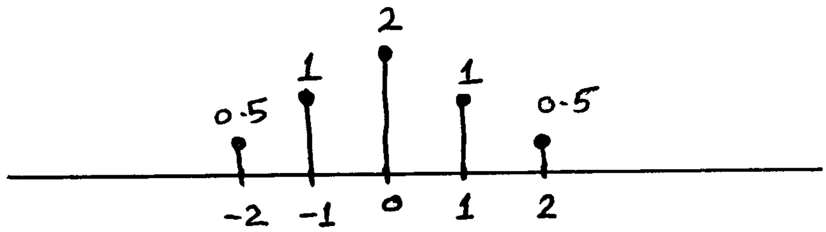
Time Domain Analysis

Consider $x(n)$ as shown in fig.



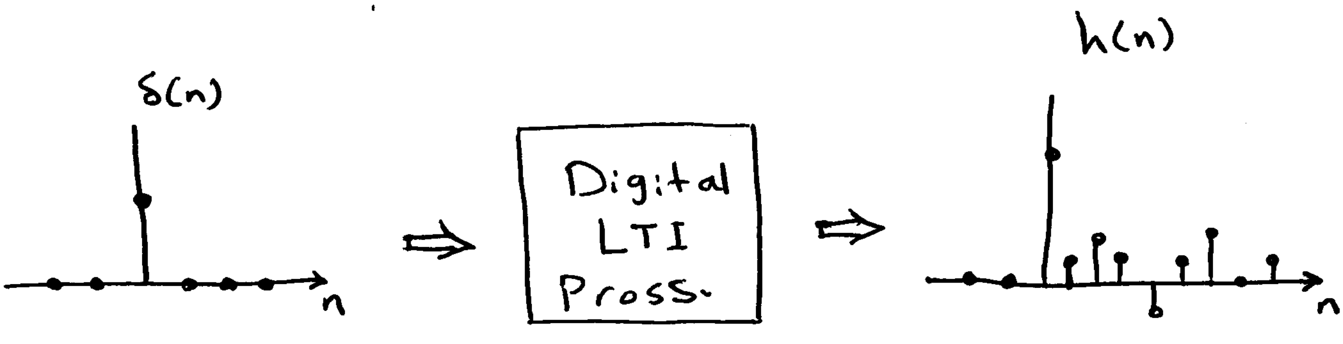
we can define the complete signal $x(n)$ as

$$x(n) = x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2)$$

or $$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

Desecribing Digital LTI Processor

1- The Impulse Response

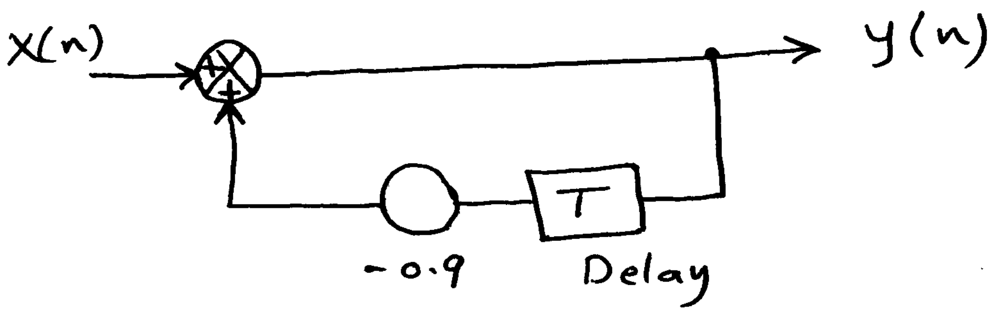


Any output signal observed after $n=0$ must be the characteristic of the Processor it self

The response is called impulse response of the system and given by symbol $h(n)$

EX Find the first four sample values of the impulse response $h(n)$ for each of the following digital Processors:-

a)



Solution:-

$$y(n) = -0.9\, y(n-1) + x(n)$$

The impulse response is

$$h(n) = -0.9\, h(n-1) + \delta(n)$$

The system cleary causal so that $h(n)=0$

for $n<0$, hence

$$h(0) = -0.9\, h(-1) + \delta(0) = 0+1 = 1$$

$$h(1) = -0.9\, h(0) + \delta(1) = -0.9\times 1 + 0 = -0.9$$

$$h(2) = -0.9\, h(1) = 0.81$$

$$h(3) = -0.9\, h(2) = -0.729$$

b) system $y(n) = x(n) + x(n-1) + x(n-2) + \cdots$

sol

system is causal so $h(n)=0$

for $n<0$

$$h(0) = \delta(0) + \delta(-1) + \delta(-2) + \cdots = 1$$

$$h(1) = \delta(1) + \delta(0) + \delta(-1) + \cdots = 1$$

$$h(2) = \delta(2) + \delta(1) + \delta(0) + \cdots = 1$$

$$h(3) = \delta(3) + \delta(2) + \delta(1) + \delta(0) + \cdots = 1$$

2- The Step Response:-

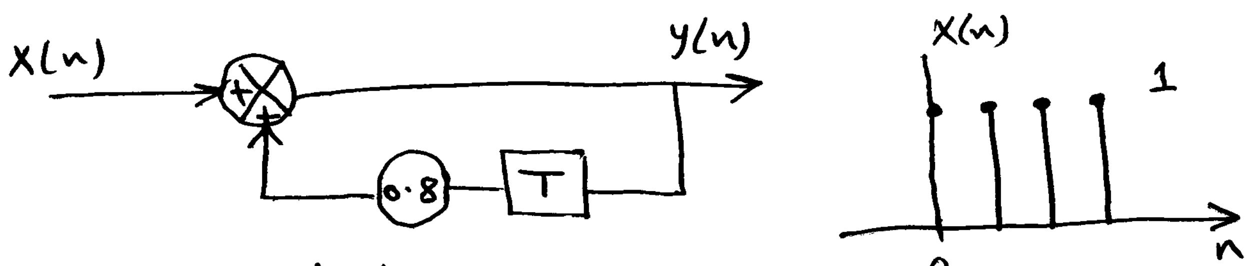
The step response of LTI Processor is the running sum of its impulse response and its denoted as $S(n)$

$$S(n) = \sum_{m=-\infty}^{n} h(m)$$

Alternatively, $h(n)$ is the first order differnce of $S(n)$

$$h(n) = S(n) - S(n-1)$$

<u>EX</u> Find and sketch the first few sample values of the impulse and step response of A system given in fig below Also determine the finil value of $S(n)$ as $n \to \infty$. Use your results to find the response to the rectangular Pulse input shown in Part (b) of the fig



(a)

Sol

$$y(n) = 0.8\, y(n-1) + x(n)$$

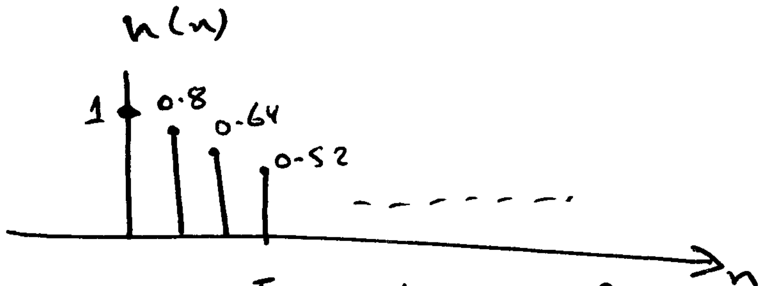
$$h(n) = 0.8\, h(n-1) + \delta(n)$$

$h(0) = 1$

$h(1) = 0.8$

$h(2) = 0.64$

$h(3) = 0.52$ and so on ...



Impulse response

Unit step response equals to the running sum of h(n) Hence its first few value are

$S(0) = h(0) = 1$

$S(1) = h(0) + h(1) = 1.8$

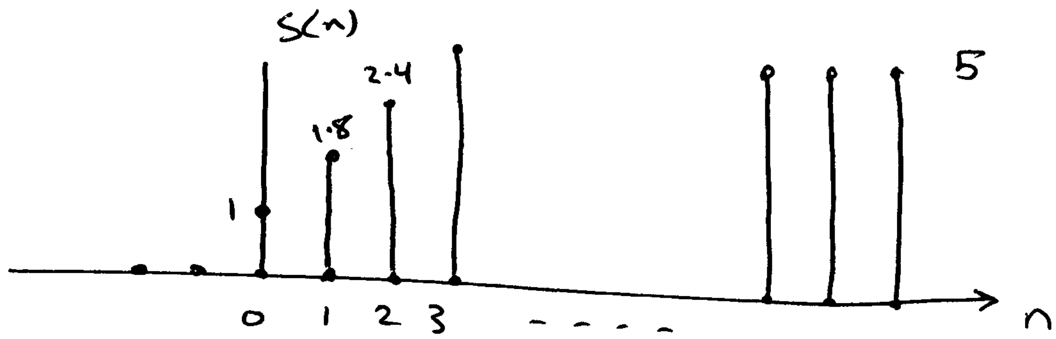
$S(2) = h(0) + h(1) + h(2) = S(1) + h(2) = 2.44$

$S(3) = S(2) + h(3) = 2.952$

$S(4) = S(3) + h(4) = 3.3616$ and so on --

$$S(\infty) = 1 + 0.8 + 0.8^2 + 0.8^3 + \cdots$$

$$= \frac{1}{1-0.8} = 5$$



* to find the response to the rectangular input

$$x(n) = u(n) - u(n-4)$$

$$y(n) = s(n) - s(n-4)$$

n	0	1	2	3	4	5	6
s(n)	1	1.8	2.44	2.952	3.362	3.689	3.952
− s(n−4)					−1	−1.8	−2.44
y(n)	1	1.8	2.44	2.952	2.362	1.889	1.511