2.7. Signal flow graph

A signal flow graph is adiagram that represent aset of Simultaneous Linear algebraic equations.

Definitions :

Node :- Anode is apoint representing avariable or Signal.

transnzittance : the transmittance is areal gain or complex gain between two nodes. such gain can be expressed in terms of the transfer function between two nodes

Branch : is adirected Line segment joining two nodes. The gain of abranch is atransmittance.

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Input node or source :- is anode that has only out going branches. This corresponds to an independent Variable.

out Put node or Sink :- is anode that has only in coming branches. This corresponds to a dependent Variable.

Mixed node :- is anode that has both incoming and outgoing branches.

Path :- is atraversal of connected branches in the direction of the branch arrows. if no node is crossed more than once the Path is open. if the Path ends at the same node from which it began and dose not cross

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any other node more than once, it is closed if a path crosses some node more than once but ends at a different node from which it began, it is neither open nor closed.

Loop :- is a closed path.

Loop gain :- is the product of the branch transmittance of a loop.

non touching loops :- if they do not possess any common nodes.

Forward Path :- is a path from an input node to an output node that dose not cross any nodes more than once.

Forward Path gain :- is the product of the branch transmittance of a forward path.

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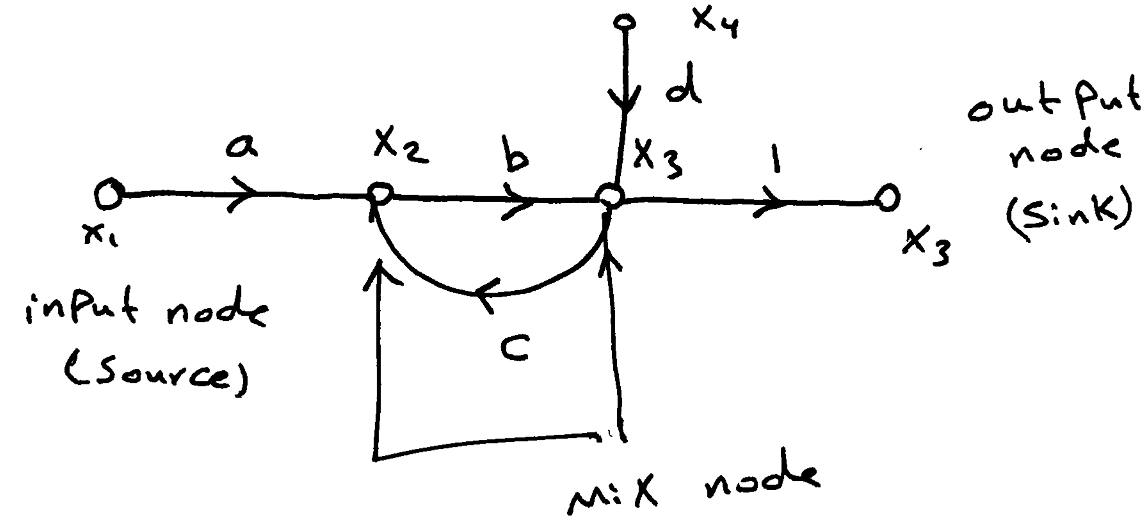
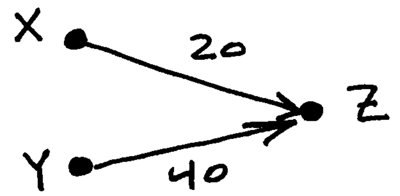


fig. Signal flow graph

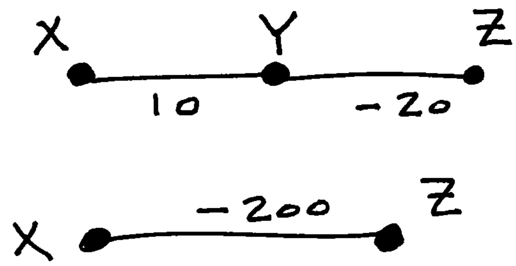
Signal flow graph algebra :

1 - Addition rule



$$Z = 20X + 40Y$$

2- The multiplicatione rule



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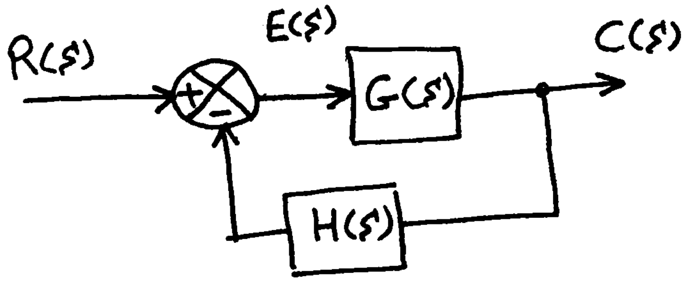
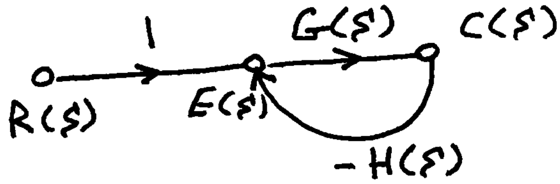
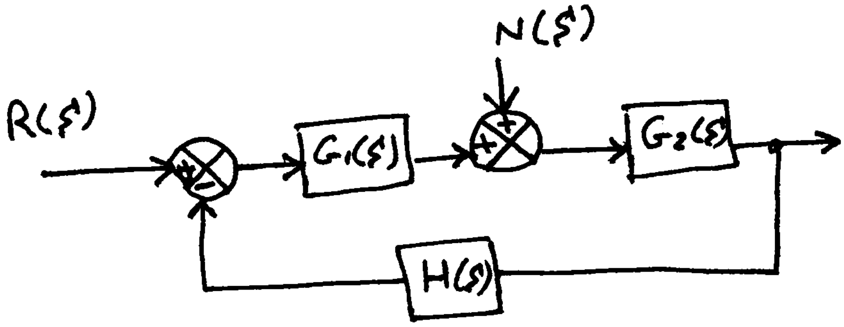
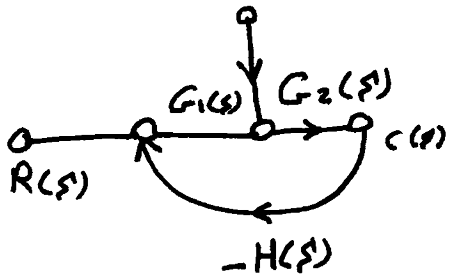
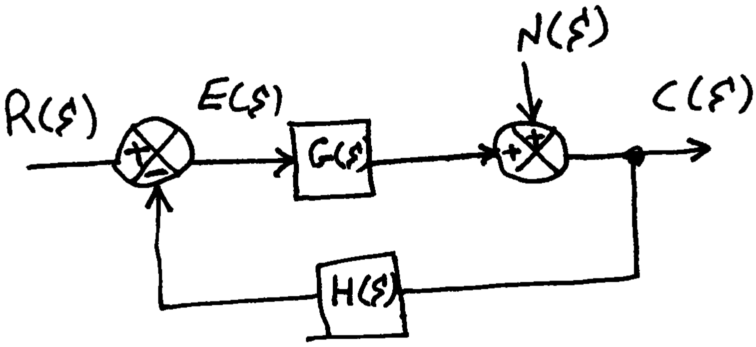
Construction of signal flow graph :-

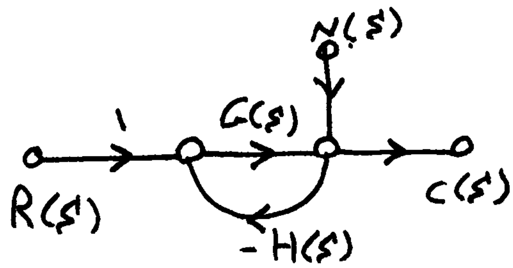












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Mason's Gain Formula for signal flow graph

In many practice cases, we wish to determine the relationship between an input variable and out put variable of the signal flow graph. The transmittance between an input node and an output node is the overall gain, or overall transmittance between these two nodes.

Mason's gain formula which is applicable to the overall gain, is given by

$$P = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$$

P_K = Path gain or transmittance of K_{th} forward path

Δ = determinint of graph

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$\Delta = 1 -$ (Sum of all individual loop gain) + (Sum of gain Products of all Possible combinations of two nontouching loops) − (Sum of gain Product of all Possible combinations of three nontouching Loops) + ...

$\sum_{a} L_a$ = Sum of all individual loops gain

$\sum_{b,c} L_b L_c$ = sum of gain Products of all Possible combinations of two nontouching loops

$\sum_{d,e,f} L_d L_e L_f$ = sum of gain Products of all Possible combinations of three nontouching loops.

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ΔK = cofactor of the K_{th} forward path determinant of the graph with the loops touching the Kth forward path removed, that is the cofactor Δ_K is obtained from Δ by removing the loops that touch path P_K

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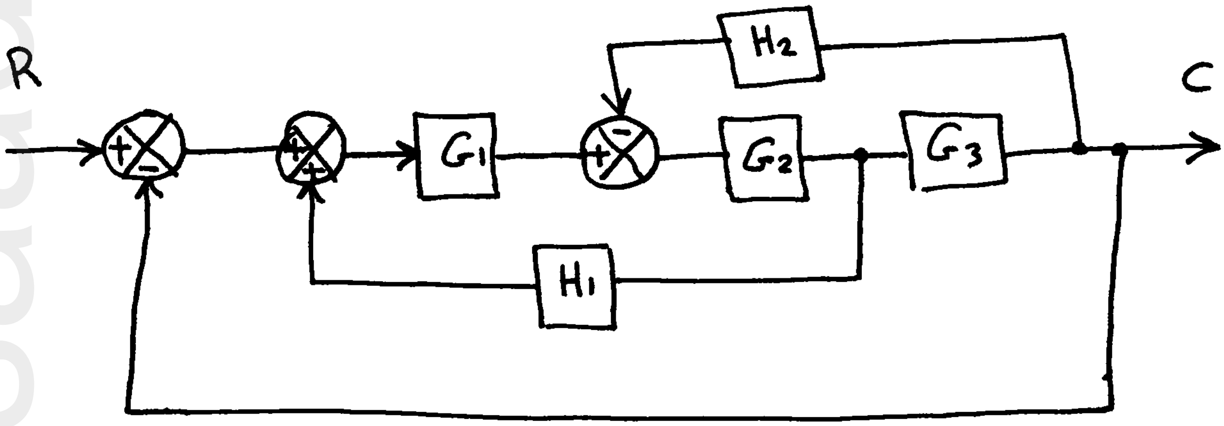
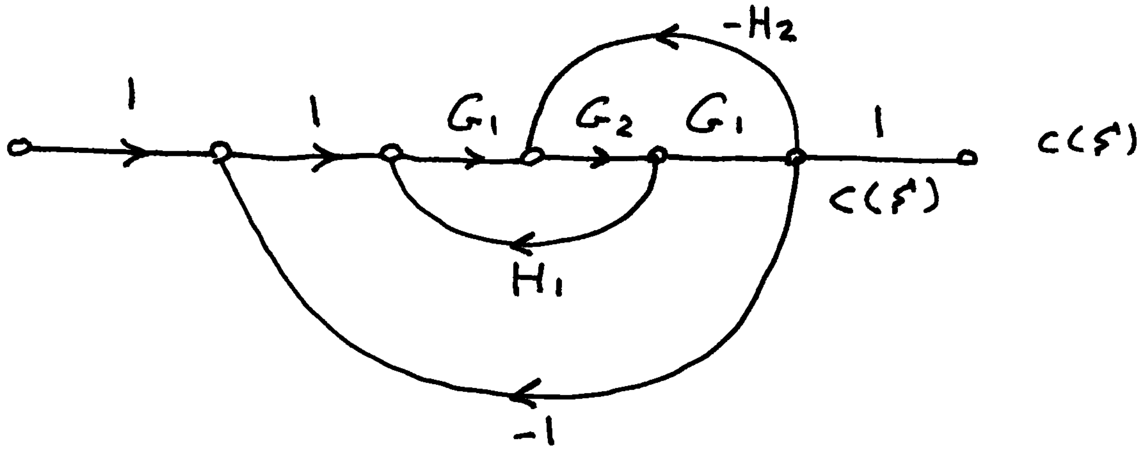
Example :

obtain the close loop transfer function $C(s)/R(s)$ by using Mason's gain formula.



Solution :- Signal flow graph for system



* Forward Path

$$P_1 = G_1 G_2 G_3$$

* Loops

$$L_1 = G_1 G_2 H_1$$

$$L_2 = -G_2 G_3 H_2$$

$$L_3 = -G_1 G_2 G_3$$

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$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$= 1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3$$

$$\Delta_1 = 1$$

$$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1}{\Delta}$$

$$= \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

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