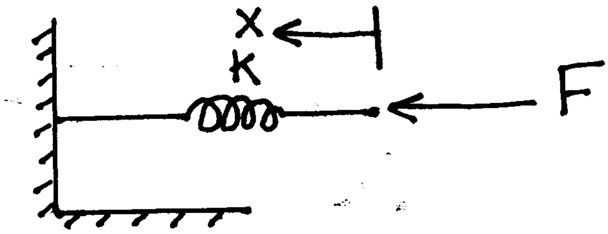
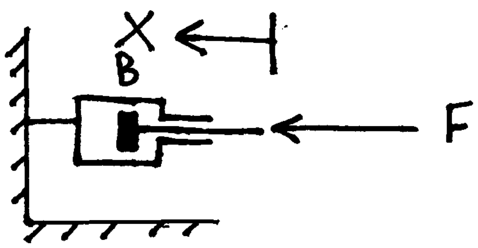
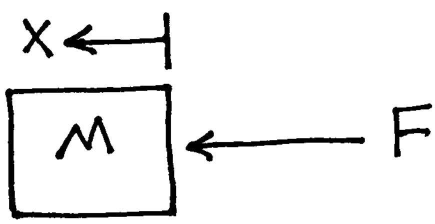
2.4. Modeling of Mechanical system

A- Mechanical translation system :-

Spring		$F = KX$
Dashpot		$F = B\frac{dx}{dt}$
Mass		$F = M\frac{d^2x}{dt^2}$

* Mechanical translation symbols and units

f : force (Newton)
X : Distance (Meter)
V : Velocity (Meter/sec)
a : acceleration (Meter/sec^2)
M : mass (Kilogram)
K : stiffness coefficient (Newton/meter)
B : Damping coefficient (Newton/(meter/sec))

1. Spring :

The elastance, or stiffness K Provides arestoring force as represented by aspring. the reaction force (f) on each end of the spring is the same and is equal to the Product of stiffness K and the amount of deformation of the spring. end 1 has the Position X_1 and end 2 has the Position X_2 measured from the respective equilibrium Positions.

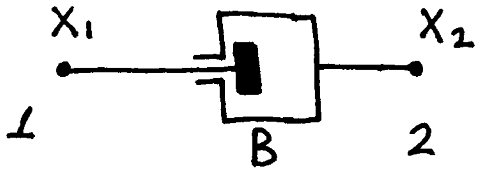
The force equation in accordance with the "Hooks Law" is

$$f = K(X_1 - X_2)$$

* if the end 2 is stationary, then $X_2 = 0$ and the above equation become

$$f = K X_1$$

2. dashpot :



The reaction damping force f is approximate by the product of damping B and the relative Velocity of the two ends of the dasphot. The direction of the force depends on the relative Magnitude and direction of this force depend on the relative magnitudes and direction of the Velocities

$$f = b \cdot \frac{d(X_1 - X_2)}{dt} \quad \text{time domain}$$

$$F = b\,s\,(X_1 - X_2) \quad s \text{ domain}$$

3. Mass :

A force applied to the mass produces an acceleration of the mass. The reaction force f is equal to the product of mass and acceleration and is opposite in direction to the applied force in term of displacement X, the force equation is

$$f = Ma = M\frac{d}{dt}V = M\frac{d^2}{dt^2}X$$

where :-

a: is the acceleration

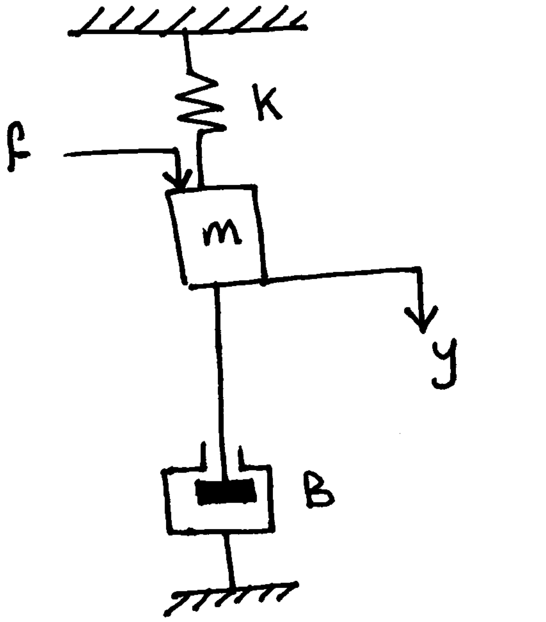
V: is the Velocity

In S Domain

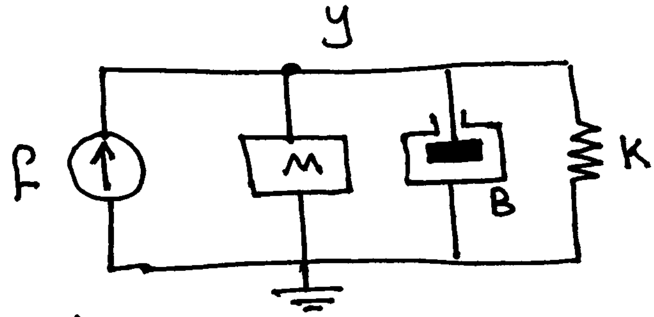
$$F = Ma = MSV = MS^2X$$

Example :

Obtain the transfer function $Y(s)/F(s)$ of the mechanical system below



Solution :-



$$f = m\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Ky$$

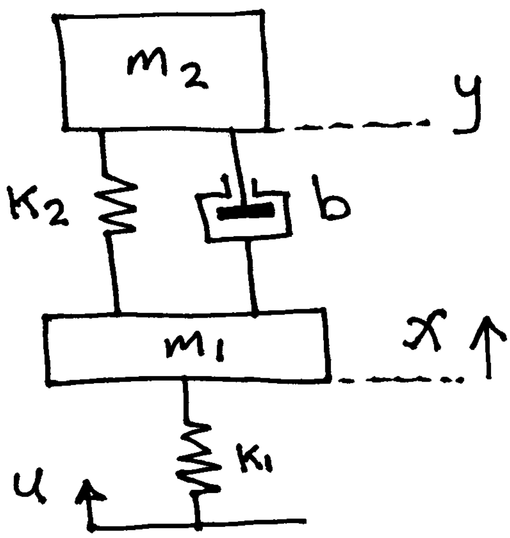
$$m s^2 Y(s) + B s Y(s) + K Y(s) = F(s)$$

$$(m s^2 + B s + K)\, Y(s) = F(s)$$

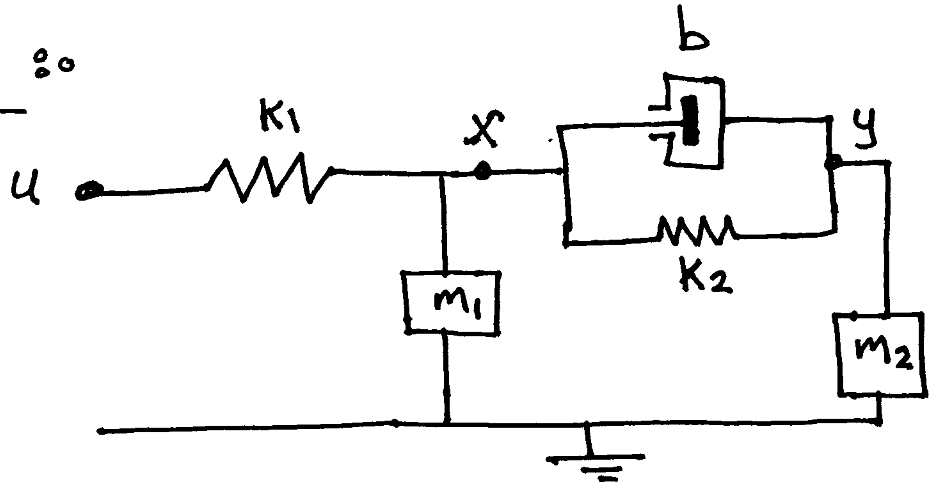
$$T.f = \frac{Y(s)}{F(s)} = \frac{1}{ms^2 + Bs + K}$$

Example :

For the following system, obtain an expression for the transfer function in form $\frac{Y(s)}{U(s)}$



Solution :



at Point y :

$$m_2 \frac{d^2 y}{dt^2} + b \frac{d}{dt}(y-x) + K_2 (y-x) = 0$$

at Point x :

$$m_1 \frac{d^2 x}{dt^2} + b \frac{d}{dt}(x-y) + K_2 (x-y) + K_1 (x-u) = 0$$

by taking Laplace transform

$$* \quad m_2 s^2 Y(s) + bs\,(Y(s) - X(s)) + K_2 (Y(s) - X(s)) = 0$$

$$m_2 s^2 Y(s) + bs\,Y(s) - bs\,X(s) + K_2 Y(s) - K_2 X(s) = 0$$

$$Y(s)\left[m_2 s^2 + bs + K_2\right] = X(s)\left[bs + K_2\right] \quad \text{---} \; 1$$

$$* \quad m_1 s^2 X(s) + bs\,(X(s) - Y(s) + K_2 (X(s) - Y(s)$$
$$+ K_1 (X(s) - U(s) = 0$$

$$m_1 s^2 X(s) + bs\,X(s) - bs\,Y(s) + K_2\,X(s) - K_2 Y(s)$$
$$+ K_1 X(s) - K_1 U(s)$$

$$X(s)\left[m_1 s^2 + bs + K_2 + K_1\right] = Y(s)\left[bs + K_2\right] + K_1 U(s)$$
$$\text{---} \; 2$$

from eq 1

$$X(s) = \frac{m_2 s^2 + bs + K_2}{bs + K_2} Y(s) \quad \text{----} (3)$$

Put eq 3 in eq 2 we get :-

$$(m_1 s^2 + bs + K_1 + K_2) \frac{m_2 s^2 + bs + K_2}{bs + K_2} Y(s)$$

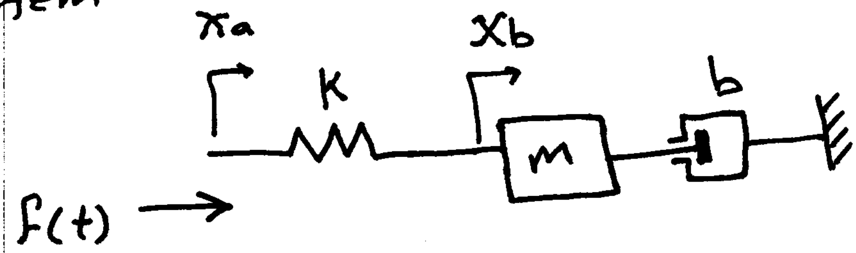
$$= (bs + K_2) Y(s) + K_1 U(s)$$

$$\frac{Y}{U} = \frac{K_1 (bs + K_2)}{m_1 m_2 s^4 + (m_1 + m_2) b s^3 + (K_1 m_2 + K_2 m_1 + K_2 m_2) s^2 + K_1 b s + K_1 K_2}$$

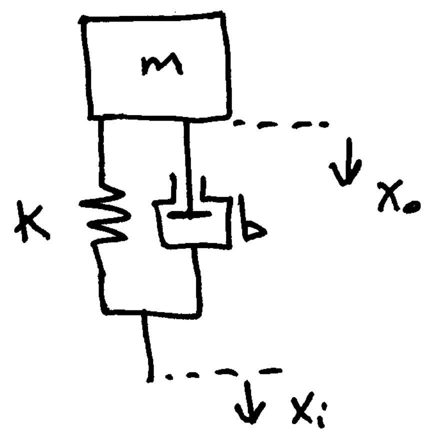
Saif Asaad

H.W

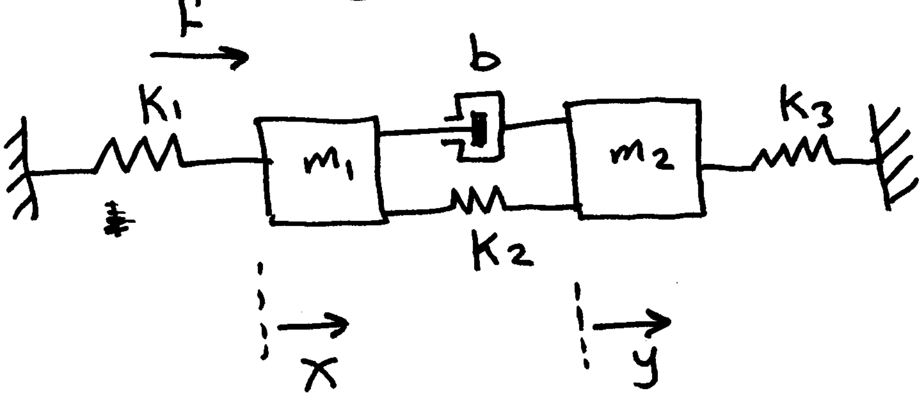
Find the transfer function in form of $Xb(s)/F$ for the following mechanical system



H.W For the following system find the transfer function of $\frac{X_o(s)}{X_i(s)}$



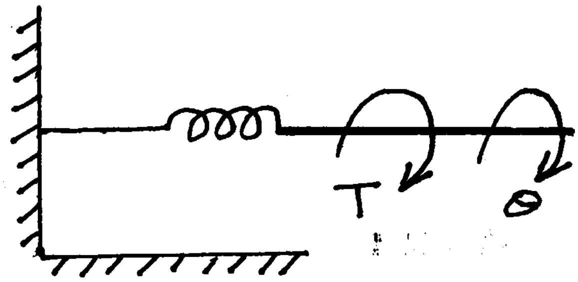
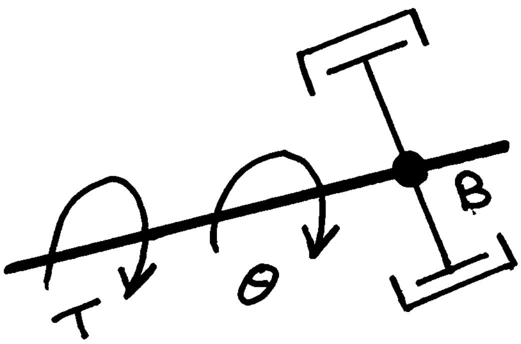
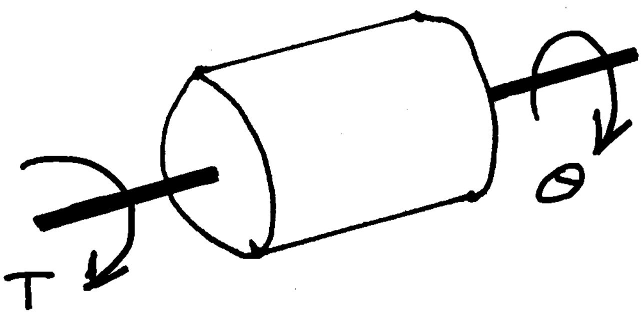
H.W For the following mechanical system



a) Drive the differential equation

b) Determine the transfer function in form $\frac{Y(s)}{F(s)}$

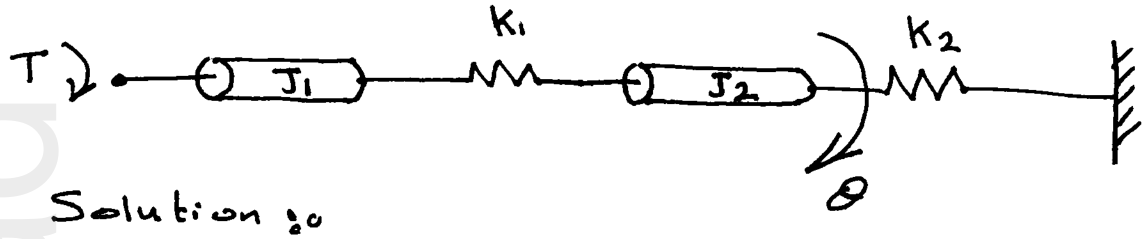
B- Mechanical Rotational system

Spring		$T = K\theta$
Dashpot		$T = B\frac{d\theta}{dt}$
Moment of inertia		$T = J\frac{d^2\theta}{dt^2}$

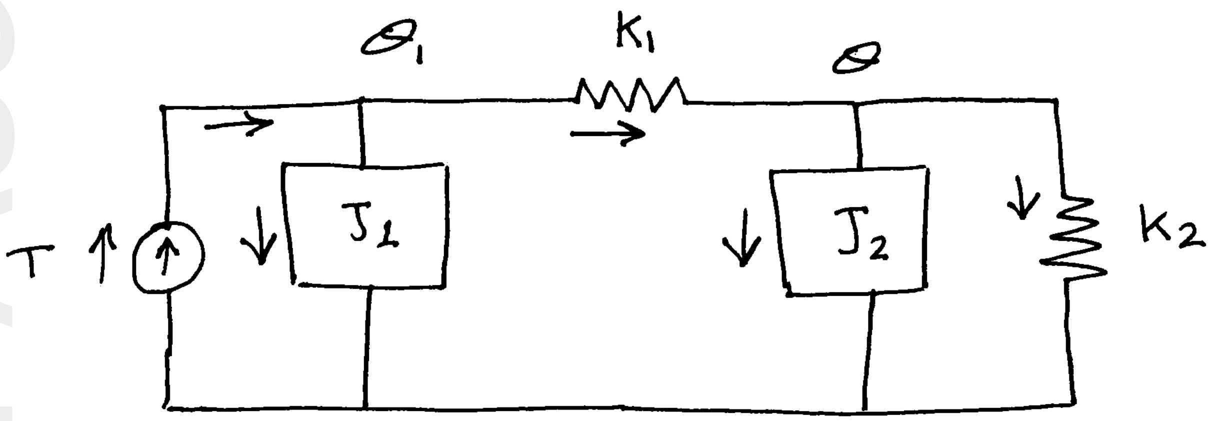
Mechanical Rotational symbols and units

T	Torque	Newton - meters
θ	Angle	Radians
ω	Angular Velocity	Radians / second
α	Angular acceleration	Radians / $second^2$
j	Moment of inertia	Kilogram - $meters^2$
K	Stiffness coefficent	$\frac{\text{Newton - meters}}{\text{radian}}$
B	Damping coefficent	$\frac{\text{Newton - meters}}{\text{radians / second}}$

Example : For the following mechanical system Find T.F in term of $\frac{\theta}{T}$



Solution :



at Point θ_1

$$T = J_1 s^2 \theta_1 + K_1(\theta_1 - \theta)$$

at Point θ

$$0 = K_2 \theta + J_2 s^2 \theta + K_1(\theta - \theta_1)$$

$$T = (J_1 s^2 + K_1)\,\theta_1 - K_1\,\theta \quad \text{---} (1)$$

$$K_1\,\theta_1 = (J_2 s^2 + K_1 + K_2)\,\theta$$

$$\therefore\ \theta_1 = \frac{J_2 s^2 + K_1 + K_2}{K_1} \quad \text{——} *$$

نعوض في معادلة رقم ١

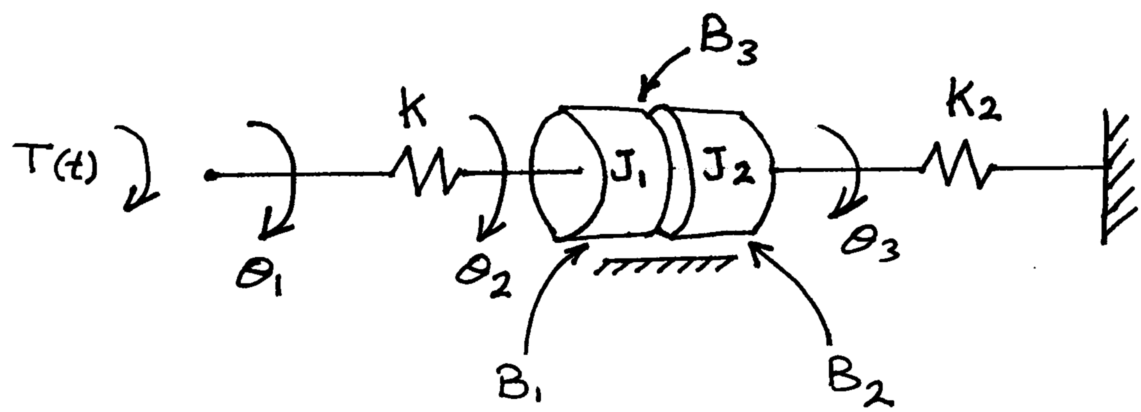
$$T = (J_1 s^2 + K_1)\,\frac{J_2 s^2 + K_1 + K_2}{K_1}\,\theta - K_1\,\theta$$

$$T = \frac{J_1 J_2 s^4 + (J_1 K_1 + J_1 K_2 + J_2 K_1) s^2 + K_1 K_2}{K_1}\,\theta$$

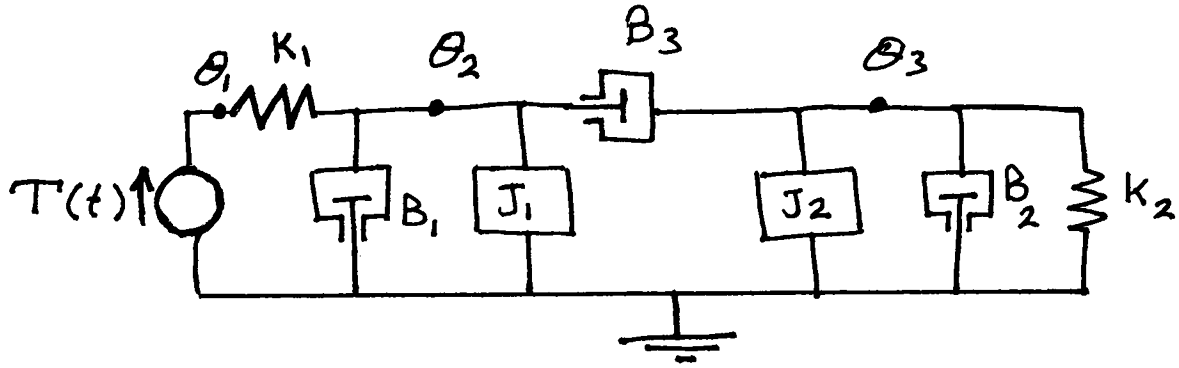
$$\frac{\theta}{T} = \frac{K_1}{J_1 J_2 s^4 + (J_1 K_1 + J_1 K_2 + J_2 K_1) s^2 + K_1 K_2}$$

Example :

Obtain mathematical model of the mechanical system shown in the fig. below



Solution :



Node θ_1

$$T = K_1(\theta_1 - \theta_2)$$

$$= K_1\theta_1 - K_1\theta_2$$

Node θ_2

$$0 = K_1(\theta_2 - \theta_1) + b_1\dot{\theta}_2 + b_3(\dot{\theta}_2 - \dot{\theta}_3) + J_1\ddot{\theta}_2$$

$$0 = (s^2 J_1 + s(b_1 + b_3) + K_1)\theta_2 - s b_3 \theta_3 - K_1\theta_1$$

Node 3

$$0 = J_2\ddot{\theta}_3 + b_2\dot{\theta}_3 + K_2\theta_3 + b_3(\dot{\theta}_3 - \dot{\theta}_2)$$

$$0 = (s^2 J_2 + s(b_2 - b_3) + K_2)\theta_3 - s b_3 \theta_2$$