Routh Stability Criterion :-

The Routh criterion is a method for determing continuous system stability, for systems with nth-order characteristic equation of the form:-

$$a_0 S^n + a_1 S^{n-1} + \dots + a_{n-1} S + a_n = 0$$

The criterion is applied using a Routh table defined as follows:-

S^n	a_0	a_2	a_4	a_6	---
S^{n-1}	a_1	a_3	a_5	a_7	---
S^{n-2}	b_1	b_2	b_3	b_4	---
S^{n-3}	c_1	c_2	c_3	c_4	---
S^{n-4}	d_1	d_2	d_3	d_4	---
$\vdots$	$\vdots$				
S^1	f_1				
S^0	g_1				

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{a_1 a_5 - a_1 b_3}{b_1}$$

$$d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1}$$

$$d_2 = \frac{c_1 b_3 - b_1 c_3}{c_1}$$

The table above is continued horizontally and vertically until only zeros are obtained. any row can be multiplied by a positive constant before the next row is computed without disturbing the properties of the table.

* The Routh criterion: all the roots of the characteristic equation negative real parts if any only if the elements of the first coulumn of the Routh table have the same sign otherwise the number of roots with positive real parts is equal to the number of changes of sign

<u>EX 5.1</u> The characteristic equation of afeed back control system is

$$s^3 + 6s^2 + 12s + 8 = 0$$

check its stability using Routh criterion ??

Solution

$$\begin{array}{c|ccc} s^3 & 1 & 12 & 0 \\ s^2 & 6 & 8 & 0 \\ s^1 & \frac{64}{6} & 0 & \\ s^0 & 8 & & \end{array}$$

* Since there are no change of sign in the first column of the table, all the roots of the equation have negative real parts and hence the system is stable

EX 5.2 The characteristic equation of a feedback control system is

$$s^3 + 3s^2 + 3s^1 + 1 + K = 0$$

Determine the range of K for which the system is stable

$$\begin{array}{c||ccc}
s^3 & 1 & 3 & 0 \\
s^2 & 3 & 1+K & 0 \\
s^1 & \frac{8-K}{3} & 0 & \\
s^0 & 1+K & &
\end{array}$$

* no sign changes in the first coulumn it is ~~nes~~ necessary that the conditions $8-K>0$ & $1+K>0$ be satisfied.

thus the range of K is

$$-1 < K < 8$$ for system stable

Special case 8o

if a first - coulmn term in any row is Zero but the remaining terms are not Zero or there is no remaining term then the Zero term is replaced by a very small Positive number ϵ and the result of the array is evaluated.

EX 5.3

The characteristic equation of a feed-back control system is $s^3+2s^2+s^1+2=0$

check its stability using Routh criterion ??

Solution

s^3	1	1
s^2	2	2
s^1	$0 \cong \epsilon$	
s^0	2	

Since There are no change in the first coulmn Sign of table all the roots of the equation have negative real Parts and hence the system is stable

EX 5.4 The characteristic equation of a feedback Control system is

$$s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0$$

Check its stability using Routh cretirion ??

s^5	1	24	-25
s^4	2	48	-50 ← Auxilary Polynomial $P(s)$
s^3	0	0	

The auxilary Polynomial $P(s)$ is

$$P(s) = 2s^4 + 48s^2 - 50$$

$$\frac{dP(s)}{ds} = 8s^3 + 96s$$

$\therefore$

s^5	1	24	-25
s^4	2	48	-50
s^3	8	96	← cofficent of $\frac{dP(s)}{ds}$
s^2	24	-50	
s^1	112.7	0	
s^0	-50		

we see that there is one change in sign in the first coulmn of the new array. So the original equation has one root with apositive real part and hence the system is unstable

EX 5.5

The characteristic equation of a feed back control system is

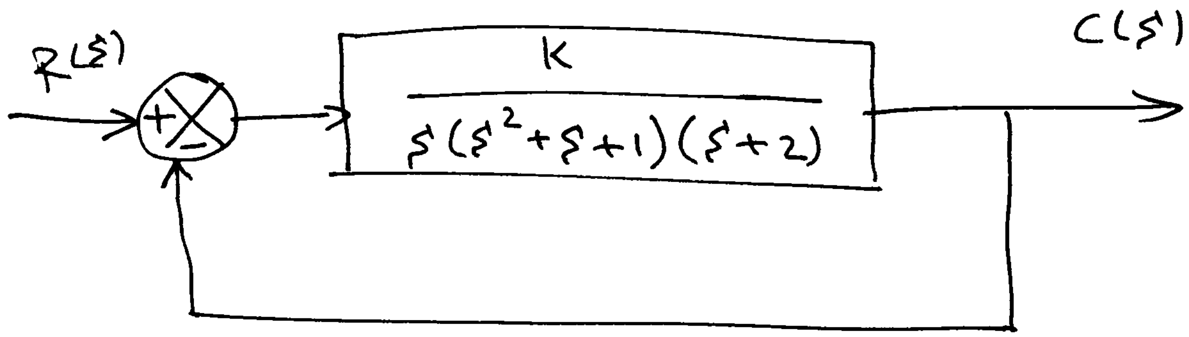
$$2s^3 + 4s^2 + 4s + 12 = 0$$

check it stability using Routh's criterion?

$$\begin{array}{c|cc} s^3 & 2 & 4 \\ s^2 & 1 & 3 \\ s^1 & -1 & 0 \\ s^0 & 3 & \end{array}$$

Since there are two changes of sign in the first ~~cont~~ column of the Routh table, the equation above has two roots with positive real parts and the system is not stable.

H.W consider the system shown



Determine the range of K for the system is stable?

ANS

$$\frac{14}{9} > K > 0$$

H.W (2) Find the range of K for stable system

$$\frac{Y(s)}{R(s)} = \frac{K(s+2)}{s(s+5)(s^2+2s+5)+K(s+2)}$$

Ans

$$0 < K < 28.1$$

H.W For the system shown determine the range of K for system is stable?