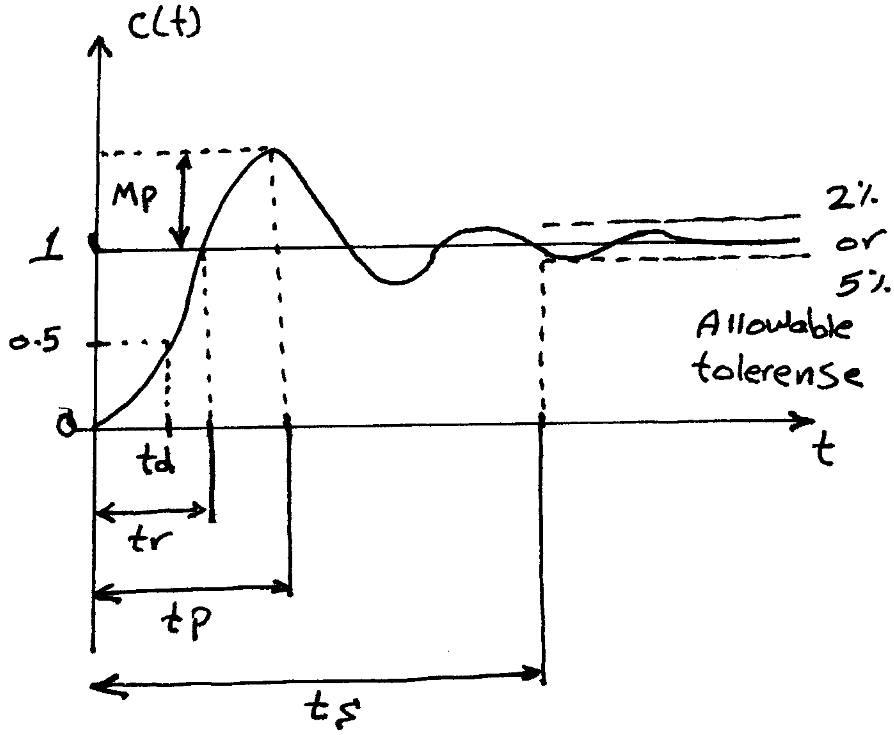
Definition of transient-Response Specifications:-

The transient-response of a practical control system often exhibits damped oscillation before reaching steady state. In specifying the transient response characteristics of a control system to a unit step input it is common to specify the following:-

1. Delay time, t_d
2. Rise time, t_r
3. Peak time, t_P
4. Maximum overshoot M_P
5. Setting time, t_s



1- <u>Delay time, td :-</u> The delay time is the time required for the response to reach half the final value.

2- <u>Rise time, tr :-</u> is the time required for the response to rise from 10% to 90%, 5% to 95% or 0 to 100% of its final value. For underdamped second-order systems the 0 to 100% rise time is normally used. For overdamped systems the 10% to 90% rise time is commonly used.

3- **Peak time, t_p:** The Peak time is the time required for the response to reach the first Peak of the overshoot.

4- **Maximum (Percent) over shoot:-** The max. overshoot (MP) is the maximum Peak value of the response curve measured from unity

$$\text{Maximum Percent overshoot} = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100\%$$

* The amount of the Maximum (Percent) overshoot direct indicates the relative Stability of the system

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

5- setting time, ts : The Setting time is the time required for the response curve to reach and stay with in arange final value of size specification by absolute Persentage of the final Value (usually 2% or 5%).

Notes

$$tr = \frac{\tan^{-1}\left(\sqrt{1-\zeta^2}/-\zeta\right)}{wd}$$

$$tr = \frac{\pi - \beta}{wd}$$

where wd : is the damped natural

$$wd = wn\sqrt{1-\zeta^2}$$

Percent max. over shoot (P.o.S)

$$P.o.S = 100\, e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

tP = time to first over shoot $= \frac{\pi}{wd}$

Example :- For the system shown in fig. determine

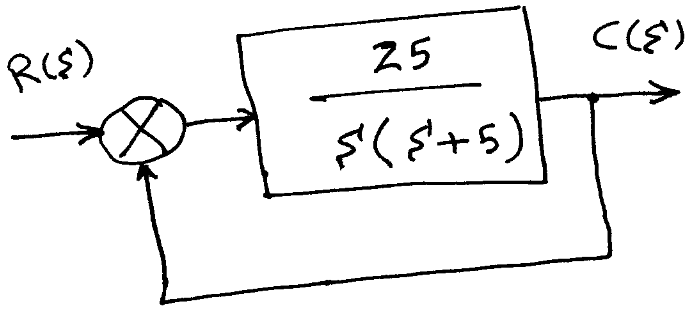
1. natural freq.

2. damping ratio

3. damped natural frequency.

4. rise time.

5. Percent over shoot



Sol

$$\frac{C(s)}{R(s)} = \frac{25}{s^2 + 5s + 25} \Leftarrow \text{ch/c equ.}$$

1) $s^2 + 5s + 25 = 0 \Rightarrow w_n^2 = 25 \Rightarrow w_n = 5$

2) $2 \zeta w_n = 5 \Rightarrow \zeta = \frac{5}{10} = 0.5$ damping ratio

3) damping natural freq. $w_d = w_n \sqrt{1-\zeta^2}$

$$= 5\sqrt{1-0.5^2}$$

$$= 4.33 \text{ rad/s}$$

4) rising time $t_r = \dfrac{\tan^{-1}(\sqrt{1-\zeta^2}/-\zeta}{w_d}$

$$= \frac{\tan^{-1}(\sqrt{0.75}/-0.5)}{4.33}$$

$$= 0.4838$$

5) Percent over shoot $P_{:0.5} = 100\, e^{-\zeta\pi\sqrt{1-\zeta^2}}$

$$= 100\, e^{-\pi/\sqrt{3}}$$

$$= 16.3\%$$

Example:- obtain an expression for unit step time response of a control system whose transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{1.68}{s^3 + 12.3 s^2 + 3.74 s + 1.68}$$

Sol:-

$$\frac{C(s)}{R(s)} = \frac{1.68}{(s+12)(s^2+0.3s+0.14)}$$

$r(t)$: unit step $\Rightarrow R(s) = \frac{1}{s}$

$$\therefore C(s) = \frac{1.68}{s(s+12)(s^2+0.3s+0.14)}$$

by using Partial fraction

$$C(s) = \frac{K_1}{s} + \frac{K_2}{s+12} + \frac{K_3 s + K_4}{s^2+0.3s+0.14}$$

$K_1 = 1$

$K_2 = -0.0009$

$$K_{a+jb} = \left[\frac{1.68}{s^2 + 12s} \right]_{a+jb}$$

$$-2a = 0.3 \Rightarrow a = -0.15$$

$$a^2 + b^2 = 0.14$$

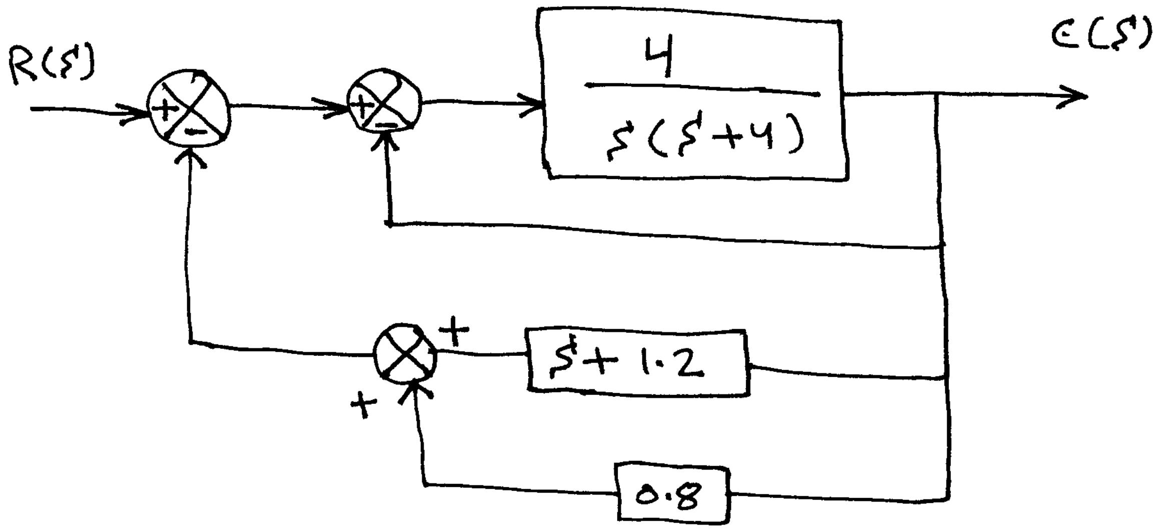
$$\therefore K_{a+jb} = 0.38 \angle -115$$

general solution

$$c(t) = \frac{1}{b} |K_{a+jb}| e^{-at} \sin(bt + \alpha) + K_1 e^{r_1 t} + K_2 e^{r_2 t}$$

$$c(t) = 1 - 0.0009 e^{-12t} - 1.006 e^{-0.15t} \cos(0.34t) - 0.47 e^{-0.15t} \sin(0.34t)$$

Example For the control system shown, find the output response $c(t)$ if the input is a step function of magnitud 2



Sol

$$\frac{C(s)}{R(s)} = \frac{4}{(s+2)(s+6)} \quad , \quad R(s) = \frac{2}{s}$$

$$c(s) = \frac{8}{s\,(s+2)(s+6)}$$

using Partial fraction

$$c(s) = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+6}$$

$$K_1 = \frac{2}{3}$$

$$K_2 = -1$$

$$K_3 = \frac{1}{3}$$

Taking inverse laplace transform on both sides

$$\therefore \quad c(t) = \frac{3}{2} - e^{-2t} + \frac{1}{3} e^{-6t}$$

Example:- Aunity feed back control system has its open loop T.f given by $G(s) = \frac{4s+1}{4s^2}$

Determine the output time response when the system is subjected to

a) unit impulse input function

b) unit step input function.

Sol

$$\frac{C(s)}{R(s)} = \frac{s + 0.25}{(s + 0.5)^2}$$

a) for impulse input $R(s) = 1$

$$\therefore C(s) = \frac{s + 0.25}{(s + 0.5)^2} = \frac{K_1}{(s + 0.5)} + \frac{K_2}{(s + 0.5)^2}$$

$K_1 = 1$ & $K_2 = -0.25$

$$\therefore C(s) = \frac{1}{s + 0.5} - \frac{0.25}{(s + 0.5)^2}$$

by taking Inverse laplace transformed

$$c(t) = e^{-0.5t} - 0.25\, t\, e^{-0.5t}$$

$$= e^{-0.5t}(1 - 0.25t)$$

b) for unit step Input, $R(s) = \frac{1}{s}$

$$\therefore C(s) = \frac{1}{s} \frac{s+0.25}{(s+0.5)^2} = \frac{K_1}{s} + \frac{K_2}{s+0.5} + \frac{K_3}{(s+0.5)^2}$$

$K_1 = 1$

$K_2 = -1$

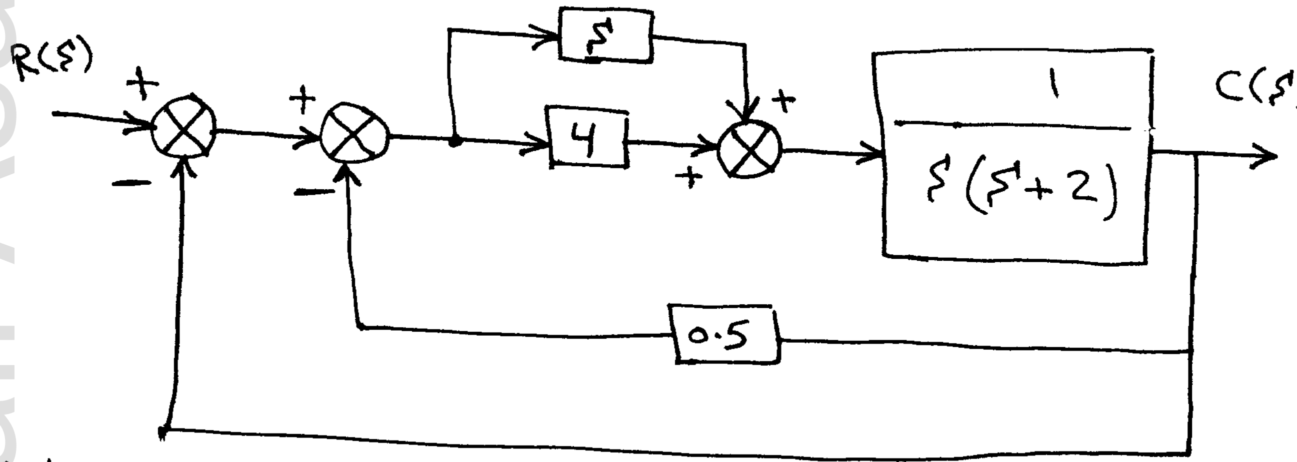
$K_3 = 0.5$

$$\therefore C(s) = \frac{1}{s} - \frac{1}{s+0.5} + \frac{0.5}{(s+0.5)^2}$$

$$\therefore c(t) = 1 - e^{-0.5t} + 0.5t\, e^{-0.5t}$$

$$c(t) = 1 - e^{-0.5t}(1 - 0.5t)$$

Example :- For the control system shown, obtain the o/p c(t) for unit step input



Sol

$$\frac{C(s)}{R(s)} = \frac{s+4}{s^2+3.5\,s+6} \quad , \quad R(s) = \frac{1}{s}$$

$$-2a = 3.5 \; , \quad a = -1.75 \; ; \quad a^2+b^2 = 6$$

$$b = 1.71$$

$a \mp jb = -1.75 \pm j1.71$ complex conjugate Poles

$$c(s) = \frac{s+4}{s(s^2+3.5s+6)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2+3.5s+6}$$

General Solution

$$c(t) = \frac{1}{b} \left| K_{a+jb} \right| e^{at} \sin(bt+\alpha) \, K_1 e^{r_1 t}$$

$$K_1 = \lim_{s' \to 0} s' \frac{s'+4}{s'(s'^2 + 3.5 s' + 6)} = 0.667$$

$$K_{a+jb} = \left. \frac{s'+4}{s'} \right|_{s' = -1.75 + j1.71}$$

$$= \frac{2.25 + j1.75}{-1.75 + j1.71} = 1.164 \angle -97.73$$

$$c(t) = 0.68 e^{-1.75t} \sin(1.71t - 97.73 \times \pi/180) + 0.667$$

$$= 0.667 - 0.09 e^{-1.75t} \sin(1.71t - 0.67 e^{-1.75t}$$

$$\cos(1.71t))$$

Example For a unity feed back control system shown, Determine

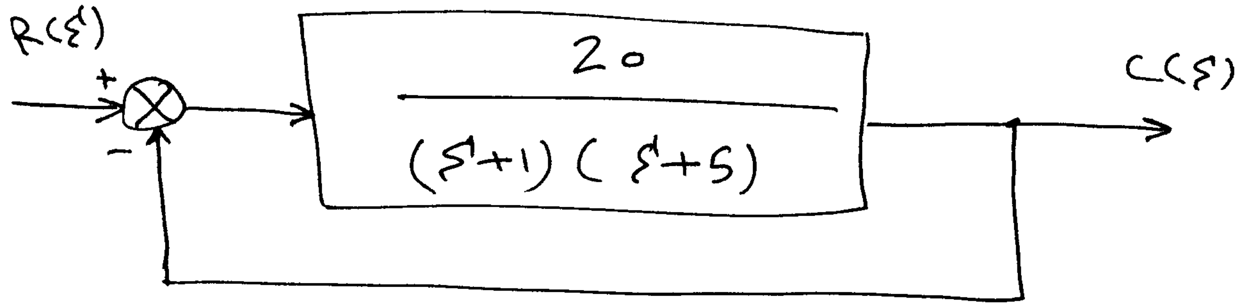
a) characteristic equation

b) w_n , ζ , w_d , t_p , M_p

c) the time at which the first under shoot occurs

d) the time period of oscillation

e) no. of cycles completed before reaching the steady state.



ch/c equation $\Rightarrow 1 + GH = 0$

$$s^2 + 6s + 25 = 0$$

$w_n^2 = 25 \Rightarrow w_n = 5$

$2\zeta w_n = 6 \Rightarrow \zeta = \frac{6}{10} = 0.6$

$w_d = w_n\sqrt{1-\zeta^2} = 4$ rad/sec

time at which Max. overshoot occurs

$$t_p = \frac{\pi}{w_d} = 0.78 \text{ Sec}$$

The max over shoot is given by $M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$

$$= 9.4$$

The time at which the first undershoot

occurs is $t = \frac{2\pi}{w_d} = \frac{2\pi}{4} = 1.57 \text{ Sec}$

The Period of oscillation $= \frac{2\pi}{w_d} = 1.57 \text{ Sec}$

the steady state reached in time

$$t_s = 4/\zeta w_n$$

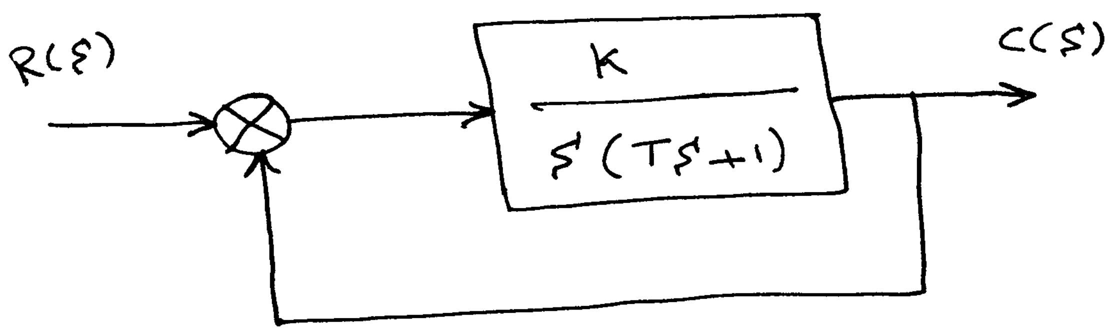
$\therefore$ the no. of oscillation completed

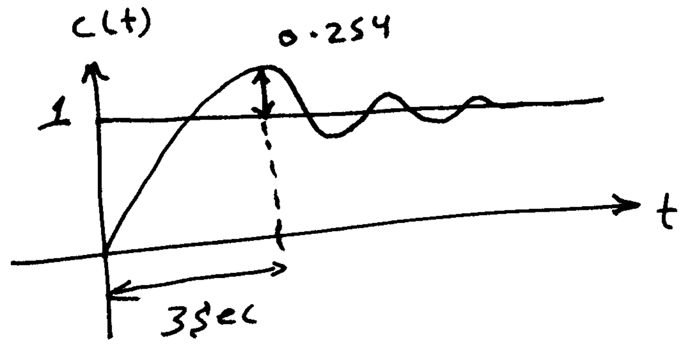
before reaching steady state is

$$w_d \times \frac{4}{\zeta w_n} = 4 \frac{4}{0.6 \times 5} = 5.3 \text{ cycle}$$

Saif Asaad

H.W 1 when the system shown below is subjected to unit step input, the system output response as shown in fig Determine the Values of K & T from the response





$$MP = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$\zeta = \underline{\underline{??}}$

$\downarrow$

$tp = \frac{\pi}{wd}$ $\quad wd = ✓$

$\downarrow$

$wd = wn\sqrt{1-\zeta^2}$ $\quad wn = ✓$

Ans

$\zeta = 0.4$

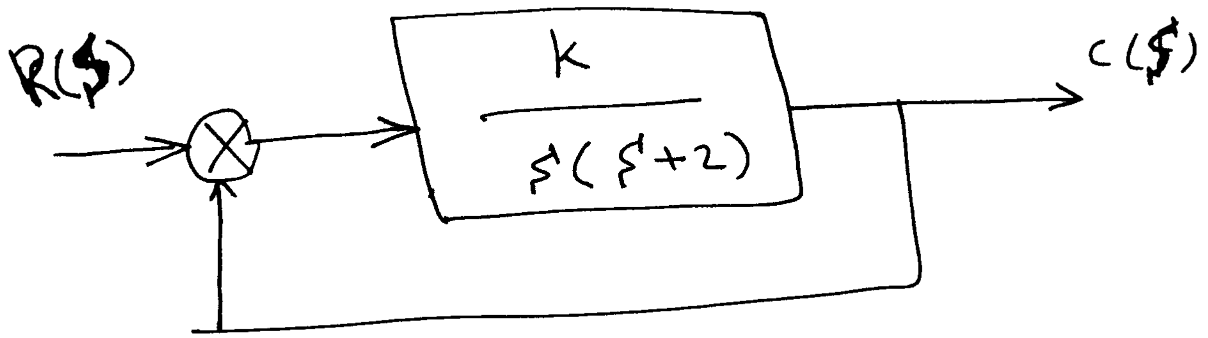
$wn = 1.14$

$K = 1.42$

$T = 1.09$ Sec

H.W 2

For unity feed back system shown in fig determine the value of K so that c(t) has an over shoot of no more than 10% in response to a unit step



Ans

$K = 3.2$

H.W 3

For the electric circuit shown in fig find the following assume all initial condition is Zeros

1) The T.f $\frac{E_o(s)}{E_i(s)}$

2) The damping ratio ζ and undamped natural freq ω_n of system

3) The value of R that will result in $e_o(t)$ having an overshoot of no more than 25%

assume $e_i(t)$ is a unit step, $L = 10\,mH$

$C = 4\,\mu f$