^\circ$), $d = \frac{\lambda}{4}$

75



Solution

$$AF = \frac{\sin \frac{N}{2} \psi}{\sin \frac{\psi}{2}} = \frac{\sin \frac{2}{2} \psi}{\sin \frac{\psi}{2}} = \frac{\sin \psi}{\sin \frac{\psi}{2}}$$

$$= \frac{2 \sin \frac{\psi}{2} \cos \frac{\psi}{2}}{\sin \frac{\psi}{2}} = 2 \cos \frac{\psi}{2}$$

Resultant pattern = Unit pattern \times AF

$$f(\theta) = \cos \theta \times 2 \cos \frac{\psi}{2}$$

$$= \cos \theta \times 2 \cos \left[\frac{1}{2} (\beta d \cos \theta + \psi) \right]$$

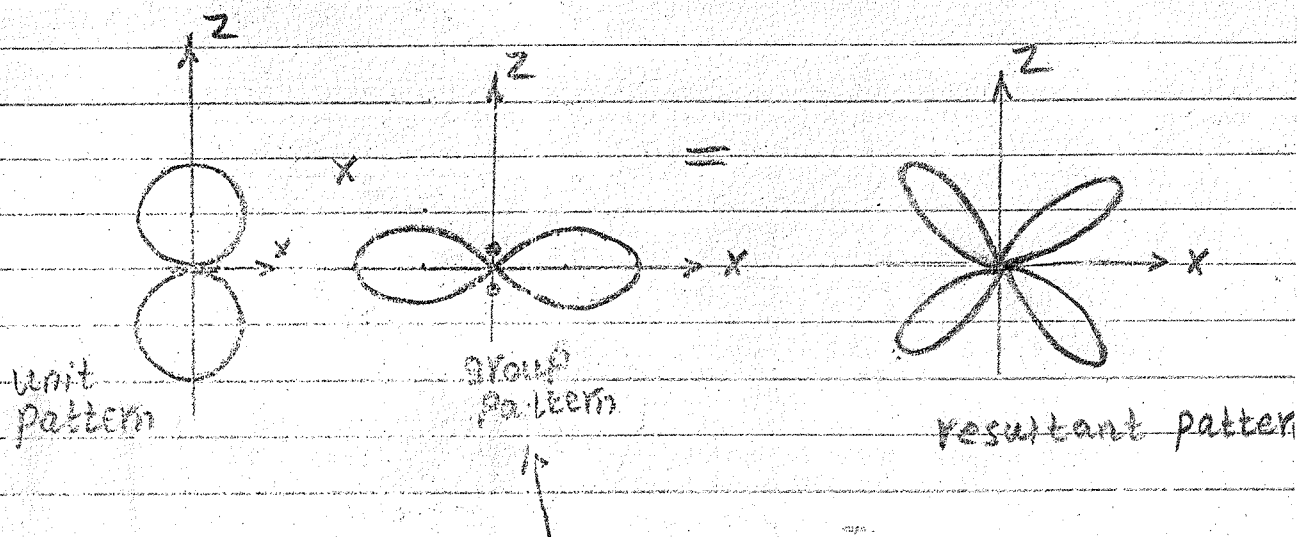
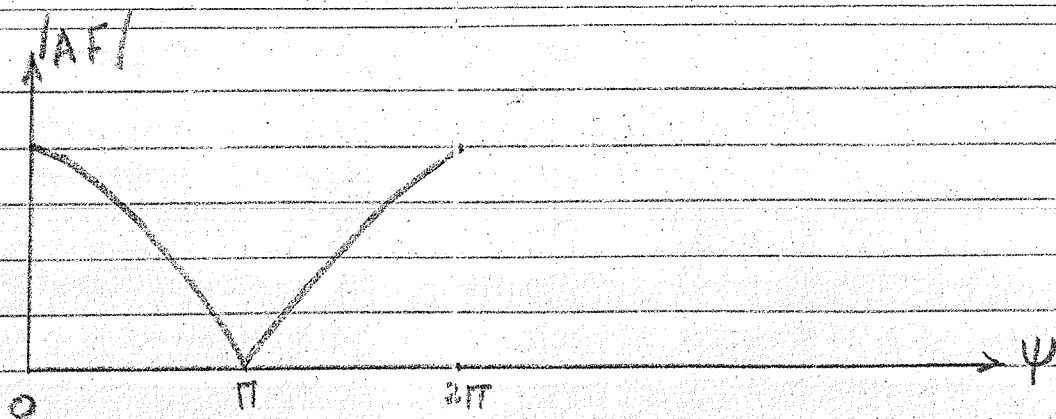
a) $\psi = 0$ and $d = \frac{\lambda}{2}$

$$f(\theta) = \cos \theta \times 2 \cos \left[\frac{1}{2} \left(\frac{2\pi}{\lambda} \frac{\lambda}{2} \cos \theta + 0 \right) \right]$$

$$= 2 \cos \theta \times \cos \left[\frac{\pi}{2} \cos \theta \right]$$

$$\frac{f(\theta)}{f(\theta)_{\max}} = \cos \theta \times \cos \left[\frac{\pi}{2} \cos \theta \right]$$

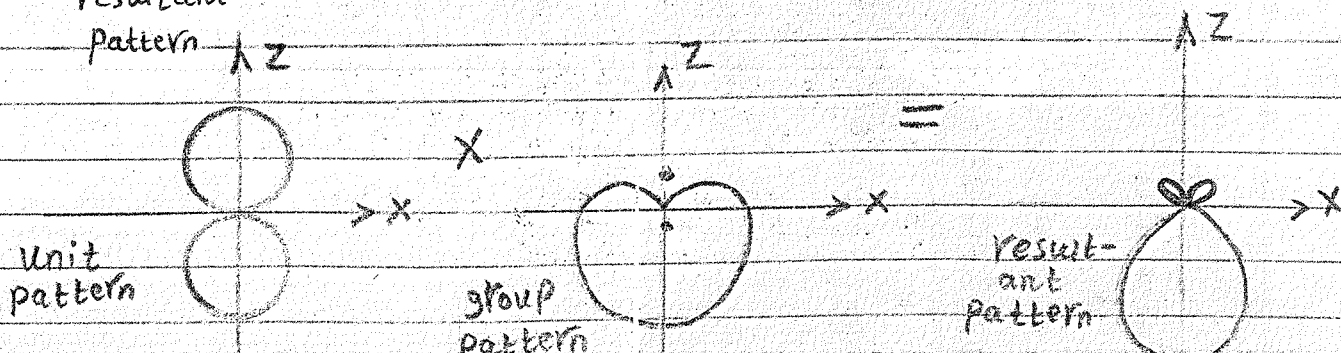
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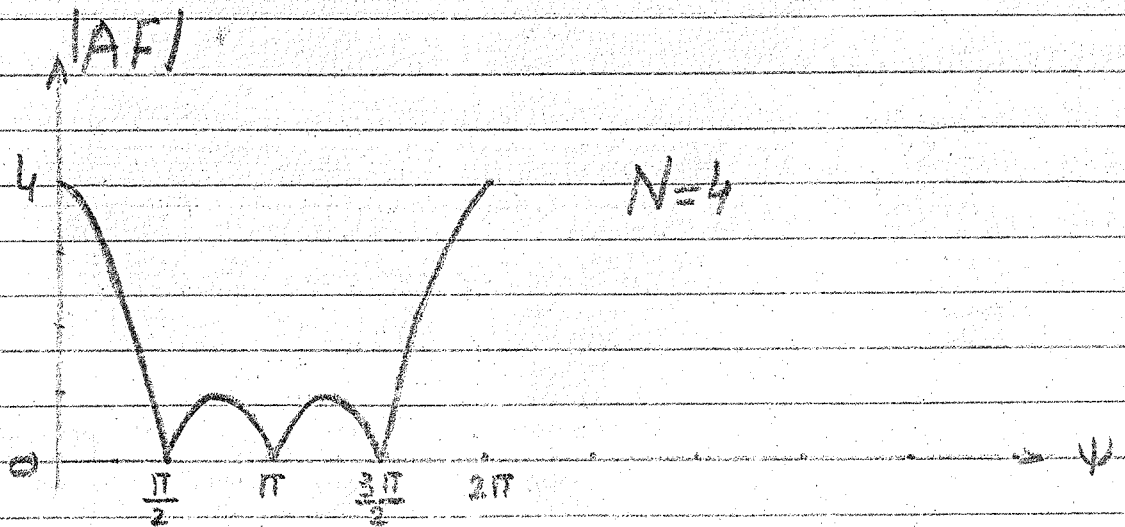
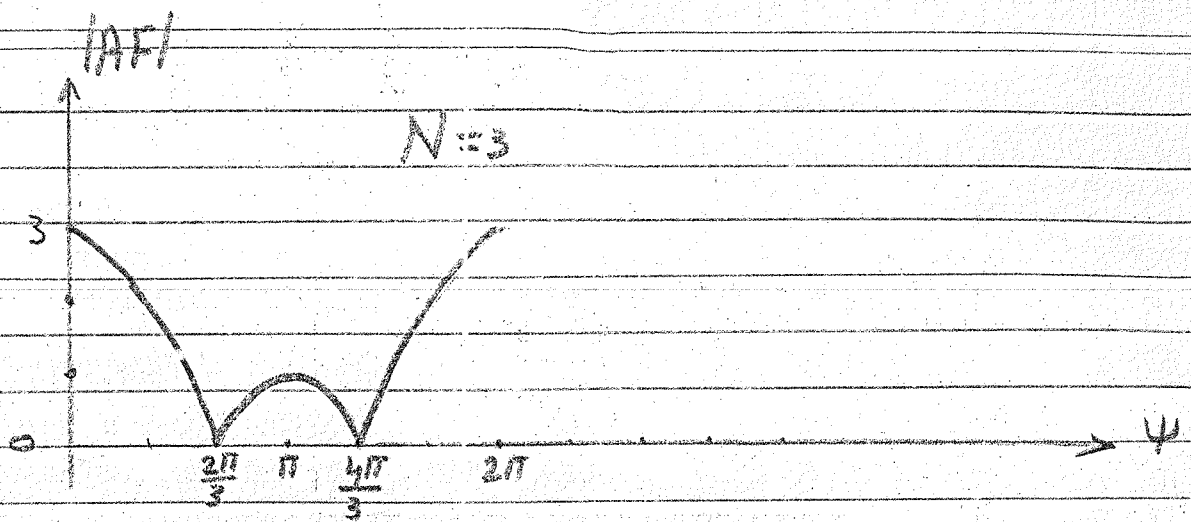
b) If $\psi = \frac{\pi}{2}$, $d = \frac{\lambda}{4}$, and $\beta d = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2}$

$$f(\theta) = \cos \theta \cos \left[\frac{\pi}{4} (\cos \theta + 1) \right]$$

↓ = unit pattern × group pattern
resultant pattern



(11)



$$N=3 \quad AF = \frac{\sin[N\frac{\psi}{2}]}{\sin\frac{\psi}{2}} = \frac{\sin[\frac{3}{2}\psi]}{\sin\psi/2}$$

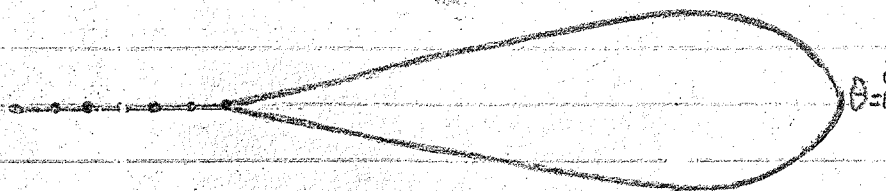
$$N=4 \quad AF = \frac{\sin[2\psi]}{\sin\frac{\psi}{2}} = \frac{\sin 2\psi}{\sin\psi/2}$$

End-Fire Array Antenna

Instead of having the maximum radiation broadside to the axis of the array, it may be desirable to direct it along the axis of the array (end-fire). As a matter of fact, it may be necessary that it radiates toward only one direction (either $\theta=0^\circ$ or 180°).

To direct the maximum toward $\theta=0^\circ$,

$$\psi = \beta d \cos \theta + \varphi \Big|_{\theta=0} = \beta d + \varphi = 0 \Rightarrow \varphi = -\beta d$$

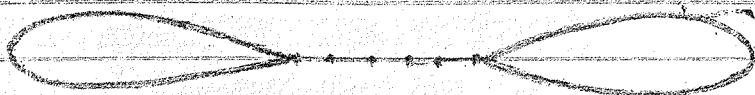


If the maximum is desired toward $\theta=180^\circ$, then

$$\psi = \beta d \cos \theta + \varphi \Big|_{\theta=180^\circ} = -\beta d + \varphi = 0 \Rightarrow \varphi = \beta d$$



If the element separation is $d = \lambda/2$, end fire radiation exists in both directions ($\theta=0^\circ$ and $\theta=180^\circ$).



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Planar Array Antenna

Planar array consist of radiators can be positioned along a rectangular grid to form a rectangular or planar array. planar array used to scan the main beam of the antenna toward any point in space.

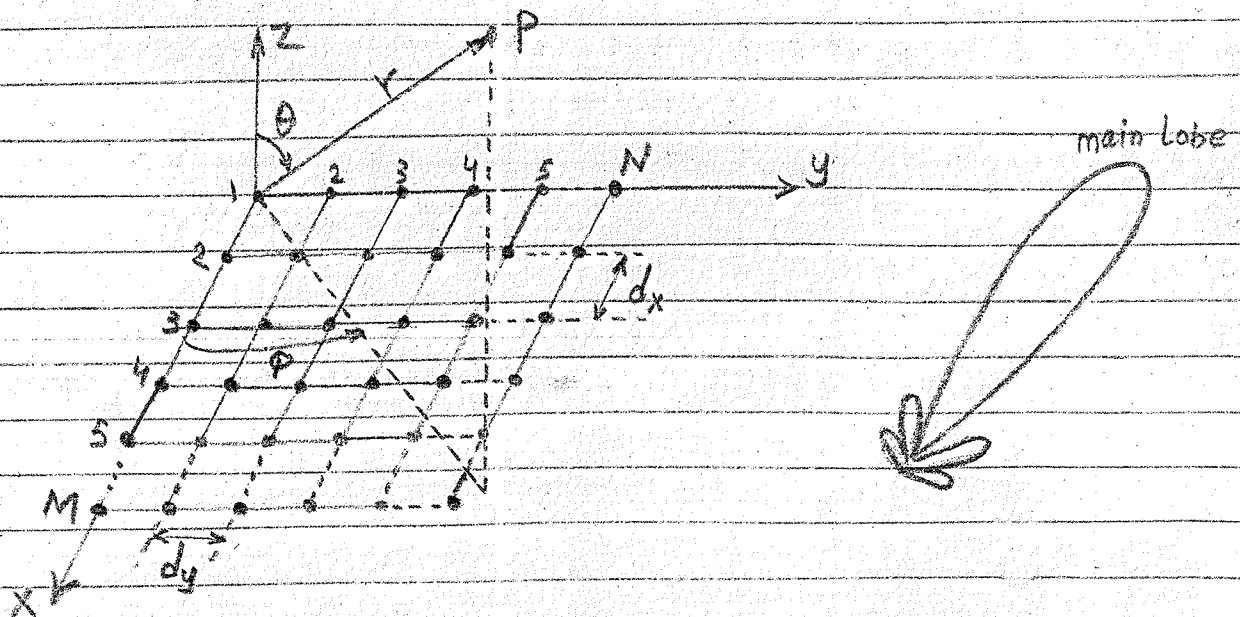
The normalized form of AF can be written as:

$$AF_n(\theta, \varphi) = \left\{ \frac{1}{M} \frac{\sin\left(\frac{M}{2} \psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1}{N} \frac{\sin\left(\frac{N}{2} \psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)} \right\}$$

$$\psi_x = \beta d_x \sin\theta \cos\varphi + \varphi_x$$

$$\psi_y = \beta d_y \sin\theta \sin\varphi + \varphi_y$$

When the spacing between elements is equal or greater than $\lambda/2$, multiple maxima of equal magnitude can be formed. The principal maximum is referred to as the major lobe and the remaining as the grating lobes. To avoid grating lobes in the X-Z and Y-Z planes, the spacing between the elements in the X- and Y- directions, respectively, must be less than $\lambda/2$ ($d_x < \frac{\lambda}{2}$ and $d_y < \frac{\lambda}{2}$)



(80)

For Rectangular array, the major Lobe and grating lobes are given by

$$\beta d_x \sin \theta \cos \varphi + \varphi_x = \pm 2m\pi \quad m = 0, 1, 2, \dots$$

$$\beta d_y \sin \theta \sin \varphi + \varphi_y = \pm 2n\pi \quad n = 0, 1, 2, \dots$$

$$AF = \sum_{xm} \sum_{ym} e^{j(m-1)(\beta d_x \sin \theta \cos \varphi + \varphi_x)} e^{j(n-1)(\beta d_y \sin \theta \sin \varphi + \varphi_y)}$$

where

$$\sum_{xm} = \sum_{m=1}^M k_{m1} e^{j(m-1)(\beta d_x \sin \theta \cos \varphi + \varphi_x)}$$

$$\sum_{yn} = \sum_{n=1}^N k_{n1} e^{j(n-1)(\beta d_y \sin \theta \sin \varphi + \varphi_y)}$$

k_{m1}, k_{n1} - excitation coefficients of each element

If $k_{m1} = k_{n1} = k = 1$:

$$\sum_{xm} = \sum_{m=1}^M e^{j(m-1)(\beta d_x \sin \theta \cos \varphi + \varphi_x)}$$

$$\sum_{yn} = \sum_{n=1}^N e^{j(n-1)(\beta d_y \sin \theta \sin \varphi + \varphi_y)}$$

The phases φ_x, φ_y are independent of each other, and they can be adjusted so that the main beam of \sum_{xm} is not the same as that of \sum_{yn} . However, in most practical applications it is required that the conical main beams of \sum_{xm} and \sum_{yn} intersect and their maxima be directed toward the same direction. If it is desired to have only one main beam that is desired along $\theta = \theta_0$ and $\varphi = \varphi_0$, the progressive phase shift between the element in the x - and y -direction must be equal to

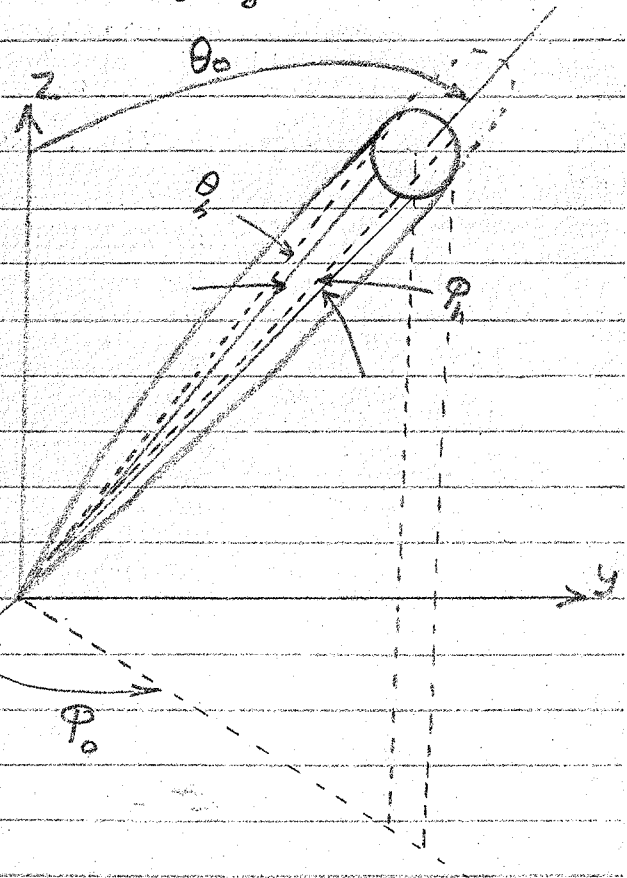
$$\varphi_x = -\beta d_x \sin \theta_0 \cos \varphi_0$$

$$\varphi_y = -\beta d_y \sin \theta_0 \sin \varphi_0$$

Beamwidth

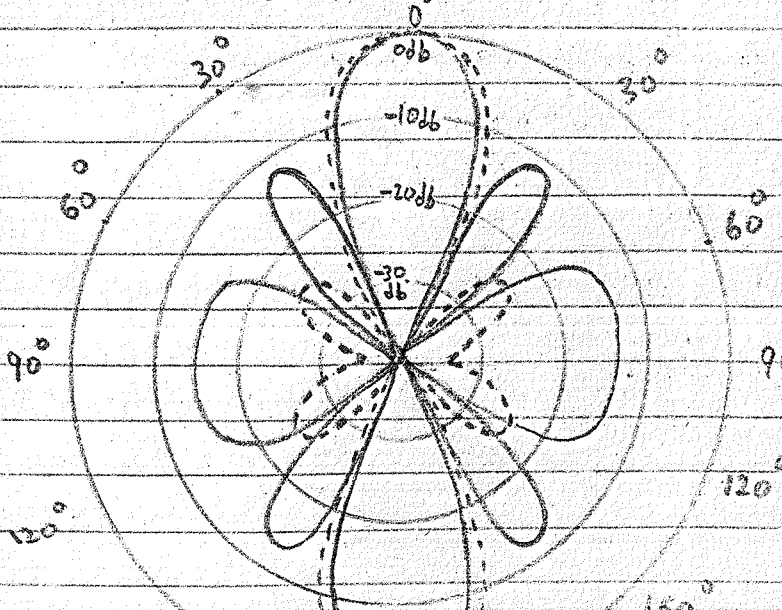
The maximum of the conical main beam of the array is assumed to be directed toward θ_0, ϕ_0 .

To define a beamwidth, two planes are chosen. One is the elevation plane defined by the angle $\phi = \phi_0$ and the other is a plane that is perpendicular to it. The corresponding half-power beamwidth of each is designated, respectively, by θ_h and ϕ_h . For example, if the array maximum is pointing along $\theta_0 = \frac{\pi}{2}$ and $\phi_0 = \frac{\pi}{2}$, θ_h represents the beamwidth in the $y-z$ plane and ϕ_h , the beamwidth in the $x-y$ plane.



Example

Sketch the two-dimensional antenna patterns of a planar array of isotropic elements with a spacing of $d_x = d_y = \lambda/2$, and equal amplitude and phase excitations.



— $\phi = 0^\circ$ (x-z plane)
 — $\phi = 90^\circ$ (y-z plane)
 - - - $\phi = 45^\circ$

$$d_x = d_y = \frac{\lambda}{2}$$

$$\phi_x = \phi_y = 0$$

$$M = N = 5$$

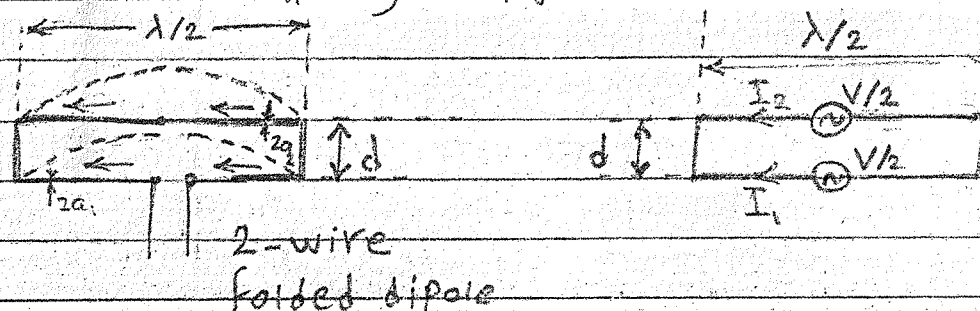
(94)

(82)

Folded Dipole Antenna

A simple $\lambda/2$ dipole has a terminal resistance of about 70Ω so that an impedance transformer is required to match this antenna to a 2-wire line of 300 to 600 Ω characteristic impedance. However, the terminal resistance of the modified $\lambda/2$ dipole (folded $\lambda/2$ dipole) is nearly 300 Ω so that it can be directly connected to a 2-wire line having a characteristic impedance of the same value. This "ultra closed-spaced type of array" is called a folded dipole.

The folded dipole is equivalent to two dipoles as shown in following figure.



Let the emf V applied to the antenna terminals be divided between the 2 dipoles:

$$V/2 = I_1 Z_{11} + I_2 Z_{12}$$

where I_1 = current at terminals of dipole 1

I_2 = current at terminals of dipole 2

Z_{11} = self-impedance of dipole 1

Z_{12} = mutual impedance of dipoles 1 and 2

95 83

$$\left. \begin{array}{l} I_1 = I_2 \\ Z_{11} = Z_{12} \end{array} \right\} \text{ because } d \ll \lambda$$

$$V = 2I_1 Z_{11} + 2I_2 Z_{12}$$

$$V = 4 I_1 Z_{11}$$

Thus, the terminal impedance Z of the antenna is given by

$$Z_A = \frac{V}{I_1} \approx 4 Z_{11}$$

$$Z_A = R_A + jX_A$$

$$R_A = R_{\text{rad}} + R_L \approx R_{\text{rad}}$$

$$Z_{11} = R_{11} + jX_{11}$$

Taking $R_{11} = 73 \Omega$ for $\frac{\lambda}{2}$ dipole

$$R_A = 292 \approx 300 \Omega$$

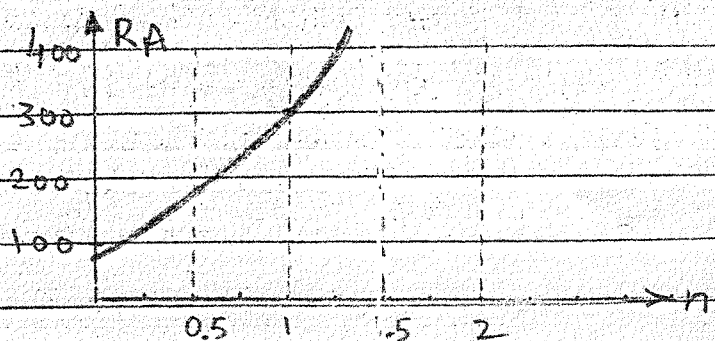
$$R_A = 4 \times 73 = 292 \Omega$$

If $a_1 \neq a_2$

$$R_A = \left(\frac{\log \frac{d}{a_1}}{\log \frac{d}{a_2}} + 1 \right) R_{11} = (n+1) R_{11}$$

Where a_1 = radius of first dipole conductor

a_2 = radius of second dipole conductor

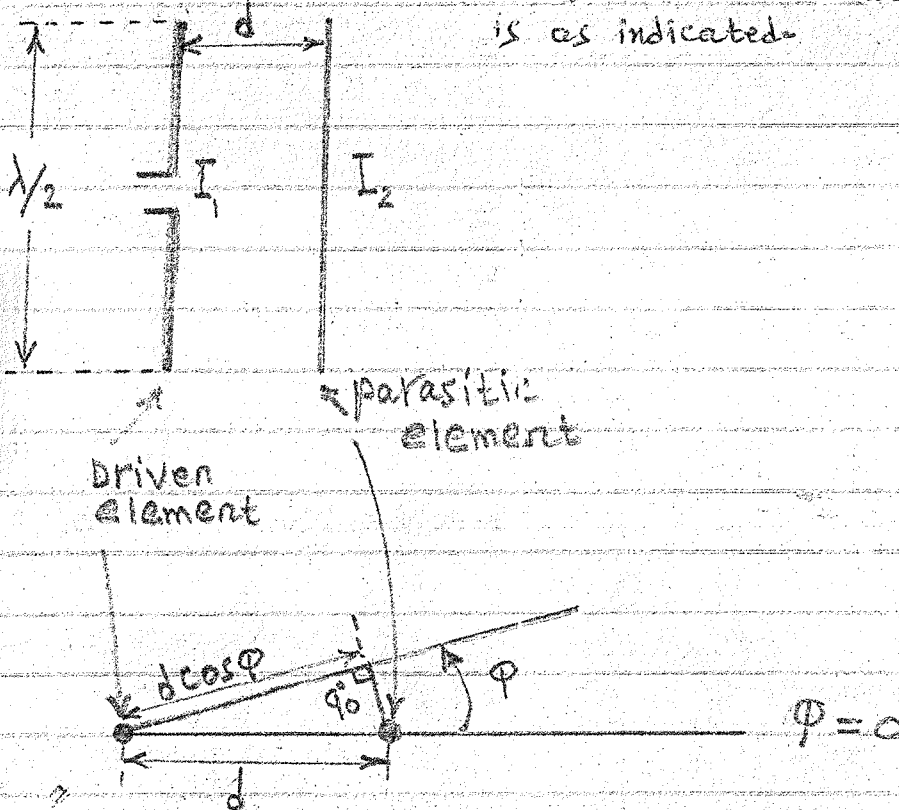


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Array with one driven dipole element and one parasitic element

Let us consider the case of an array in free space consisting of one driven (active) $\frac{\lambda}{2}$ dipole element and one parasitic element as in figure below. Suppose that both elements are vertical so that the azimuth angle ϕ is as indicated.



The circuit relations for the elements are

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$

$$0 = I_2 Z_{22} + I_1 Z_{12}, \quad Z_{21} = Z_{12}$$

From last equation the current in element "2" is

$$I_2 = -I_1 \frac{Z_{12}}{Z_{22}} = -I_1 \frac{|Z_{12}| \angle \underline{I_m}}{|Z_{22}| \angle \underline{I_2}} = -I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \angle \underline{I_m - I_2}$$

$$= I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \angle \underline{\phi}$$

(96)

(85)

Where $\xi = \pi + \Gamma_m - \Gamma_2$, in which

$$\Gamma_m = \arctan \frac{X_{12}}{R_{12}}$$

$$\Gamma_2 = \arctan \frac{X_{22}}{R_{22}}$$

where

$R_{12} + jX_{12} = Z_{12}$ = mutual impedance of elements 1 and 2, Ω

$R_{22} + jX_{22} = Z_{22}$ = self impedance of the parasitic element, Ω

The electric field intensity at a large distance from the array as a function of φ is

$$E(\varphi) = k(I_1 + I_2 / dr \cos \varphi), \text{ where } dr = \frac{2\pi}{\lambda} d$$

$$= kI_1 \left(1 + \left| \frac{Z_{12}}{Z_{22}} \right| / \cos \varphi \right)$$

$$Z_1 = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}}{Z_{22}} \cdot Z_{12} = Z_{11} - \frac{Z_{12}^2}{Z_{22}} = Z_{11} - \left| \frac{Z_{12}}{Z_{22}} \right| \cos [2\Gamma_m - \Gamma_2]$$

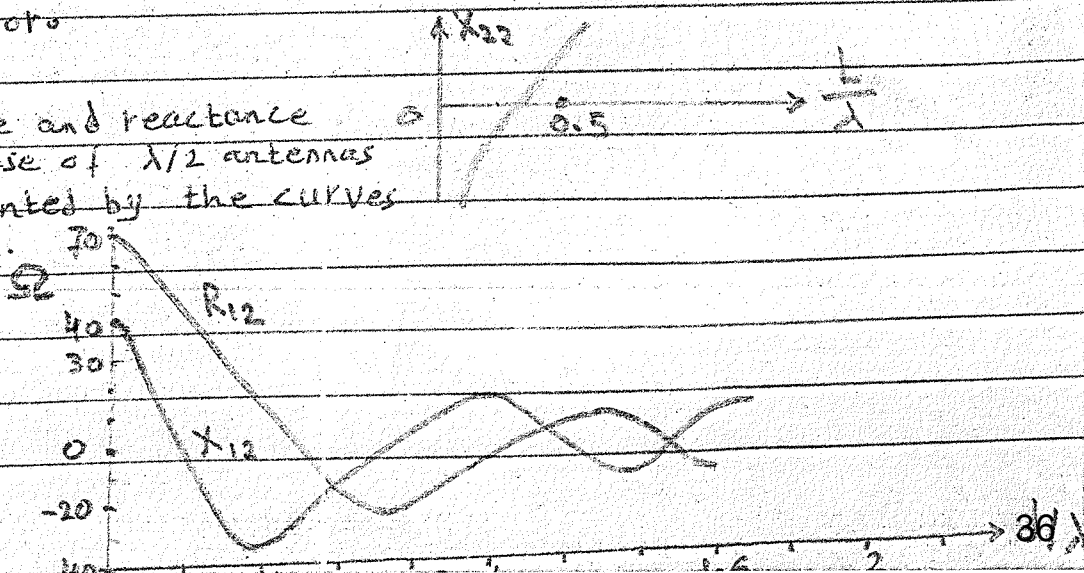
$$R_1 = R_{11} - \left| \frac{Z_{12}}{Z_{22}} \right| \cos [2\Gamma_m - \Gamma_2]$$

Adding a term for the effective Loss resistance,

$$\text{we have } R_1 = R_{11} + R_{1L} - \left| \frac{Z_{12}}{Z_{22}} \right| \cos [2\Gamma_m - \Gamma_2]$$

The tuning of parasitic element is accomplished by adjusting its length. When the parasitic element is inductive (longer than its resonant length) it act as a reflector. When it is capacitive (shorter than its resonant length) it act as a director.

The mutual resistance and reactance (R_{12}, X_{12}) for the case of $\lambda/2$ antennas ($L = \lambda/2$) are presented by the curves in following figure.



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For a power input P to the driven element,

$$I_1 = \sqrt{\frac{P}{R_{11}}} = \sqrt{\frac{P}{R_{11} + R_{1L} - \left| \frac{Z_{12}^2}{Z_{22}} \right| \cos[2\Gamma_m - \Gamma_2]}}$$

$$E(\Phi) = k \sqrt{\frac{P}{R_{11} + R_{1L} - \left| \frac{Z_{12}^2}{Z_{22}} \right| \cos(2\Gamma_m - \Gamma_2)}} * \left(1 + \left| \frac{Z_{12}}{Z_{22}} \right| \sqrt{1 + \cos \Phi} \right)$$

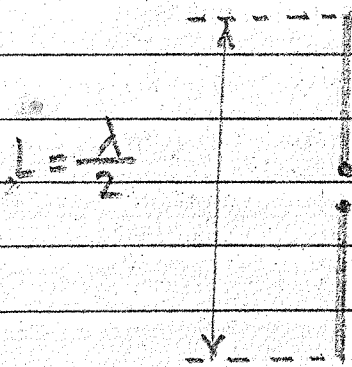
For a power input P to single vertical $\frac{\lambda}{2}$ element the electric field intensity at the same distance is

$$E_{Hw}(\Phi) = k I_0 = k \sqrt{\frac{P}{R_{00} + R_{0L}}} = k \sqrt{\frac{P}{R_{11} + R_{1L}}}$$

where R_{00} - self resistance of single $\frac{\lambda}{2}$ element, Ω

R_{0L} - Loss resistance of single $\frac{\lambda}{2}$ element, Ω

$$R_{00} = R_{11} \quad , \quad R_{0L} = R_{1L}$$



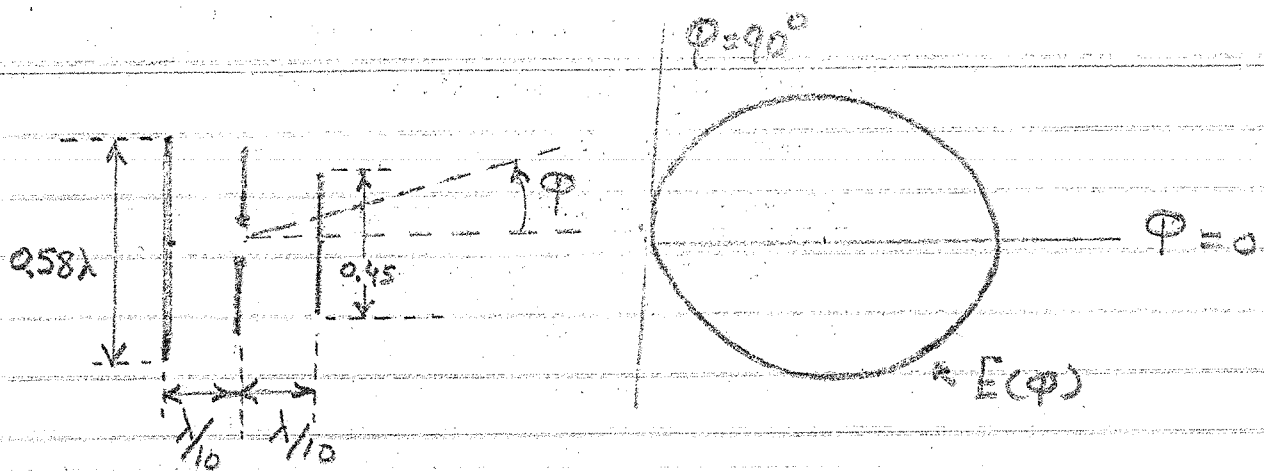
The gain in field intensity (as a function of Φ) of the array with respect to a single $\frac{\lambda}{2}$ with the same power input in the ratio of $E(\Phi)$ to $E_{Hw}(\Phi)$

$$G(\Phi) = \sqrt{\frac{R_{11} + R_{1L}}{R_{11} + R_{1L} - \left| \frac{Z_{12}^2}{Z_{22}} \right| \cos(2\Gamma_m - \Gamma_2)}} * \left(1 + \left| \frac{Z_{12}}{Z_{22}} \right| \sqrt{1 + \cos \Phi} \right)$$

(87)

(98)

Array maybe constructed with both reflector and director. Experimentally measured field pattern of 3-element array is shown in figure a



Yagi-Uda antenna

Yagi-Uda antenna consists of driven element (active element), reflector and ~~more~~ directors [reflector and directors are passive elements]. Yagi-Uda antenna is an array of a driven element (or active element where the power from the transmitter is fed or which feed received power to the receiver) and ~~an~~ parasitic elements (i.e. passive elements which are not connected directly to the transmission line but electrically coupled).

The voltages are induced in parasitic elements by the current flow in the driven element. The phase and currents flowing due to the induced voltage depend on the spacing between the elements and upon the reactance of the elements (i.e. Length).

The parasitic elements in conjunction with driven element causes the dipole impedance to fall ~~down~~ well below 73Ω (for half-wave dipole). It may be as low as 25Ω and hence it becomes necessary to use folded dipole so that input impedance could be raised to a suitable value, to match the feed cable.

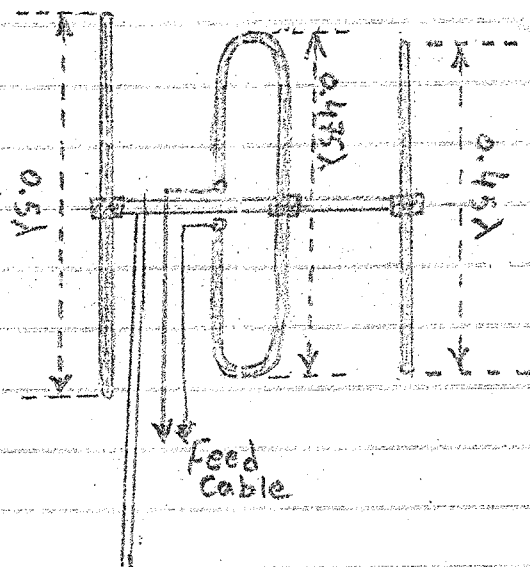
88

99

The reflector is 5% longer than the driven element and spaced between 0.1λ to 0.25λ behind the driven element. The directors are shorter than the driven element. The length of first director is 5% less than the driven element, the length of second director is 5% less than the first director and so on. The distance between directors between 0.1λ to 0.35λ .

More directors may be used if greater directivity (lower beam width) is desired. For example $N=8-10$ the beam width about 15° [N -number of directors]. $N>10$ not used because the pattern of antenna is defected.

A typical 3 elements Yagi antenna suitable for TV reception of moderate field strength as shown in figure:



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Loop Antennas

Loop antennas take many different form such as a rectangle, square, triangle, ellipse, circle. Because of the simplicity in analysis and construction, the circular loop is the most popular and has received the widest attention.

Loop antennas are usually classified into two categories, small and large. Small antennas are those whose ~~overall length~~ ~~(number of turns)~~ radius is very small compared to the wave length ($a \ll \lambda$). However, large loops are those whose circumference is about a wave length ($C \sim \lambda$).

Most of the applications of loop antennas are in the HF (3-30 MHz), VHF (30-300 MHz), and UHF (300-3000 MHz) bands.

Most of the development concerns circular loops, square loops are also discussed, and it is shown that the far fields of circular and square loops of the same area are the same when they are small but different when they are large in terms of wavelength.

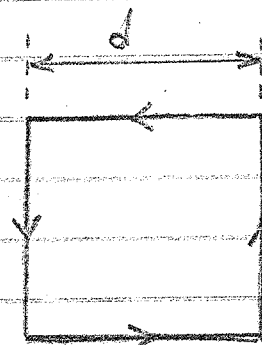
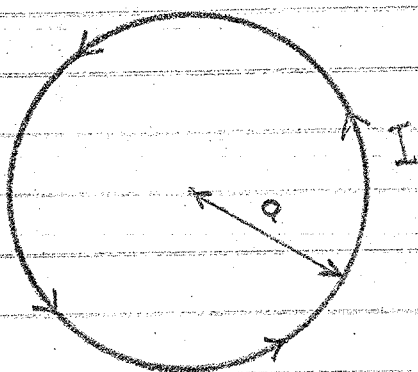
The Small Loop

Consider a circular loop of radius "a" with a uniform in-phase current as suggested by figure below. The radius "a" is very small compared to the wavelength ($a \ll \lambda$). Suppose now that the circular loop is represented by a square loop of

54 90

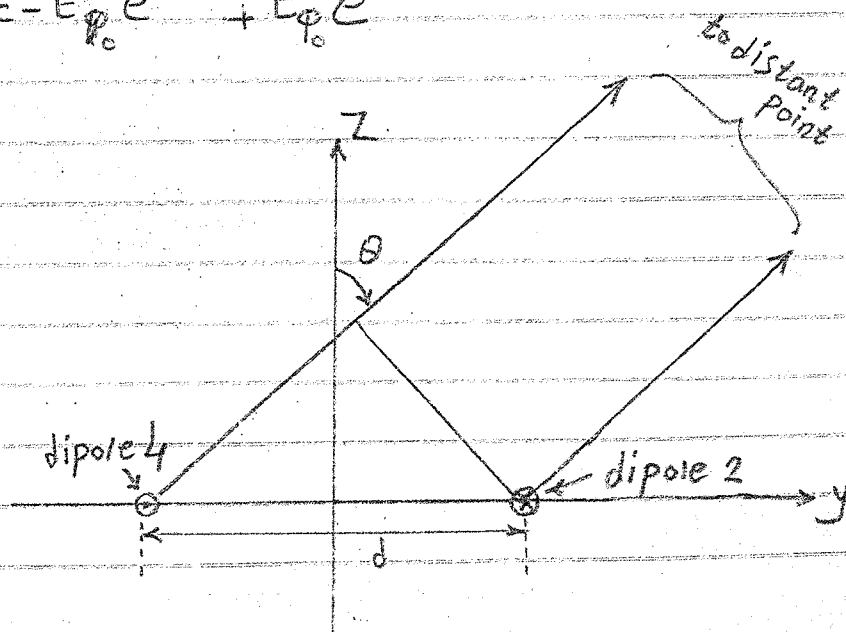
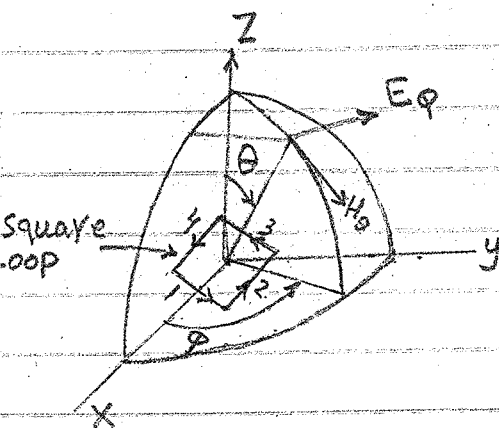
side length, also with a uniform inphase current. The Loop can be treated as four short linear dipoles, whose properties we have already investigated previously.

$$\text{Let } d^2 = \pi a^2.$$



If the loop is oriented as in figure below, its far electric field has only an E_ϕ component. To find the far-field pattern in the yz plane, it is necessary to consider two of the four small linear dipoles (2 and 4). The individual small dipoles 2 and 4 are nondirectional in the yz plane, the field pattern of the loop in this plane is the same as that for two isotropic point sources. Thus

$$E_\phi = -E_{\phi_0} e^{j\phi/2} + E_{\phi_0} e^{-j\phi/2}$$



9.1

where $E_{\phi} = -E_{\phi_0} e^{j\psi/2} + E_{\phi_0} e^{-j\psi/2}$

Where E_{ϕ_0} = electric field from individual dipole

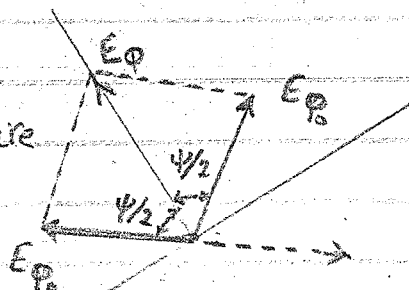
and

$$\psi = \frac{2\pi d}{\lambda} \sin\theta = d_r \sin\theta$$

It follows that

$$E_{\phi} = -2j E_{\phi_0} \sin\left(\frac{d_r}{2} \sin\theta\right)$$

The factor j indicates that the total field E_{ϕ} is in phase quadrature with the field E_{ϕ_0} of the individual dipole



If $d \ll \lambda$ can be written

$$E_{\phi} = -j E_{\phi_0} d_r \sin\theta$$

$$E_{\phi_0} = \frac{j 30 \pi [I] L}{r}$$

$$= \frac{j 30 \pi [I] L}{\lambda r} = \frac{j 60 \pi [I] L}{r \lambda} \quad \begin{matrix} L - \text{length of the short dipole} \\ L = d \end{matrix}$$

$$E_{\phi} = \frac{j 60 \pi [I] L}{r \lambda} d_r \sin\theta$$

However, the length L of the short dipole is the same as d , that is, $L = d$. Noting also that $d_r = \frac{2\pi d}{\lambda}$ and that the area A of the loop is d^2 .

$$E_{\phi} = \frac{60 \pi [I] L}{r \lambda} \frac{2\pi}{\lambda} L \sin\theta = \frac{120 \pi^2 [I] \sin\theta A}{r \lambda^2}$$

$$[I] = I_0 e^{j\omega(t - \frac{r}{c})}$$

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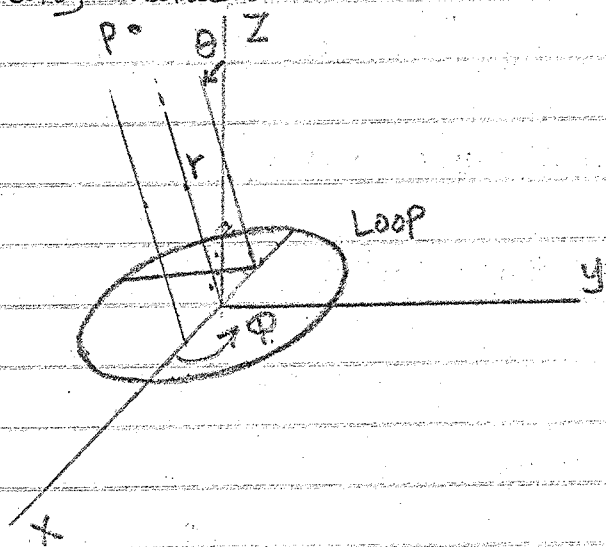
$$E_{\theta} = \frac{120 \pi^2 [I] \sin \theta}{r} \frac{A}{\lambda^2}$$

$$H_{\theta} = \frac{E_{\theta}}{120 \pi} = \frac{\pi [I] \sin \theta}{r} \frac{A}{\lambda^2}$$

The far fields of small circular and square Loops are identical ($d^2 = \pi a^2$).

The Loop antenna. General case

The general case of a Loop antenna with uniform, in-phase current will now be discussed. The size of Loop may assume any value.



The far field of the Loop with any radius "a" given by

$$E_{\theta} = \frac{60 \pi \beta a [I] J_1(\beta a \sin \theta)}{r}$$

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The peak value of E_ϕ is obtained by putting $[I] = I_0$, where I_0 is the peak value (in time) of the current on the loop.

$$H_\theta = \frac{E_\phi}{120\pi} = \frac{\beta a [I] J_1(\beta a \sin\theta)}{2r}$$

$$E_\phi = \frac{60\pi \beta a [I] J_1(\beta a \sin\theta)}{r}$$

For a loop of a given size, βa is constant and the shape of the far-field pattern is given as a function of θ by

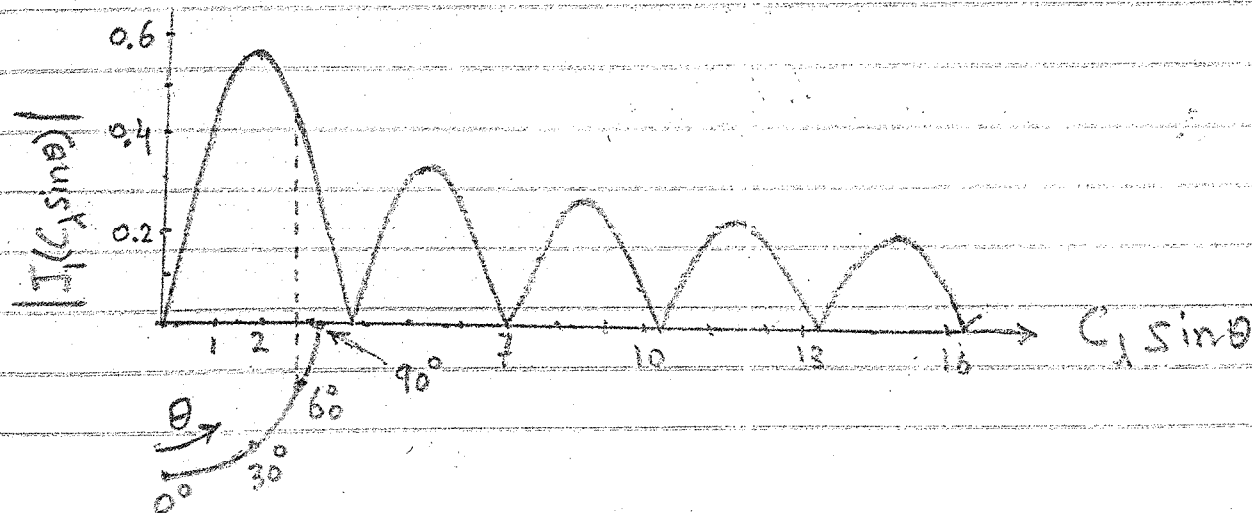
$$J_1(\beta a \sin\theta) = J_1(C_\lambda \sin\theta)$$

$$C_\lambda = \beta a = \frac{2\pi a}{\lambda}$$

$$\text{When } \theta = 90^\circ \rightarrow J_1(C_\lambda)$$

$$\theta = 0 \rightarrow J_1(0) = 0$$

The following figure illustrates the Rectified First-order Bessel curve as a function of $C_\lambda \sin\theta$:



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As an example, let us find the pattern for a loop 1λ in diameter ($2a = 1\lambda$) ($C_\lambda = \pi = 3.14$). The relative field in the direction $\theta = 90^\circ$ is then 0.285. The maximum of $J_1(C_\lambda \sin \theta)$ of 0.582 at angle θ of about 36° .

$$J_1(C_\lambda \sin 36^\circ) = J_1(\pi \sin 36^\circ) = 0.582$$

As θ decreases, the field intensity also decreases, reaching zero at $\theta = 0^\circ$.

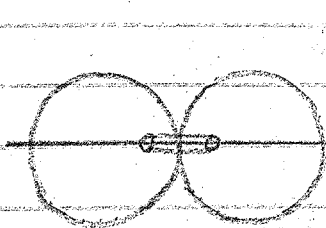
$$\frac{2\pi a}{\lambda} \sin \theta \approx \frac{0.4}{\lambda}$$

The pattern in the other four quadrants is symmetrical.

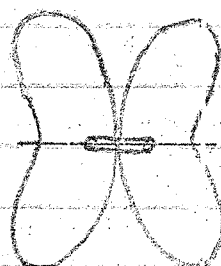
For loops which are less than 1.24λ (less than 0.585λ in diameter), $2a < 0.585\lambda$ the maximum field is in the direction $\theta = 90^\circ$.

The pattern for a loop 5λ in diameter, which is typical for large circular loops with uniform current,

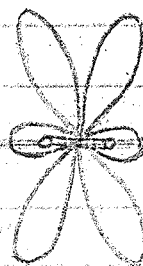
All patterns in the following figures have zero field in the direction normal to the loop ($\theta = 0^\circ$) regardless of the size of the loop.



Diameter $< \frac{\lambda}{10}$
 $C_\lambda = 0.314$



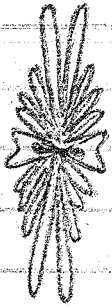
Diameter $= \lambda$
 $C_\lambda = 3.14$



Diameter $\frac{3}{2}\lambda$
 $C_\lambda = 4.71$

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Diameter = 5λ
 $G_1 = 15.7$



Diameter = 8λ
 $G_1 = 25.1$

For small arguments of the first-order Bessel function, the following approximate relation can be used:

$$J_1(x) = \frac{x}{2}$$

For small Loop the far electric and magnetic fields become are given by,

$$\begin{aligned} E_\phi &= \frac{60\pi \beta a^2 I_1 (Ba \sin\theta)}{r} = \frac{60\pi \beta^2 a^2}{2r} \sin\theta = \frac{60\pi (2\pi)^2 a^2}{2r \lambda^2} \sin\theta \\ &= \frac{120\pi^2 A \sin\theta}{r \lambda^2} = \frac{120\pi^2 I_1}{r} \sin\theta \frac{A}{\lambda^2} \end{aligned}$$

$$H_\theta = \frac{\pi I_1 \sin\theta}{r} \frac{A}{\lambda^2}$$

$$E_\phi = \frac{120\pi^2 I_1 \sin\theta}{r} \frac{A}{\lambda^2}$$

These far-field equations for a small Loop are identical with those obtained for small Square Loop.

60 96

Radiation resistance of Loops

The average pointing vector of a far field is given by

$$S = \frac{1}{2} |H|^2 \operatorname{Re} Z = \frac{1}{2} \left[\frac{\beta a I_0}{2r} J(\beta a \sin \theta) \right]^2 \cdot 120 \pi$$

The total power radiated is given by,

$$\begin{aligned} P_r &= \iint_{4\pi} S ds = \iint_{4\pi} S r^2 d\Omega = \iint_{4\pi} S r^2 d\varphi \sin \theta d\theta \\ &= \int_0^{2\pi} d\varphi \int_0^\pi S r^2 \sin \theta d\theta = 2\pi \times \frac{1}{2} \cdot 120 \pi \frac{\beta^2 a^2}{4} \int_0^\pi J_1^2(\beta a \sin \theta) \sin \theta d\theta \\ &= 30 \pi^2 (\beta a I_0)^2 \int_0^\pi J_1^2(\beta a \sin \theta) \sin \theta d\theta \end{aligned}$$

In the case of a loop that is small in terms of wavelengths, the approximation ($J(x) \approx \frac{x}{2}$) can be applied.

$$\begin{aligned} P_r &= 30 \pi^2 (\beta a I_0)^2 \int_0^\pi \left[\frac{\beta a \sin \theta}{2} \right]^2 \sin \theta d\theta = \frac{30}{4} \pi^2 (\beta a)^4 I_0^2 \int_0^\pi \sin^3 \theta d\theta \\ &= \frac{30}{4} (\pi a^2)^2 \beta^4 I_0^2 \cdot \frac{4}{3} = 10 \beta^4 I_0^2 A^2 \end{aligned}$$

$$P_r = \frac{I_0^2}{2} R_r = 10 \beta^4 I_0^2 A^2, \quad A = a^2 \pi$$

$$R_r = 20 \left(\frac{2\pi}{\lambda} \right)^4 A^2 = 20 \cdot \pi^4 \cdot 2 \left(\frac{A}{\lambda^2} \right)^2 = 31170.9 \left(\frac{A}{\lambda^2} \right)^2$$

$$\boxed{R_r \approx 31200 \left(\frac{A}{\lambda^2} \right)^2} = 20 \beta^4 a^4 \pi^2 = 197 \beta^4 a^4 = 197 C_\lambda^4$$

This relation for small single-turn loop antenna, circular or square, with uniform in-phase current. The relation ~~is~~ is about 2 percent in error when the loop perimeter is $\frac{1}{3}$.

97 (6)

A circular Loop of this perimeter has a diameter of about $\lambda/10$. Its radiation resistance by last equation is nearly 2.5Ω .

The radiation resistance of a small Loop consisting of one or more turns is given by,

$$R_r = 31200 \left(n \frac{A}{\lambda^2} \right)^2 (\Omega)$$

Let us now proceed to find the radiation resistance of a circular loop of any radius "a". Thus in general

$$\int_0^\pi J_1^2(x \sin \theta) \sin \theta d\theta = \frac{1}{x} \int_0^{2x} J_2(y) dy$$

where y is any function

$$P_r = 30 \pi^2 (\beta a I_0)^2 \int_0^\pi J_1^2(\beta a \sin \theta) \sin \theta d\theta$$

$$P_r = 30 \pi^2 \beta a I_0^2 \int_0^{2\beta a} J_2^2(y) dy$$

This is the radiation resistance for a single-turn circular Loop with uniform in-phase current and of any circumference $C_\lambda = \beta a$.

When the loop is large $C_\lambda \gg 1$, we can use the approximation $\int_0^{2C_\lambda} J_2^2(y) dy \approx 1$.

$$R_r = 60 \pi^2 \beta a = 60 \pi^2 C_\lambda = 372 C_\lambda = 3720 \frac{a}{\lambda}$$

62 98

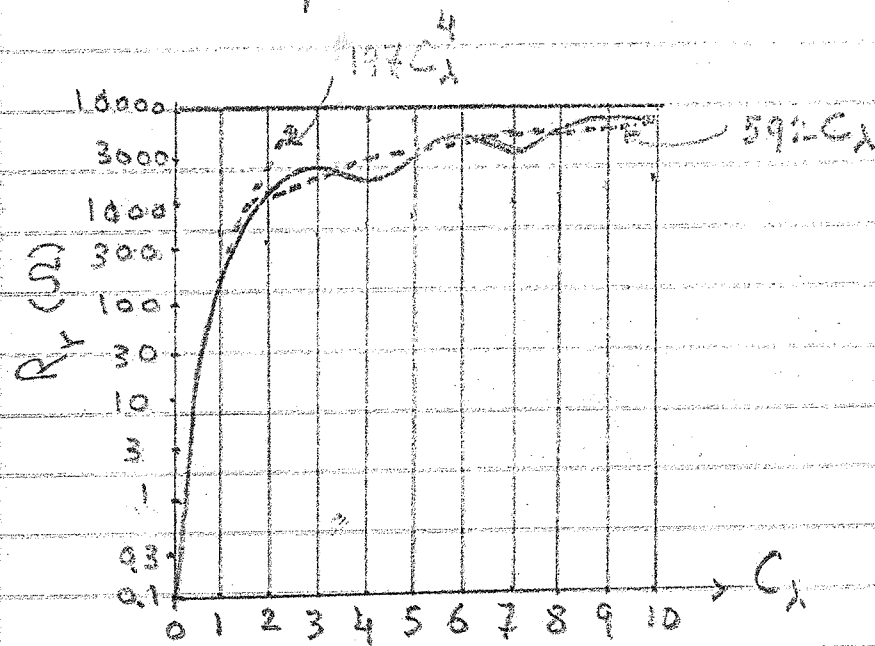
For Loop of 10λ perimeter, the radiation resistance is nearly 6000Ω .

For values of C_λ between $\frac{1}{3}$ and 5 the integral

$\int_0^{2C_\lambda} J_2(y) dy$ can be evaluated using the transformation

$$\int_0^{2C_\lambda} J_2(y) dy = \int_0^{2C_\lambda} J_0(y) dy - 2J_1(2C_\lambda)$$

A graph showing the radiation resistance of single-turn loops with uniform current is presented in figure:



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SS

Directivity of circular Loop antennas with uniform current

The directivity D of an antenna was defined as the ratio of maximum radiation intensity to the average radiation intensity.

$$U_{\max}(\theta) = S_r \cdot r^2 = \frac{15\pi (\beta a I_0)^2}{r^2} J_1^2(\beta a \sin\theta) \cdot r^2 = \frac{15\pi (\beta a I_0)^2}{1} J_1^2(\beta a \sin\theta)$$

$$U_{av} = \frac{P_r}{4\pi} = \frac{30\pi^2 \beta a I_0^2}{4\pi} \int_0^{2\beta a} J_2(y) dy$$

$$D = \frac{4\pi \times 15\pi C_\lambda^2 I_0^2 J_1^2(C_\lambda \sin\theta)}{30\pi^2 C_\lambda I_0^2 \int_0^{2\beta a} J_2(y) dy} = \frac{2C_\lambda [J_1^2(C_\lambda \sin\theta)]_{\max}}{\int_0^{2C_\lambda} J_2(y) dy}$$

This expression for ^{the} directivity of ~~any~~ a circular Loop with uniform in-phase current of any circumference C_λ .

The angle θ in last equation is the value for which the field is a maximum.

For a small loop ($C_\lambda \leq \frac{1}{3}$), the directivity expression reduces to

$$D = \frac{3}{2} \sin^2\theta = 1.5$$

For a large loop ($C_\lambda > 5$),

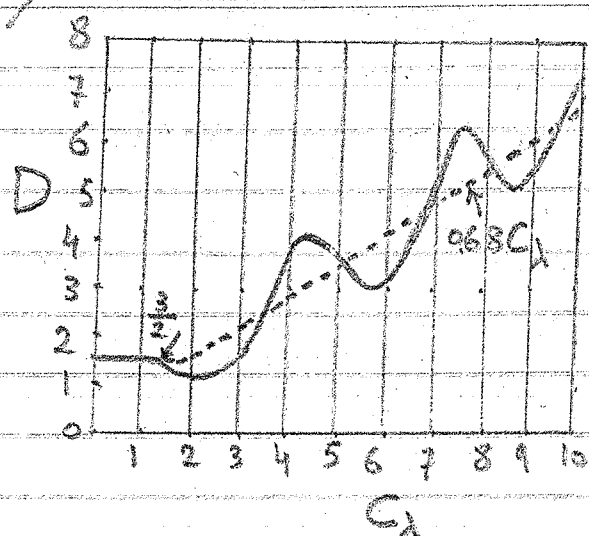
$$D = 2C_\lambda [J_1^2(C_\lambda \sin\theta)]_{\max}$$

The maximum value of $J_1(C_\lambda \sin\theta)$ is 0.582, Thus the directivity for large loop becomes

$$D = 0.68 C_\lambda$$

$$C_\lambda = \beta a$$

64 100



dashed line is based on the approximate relations

Square Loops

The far-field patterns of square and circular loops of the same area are identical when the loops are small ($A < \frac{\lambda^2}{100}$).

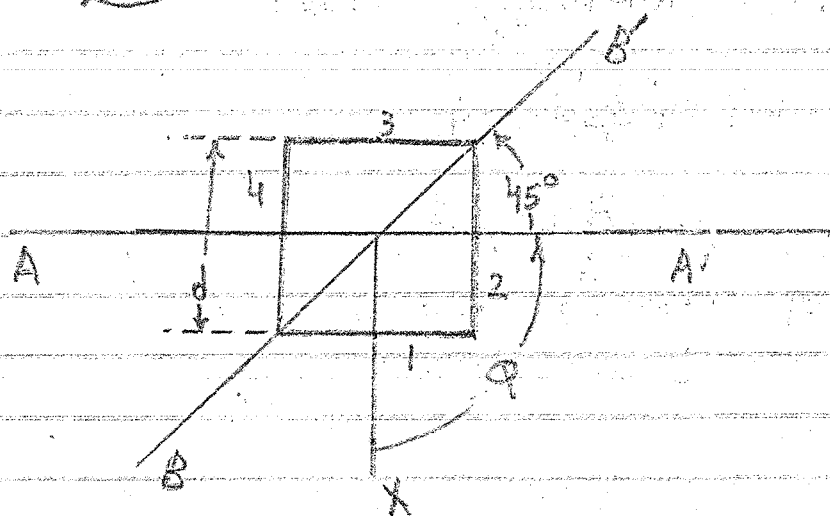
The pattern of a circular loop of any size is independent of the angle ϕ but is a function of θ . On the other hand, the pattern of a large square loop is a function of both θ and ϕ .

Referring to figure below, the pattern in a plane normal to the plane of the loop and parallel to two sides (1 and 3), as indicated by the line AA' , is simply the pattern of two point sources representing sides 2 and 4 of the loop. The pattern in a plane normal to the plane of the loop and passing through diagonal corners as indicated by the line BB' , is different.

The complete range in the pattern variation as a function of ϕ is contained in this 45° interval between AA' and BB' .

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An additional difference of large circular and square loops is in the θ pattern. Figure below shows the pattern as a function of θ for a square loop 4.44λ ($d = 4.44\lambda$). This pattern is different from the pattern of circular loop with the same area, because

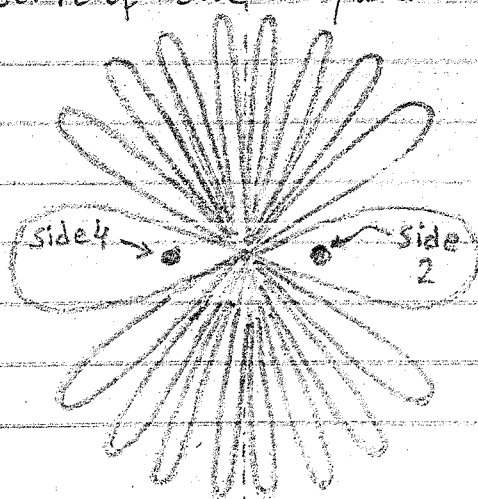
~~$$A_{sq} = [4.44\lambda]^2 = 19.7136\lambda^2$$~~

$$A_{sq} = [4.44\lambda]^2 = 19.7136\lambda^2$$

$$A_{cir} = \left(\frac{5\lambda}{2}\right)^2 \pi = 19.64\lambda^2$$

$$A_{sq} \approx A_{cir}$$

~~The~~ The difference between patterns illustrated by the difference between the Bessel function pattern of the circular loop and the trigonometric function pattern of the square loop.



$$d = 4.44\lambda$$

Radiation efficiency

$$C = k D$$

where k = efficiency factor ($0 \leq k \leq 1$)

$$k = \frac{R_r}{R_r + R_L}, \text{ where } R_r = \text{radiation resistance}$$

$R_L = \text{loss resistance}$

$$C = \frac{R_r}{R_r + R_L} D = \frac{R_r}{R_r + R_L} \frac{4\pi A_e}{\lambda^2}$$

The loop antenna which is small compared to the wavelength, the radiation resistance R_r is small and, if ohmic losses R_L are significant, radiation efficiency is reduced. For example $R_r = R_L$ the radiation efficiency is 50 percent; only half of the power input to the antenna is radiated, the other half being dissipated as heat in the antenna structure.

In spite of the low efficiency of a small loop, there are many applications where such loops are useful in receiving applications provided the received signal-to-noise ratio is acceptable.

For loops with n turns, R_r increases in proportion to n^2 while R_L increases in proportion to n .

The radiation efficiency of a multiturn loop or coil antenna can be increased by introducing a ferrite rod into the coil. The coil (horizontal to receive vertical

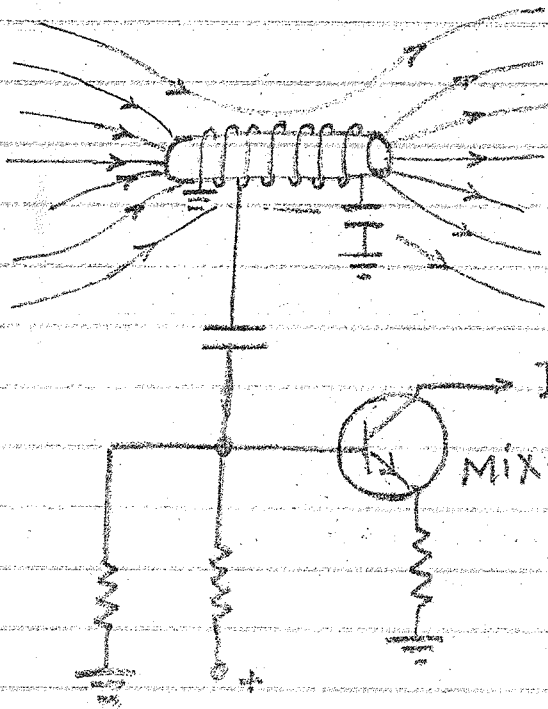
103

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polarization) serves the function of both an antenna and also (with a series capacitor) of the resonant circuit for the first (mixer) stage of broadcast receiver (500 to 1600 kHz).

The radiation resistance of a ferrite loaded loop or coil is given by

$$R_r = 31200 \mu_{er} n^2 \left(\frac{A}{\lambda} \right)^2 = 197 \mu_{er} n^2 C_{\lambda}^4 \quad (\Omega)$$



$$C_{\lambda} = \frac{C}{\lambda} = \frac{2\pi a}{\lambda} = \frac{\pi \cdot 2a}{\lambda}$$

C - circumference of circular loop

$$C = L$$

The ohmic (or loss resistance) of a small loop antenna is given by

$$R_L = \frac{L}{d} \sqrt{\frac{f \mu_0}{\pi \sigma}}, \quad L = C$$

d - wire or conductor diameter, m

σ - conductivity of wire

and the loss resistance due to ferrite rod by

$$R_f = 2\pi f \mu_{er} \frac{\mu_r''}{\mu_r'} \mu_0 n^2 \frac{A}{L}$$

58 104

where f - frequency, Hz

μ_{er} - effective relative permeability of ferrite rod, dimensionless

μ_r' - real part of relative permeability of ferrite material, dimensionless

μ_r'' - imaginary part of relative permeability of ferrite material, dimensionless

$\mu_0 = 4\pi \cdot 10^{-7}$, H/m

n - number of turns

A' - ferrite rod cross-sectional area, m^2

l - length of ferrite rod, m

The radiation efficiency factor for the ferrite rod coil antenna is then

$$k = \frac{R_r}{R_r + R_l + R_j}$$

(69) (105)

Example

A Loop antenna consists of 15 turns, each having an area 1 m^2 . A radio wave having a frequency of 10 MHz induces a sinusoidal e.m.f. of 200 mV (r.m.s) in this antenna when it is oriented for maximum response. Calculate the peak value of the magnetic field intensity H of the radio wave. $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$.

Solution

$$V_m = \omega BAN = \omega \mu_0 H AN$$

$$N = 15 \text{ turns}$$

$$A = 1 \text{ m}^2$$

$$f = 10 \text{ MHz}$$

$$\cos \theta = 1$$

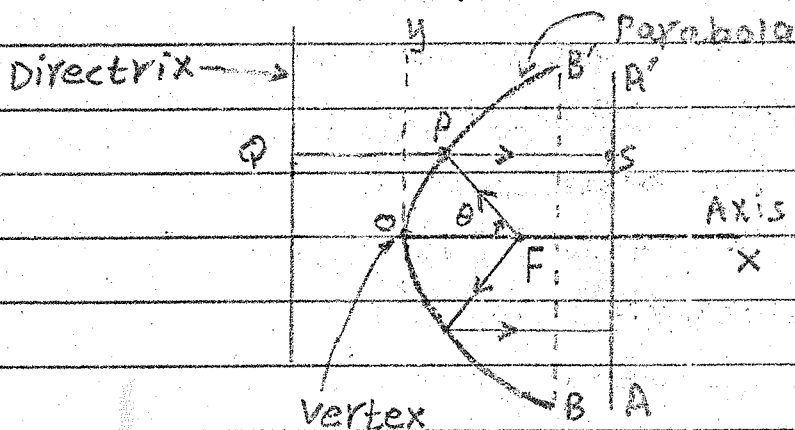
$$H = \frac{V_m}{2\pi f \mu_0 AN} = \frac{V_{rms} \sqrt{2}}{2\pi f \mu_0 AN} \quad \text{A/m}$$

$$= \frac{1.414 \times 200 \times 10^{-3}}{2\pi \times 10 \times 10^6 \times 4\pi \times 10^{-7} \times 1 \times 15} = 0.239 \text{ mA/m}$$

$$= \frac{1.414 \times 2 \times 10^{-1}}{2\pi \times 4\pi \times 15} \frac{\text{A}}{\text{m}} = \frac{1.414 \times 2 \times 10^2}{2\pi \times 4\pi \times 15} = \frac{1.414 \times 10^2}{4\pi^2 \times 15} = 0.239 \frac{\text{mA}}{\text{m}}$$

Antennas With parabolic Reflectors

A parabola may be defined as the Locus of a point which moves in such way that its distance from the fixed point (called focus) plus its distance from a straight line ~~is constant~~ is constant.



F - focus

$$y^2 = 4fx$$

$$2L = R(1 + \cos\theta)$$

$$R = \frac{2L}{1 + \cos\theta}, \text{ where}$$

$$R = FP, L = FO$$

$PF = PQ$. Let AA' be a line normal to the axis at an arbitrary distance QS from the directrix.

$$PF + PS = PF + QS - PQ = QS$$

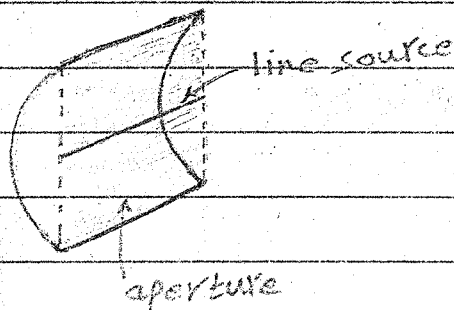
Thus, a property of a parabolic reflector is that all waves from an isotropic source at the focus that are reflected from the parabola arrive at a line AA' with equal phase. The "image" of the focus is the directrix and the reflected field along the line AA' appears as though it originated at the directrix as a plane wave. The plane BB' at which a reflector is cut off is called the aperture plane.

The equation of parabola curve is given by

$$y^2 = 4fx$$

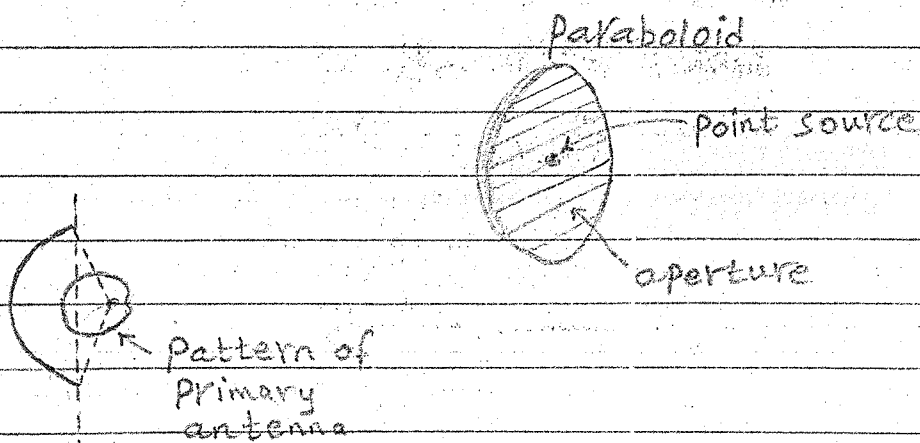
A cylindrical parabola converts a cylindrical wave radiated by an in-phase line source at the focus into a plane wave at the aperture. Parabolic cylinder formed by moving the parabola in side way. A plane sheet curved to parabolic shape in one dimension only forms the parabolic cylinder. This has a "focal line" instead of a focal point and a vertex line instead of an vertex

cylindrical parabola



The paraboloidal Reflector: The surface generated

by the revolution of parabola around its axis is called a paraboloid. The source (primary antenna) is placed at the focus of paraboloidal reflector as in following figure:



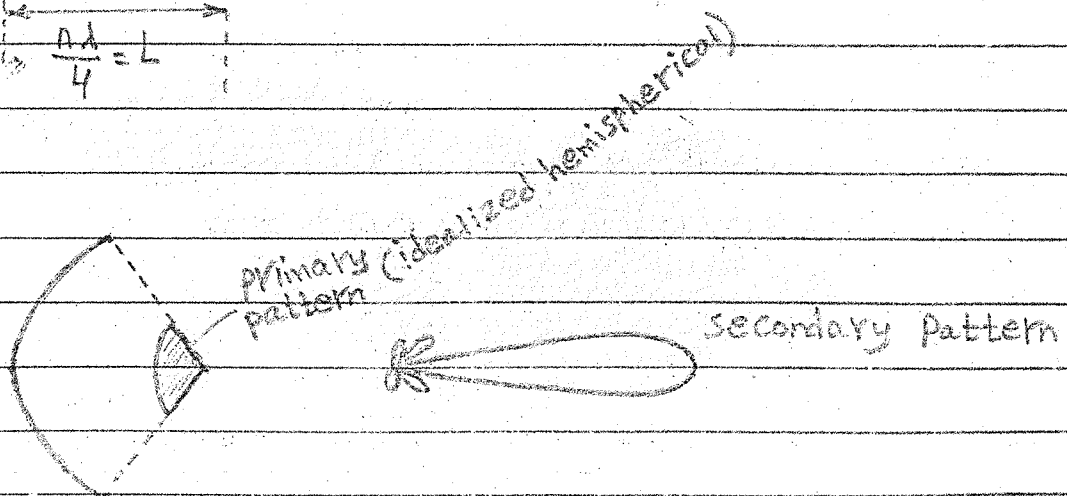
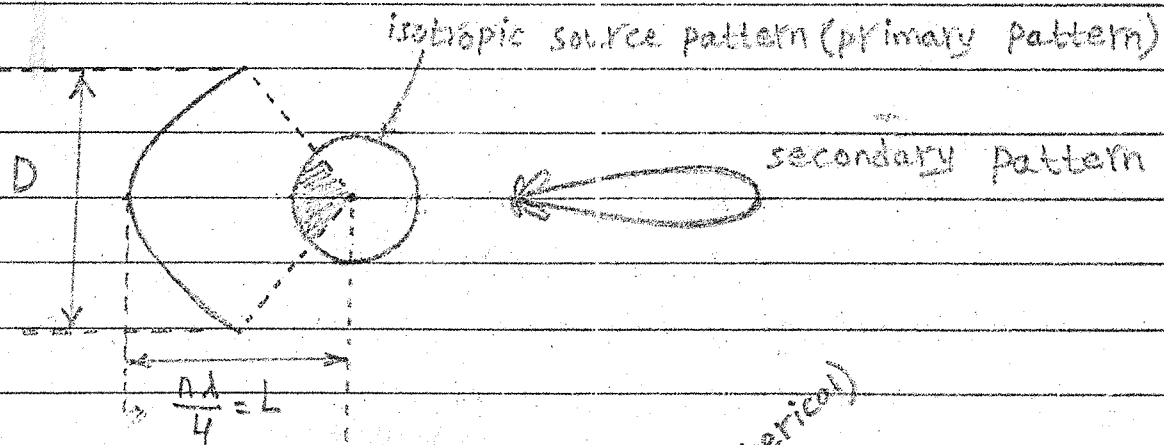
log

If the distance L between the focus and vertex of the paraboloid is an even number of $\lambda/4$, the direct radiation in the axial direction from the source will be in opposite phase and will tend to cancel the central region of the reflected wave. However, if

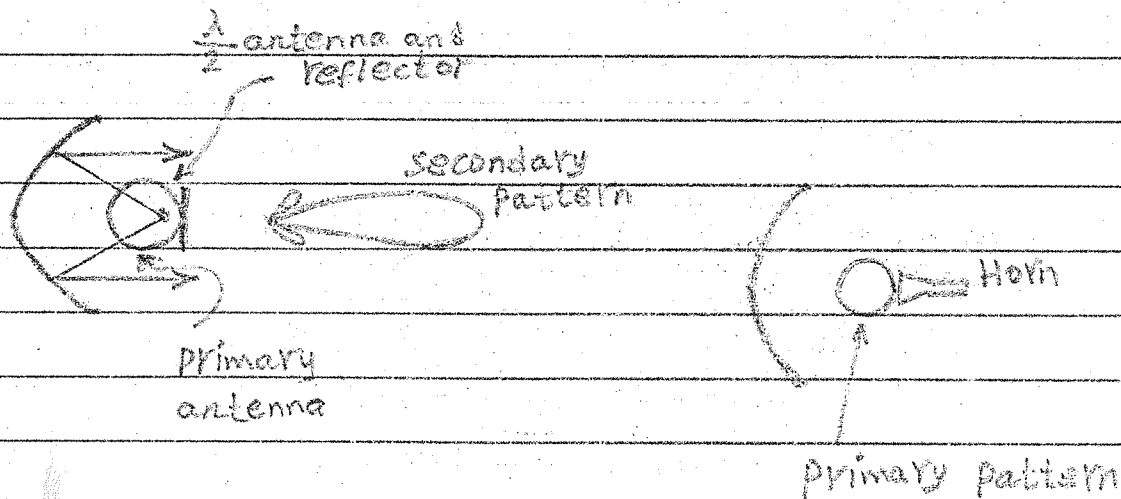
$$L = n \frac{\lambda}{4}$$

where $n = 1, 3, 5, 7, \dots$, the direct radiation in the axial direction from the source will be in the same phase and will tend to reinforce the central region of the reflected wave.

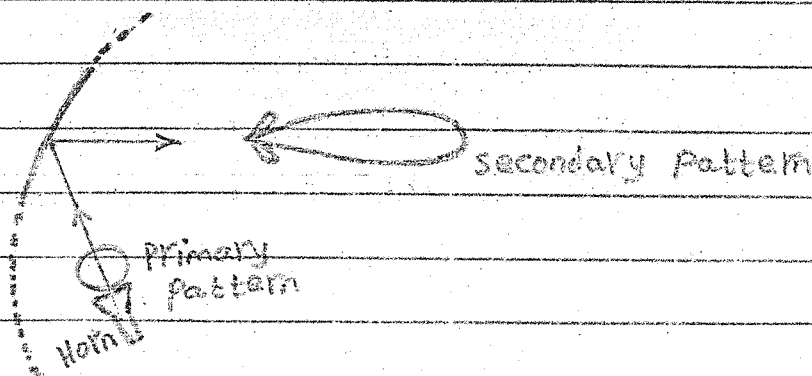
Direct radiation from the source can be eliminated by means of directional source (Primary antenna).



Suitable directional patterns may be obtained with various types of primary antennas. As examples, a $\lambda/2$ antenna with a small ground plane ~~and a small ground plane~~ and a small horn antenna are shown in figures



The presence of the primary antenna in the path of the reflected wave, as in the above figures, has two(2) principal disadvantages. There are, first, that waves reflected from the parabola back to the primary antenna produce interaction and mismatching. Second the primary antenna acts as an obstruction, blocking out the central portion of the aperture and increasing the minor lobes. To avoid both effects, a portion of the paraboloid can be used and the primary antenna displaced as in figure below. This is called an offset feed.



(III)

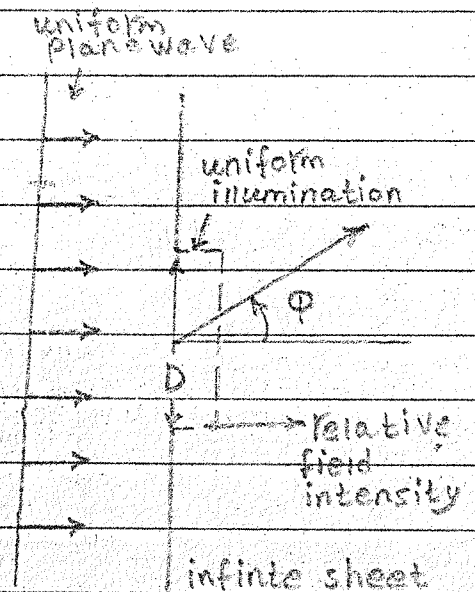
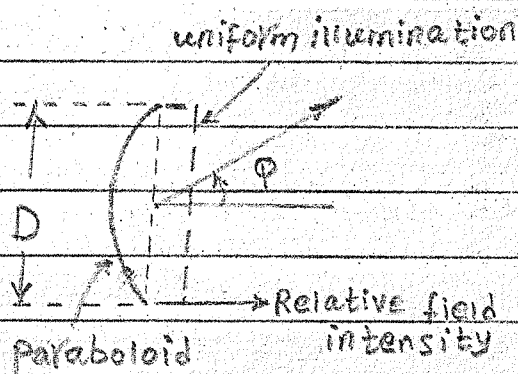
Patterns of Large circular apertures with uniform illumination

The radiation from a large paraboloid with uniformly illuminated aperture is essentially equivalent to that from a circular aperture of the same diameter D in an infinite metal plate with a uniform plane wave incident on the plate as in figures below. The radiation-field pattern for such a uniformly illuminated aperture can be calculated by applying Huygens' principle in a similar way to that for a rectangular aperture. The normalized field pattern $E(\Phi)$ as a function of Φ and D is

$$E(\Phi) = \frac{2\lambda}{\pi D} \frac{J_1(\pi D/\lambda \sin \Phi)}{\sin \Phi} \quad , \text{ where}$$

[for circular]

J_1 = first-order Bessel function



The angle Φ_0 to the first nulls of the radiation pattern are given by $\frac{\pi D}{\lambda} \sin \Phi_0 = 3.83$

$$\Phi_0 = \sin^{-1} \left(\frac{3.83 \lambda}{\pi D} \right) = \sin^{-1} \left(\frac{1.22 \lambda}{D} \right)$$

When Φ_0 is very small (aperture large)

$$\Phi_0 = \frac{1.22 \lambda}{D} \text{ (rad)} = \frac{70 \lambda}{D}$$

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The beam width between first nulls (BWFN) for large circular

$$\text{BWFN} = \frac{1.40 \lambda}{D}, \text{ where } D = \text{diameter of aperture (degree)}$$

By way of comparison, the beam width between first nulls for a large uniformly illuminated rectangular aperture or long linear array is

$$\text{BWFN} = \frac{1.15 \lambda}{L}, \text{ where } L = \text{Length of aperture (degree)}$$

The beam width between half-power points for a large circular aperture is

$$\text{HPBW} = \frac{58 \lambda}{D} \text{ (degree)}$$

And the half-power beamwidth for rectangular aperture is $\text{HPBW} = \frac{51 \lambda}{L}$ (degree)

The directivity D of a large uniformly illuminated aperture is given by

$$D = \frac{4\pi \text{ area}}{\lambda^2}$$

For a circular aperture

$$D = 4\pi \frac{\pi D^2}{4 \lambda^2} = 9.87 \left(\frac{D}{\lambda}\right)^2$$

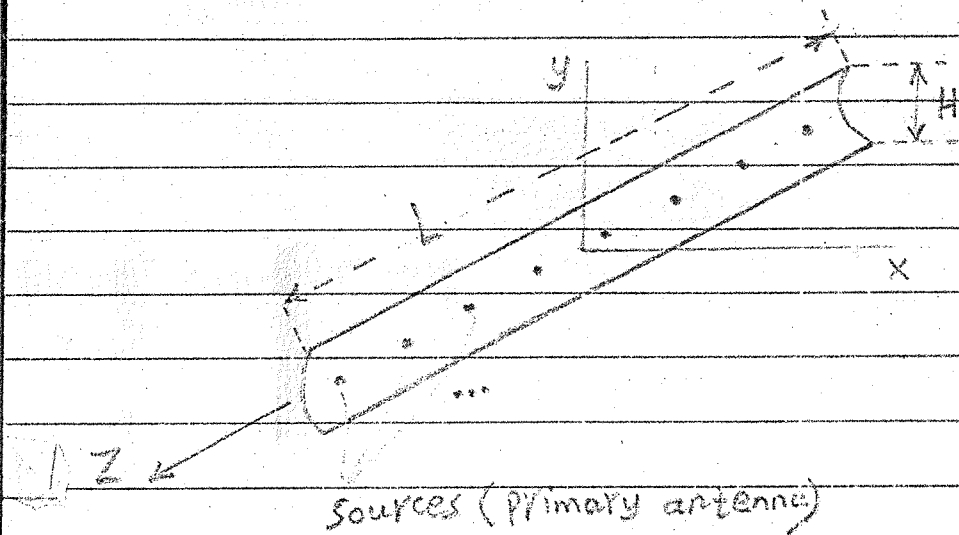
D - diameter

D - directivity

113.

The cylindrical parabolic Reflector

The cylindrical parabolic reflector is used with a line-source type of primary antenna. One type is illustrated in figure below. The field pattern (secondary pattern) is wide in one plane and narrow in the other (fan beam). Neglecting edge effects, the patterns of the antenna is those of rectangular aperture of side dimensions L by H .



$$H \gg \lambda$$

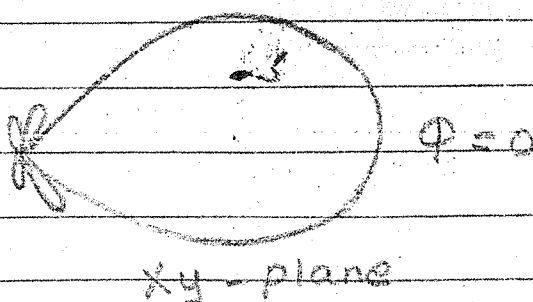
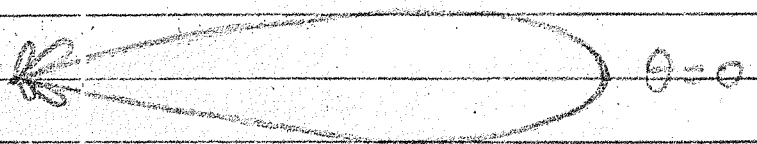
$$L \gg \lambda$$

$$E(\theta, \phi) = \frac{\sin[(a_r \sin \theta)/2]}{(a_r \sin \theta)/2} \frac{\sin[(b_r \sin \phi)/2]}{(b_r \sin \phi)/2}, \text{ where}$$

$$a_r = 2\pi a/\lambda = 2\pi L/\lambda, \quad a = L, \quad b = H$$

$$b_r = 2\pi b/\lambda = 2\pi H/\lambda$$

XZ - plane



XY - plane

Gain

$$G = \frac{4\pi}{\lambda^2} A$$