

Convolution

The relationship between the input to a linear shift-invariant system, $x(n)$ and the output $y(n)$ is given by the convolution sum

$$X(n) * h(n) = \sum_{k=-\infty}^{\infty} X(k) h(n-k)$$

Circular Convolution

Circular convolution is used for periodic sequences and is given by:-

$$y(n) = \sum_{m=0}^n h(m) x(n-m) = \sum_{m=0}^n x(m) h(n-m)$$

EX: Find the Convolution between

$$x(n) = 1, 2, 3, 4 \quad n \geq 0$$

$$h(n) = 4, 3, 2, 1 \quad n \geq 0$$

$$y(n) = \sum_{m=0}^3 x(m) h(n-m)$$

$$y(0) = \sum_{m=0}^3 x(m) h(-m)$$

$$= x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3)$$

$$= 1 \times 4 + 2 \times 1 + 3 \times 2 + 4 \times 3 = 4 + 2 + 6 + 12 = 24$$

$$y(1) = \sum_{m=0}^3 x(m) h(1-m)$$

$$= x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2)$$

$$= 1 \times 3 + 2 \times 4 + 3 \times 1 + 4 \times 2 = 3 + 8 + 3 + 8 = 22$$

$$y(2) = \sum_{m=0}^3 x(m) h(2-m)$$

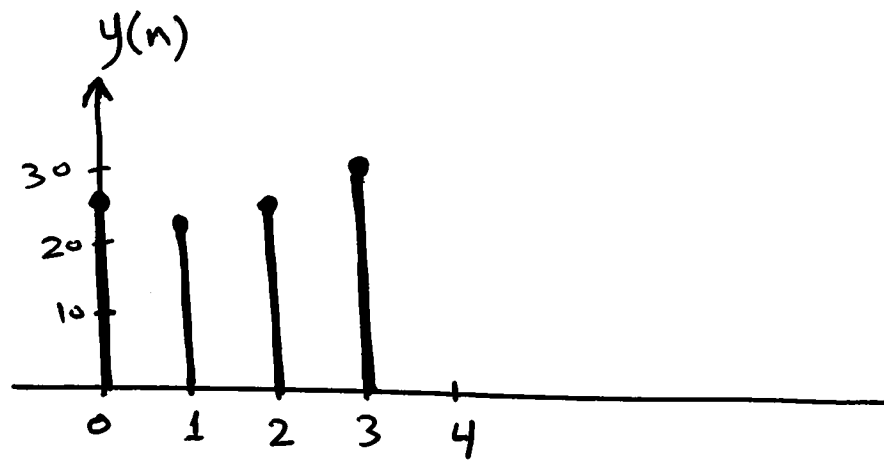
$$= x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(-1)$$

$$= 1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 1 = 2 + 6 + 12 + 4 = 24$$

$$y(3) = \sum_{m=0}^3 x(m) h(3-m)$$

$$= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0)$$

$$= 1 \times 1 + 2 \times 2 + 3 \times 3 + 4 \times 4 = 30$$



Note

if the Sequences has different lengths

$$N_1 \neq N_2$$

Add $N_2 - 1$ Zeros to N_1

Add $N_1 - 1$ Zeros to N_2

and the Sequence length become $N_1 + N_2 - 1$

Linear Convolution

Linear convolution is used for energy sequences or aperiodic sequences.

$$Y(n) = \sum_{m=0}^{2N-1} x(m) h(n-m)$$

where N = sequence length

if $x(n)$ has length N_1

$h(n)$ has length N_2 then the upper

limit of the convolution sum becomes

$$N_1 + N_2 - 1$$

EX

$$x(n) = 1, 2, 3, 4 \quad n \geq 0$$

$$h(n) = 4, 3, 2, 1 \quad n \geq 0$$

Find linear convolution $y(n)$

$$y(n) = \sum_0^7 x(m) h(n-m)$$

$$y(0) = \sum_0^7 x(m) h(-m)$$

$$\begin{aligned} &= x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3) \\ &\quad + x(4)h(-4) + x(5)h(-5) + x(6)h(-6) + \\ &\quad x(7)h(-7) \end{aligned}$$

$$= 1 \times 4 + 0 = \boxed{4}$$

$$y(1) = \sum_0^7 x(m) h(1-m)$$

$$\begin{aligned} &= x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2) \\ &\quad + x(4)h(-3) + x(5)h(-4) + x(6)h(-5) + x(7)h(-6) \end{aligned}$$

$$= 1 \times 3 + 2 \times 4 + 0 = \boxed{11}$$

$$Y(2) = \sum_{n=0}^7 X(n) h(2-n)$$

$$= X(0)h(2) + X(1)h(1) + X(2)h(0) + X(3)h(-1) \\ + X(4)h(-2) + X(5)h(-3) + X(6)h(-4) + X(7)h(-5)$$

$$= 1 \times 2 + 2 \times 3 + 3 \times 4 + 0$$

$$= 2 + 6 + 12 = \boxed{20}$$

$$Y(3) = \sum_{n=0}^7 X(n) h(3-n)$$

$$= X(0)h(3) + X(1)h(2) + X(2)h(1) + X(3)h(0) \\ + X(4)h(-1) + X(5)h(-2) + X(6)h(-3) + X(7)h(-4)$$

$$= 1 \times 1 + 2 \times 2 + 3 \times 3 + 4 \times 4 + 0$$

$$= 1 + 4 + 9 + 16 = 30$$

$$Y(4) = \sum_{n=0}^7 X(n) h(4-n)$$

$$= X(0)h(4) + X(1)h(3) + X(2)h(2) + X(3)h(1) \\ + X(4)h(0) + X(5)h(-1) + X(6)h(-2) + X(7)h(-3)$$

$$= 1 \times 0 + 2 \times 1 + 3 \times 2 + 4 \times 3 + 0$$

$$= 2 + 6 + 12 = \boxed{20}$$

$$Y(5) = \sum_0^7 x(m) h(5-m)$$

$$= x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) \\ + x(4)h(1) + x(5)h(0) + x(6)h(-1) + x(7)h(-2) \\ = 1 \times 0 + 2 \times 0 + 3 \times 1 + 4 \times 2 = 3 + 8 = \boxed{11}$$

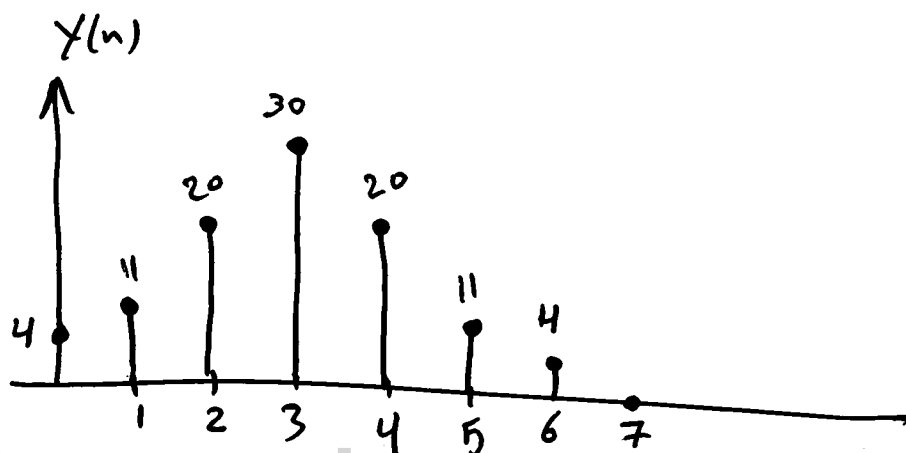
$$Y(6) = \sum_0^7 x(m) h(6-m)$$

$$= x(0)h(6) + x(1)h(5) + x(2)h(4) + x(3)h(3) \\ + x(4)h(2) + x(5)h(1) + x(6)h(0) + x(7)h(-1) \\ = 4 \times 1 = \boxed{4}$$

$$Y(7) = \sum_0^7 x(m) h(7-m)$$

$$= x(0)h(7) + x(1)h(6) + x(2)h(5) + x(3)h(4) \\ + x(4)h(3) + x(5)h(2) + x(6)h(1) + x(7)h(0) \\ = 0$$

$$Y(n) = 4, 11, 20, 30, 20, 11, 4, 0$$



Digital Convolution (method 2)

We can introduce to this method with simple example Consider an input $x(n]$ as shown in Fig (a) and $h(n]$ as shown in Fig(b)

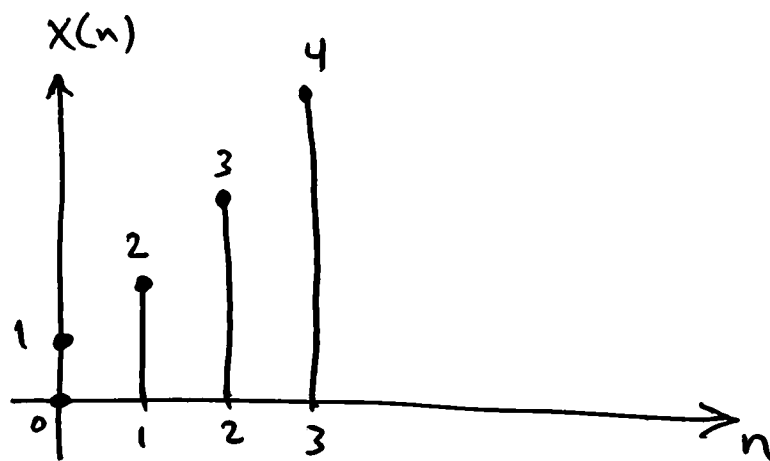
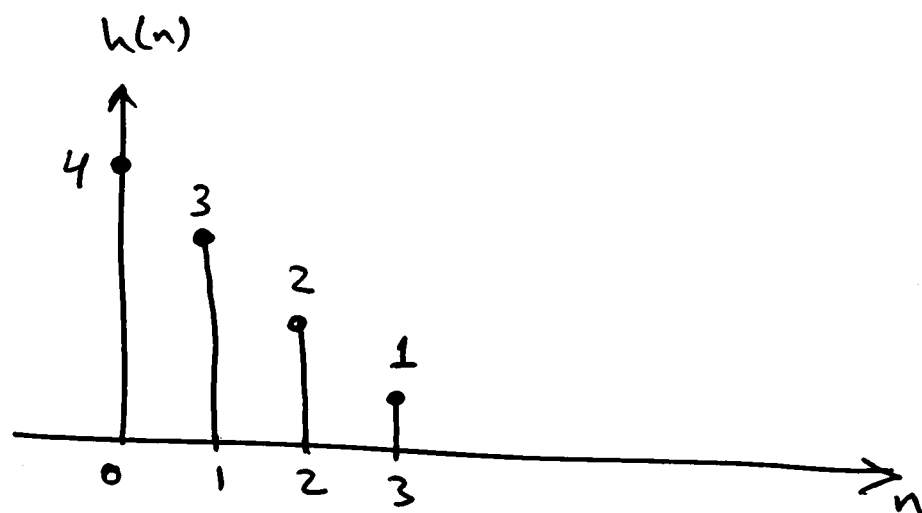
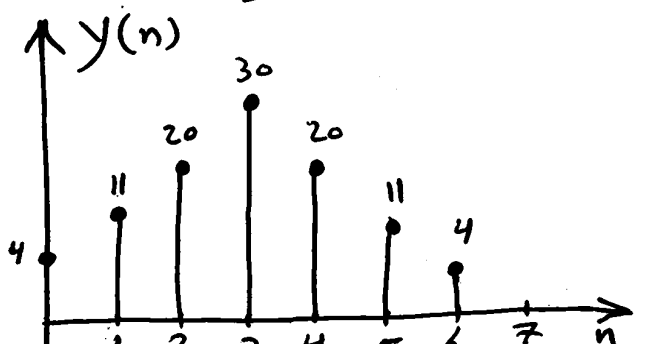
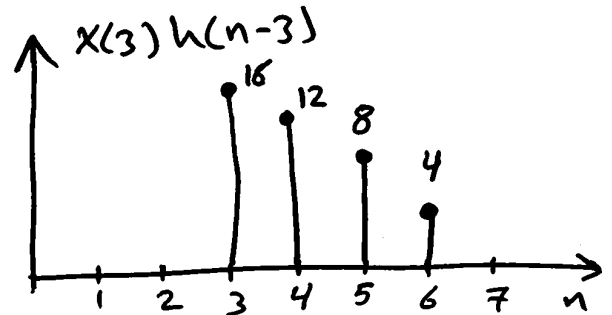
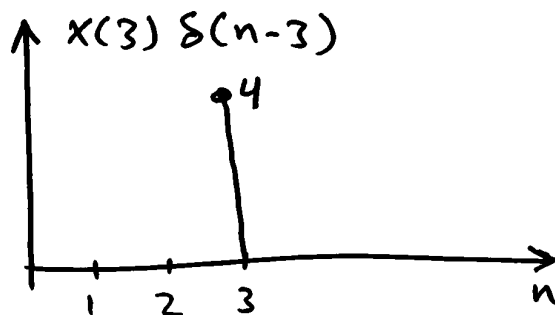
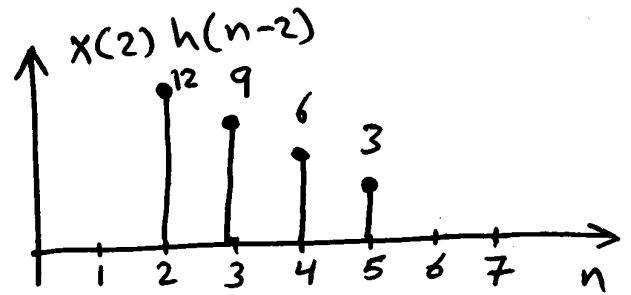
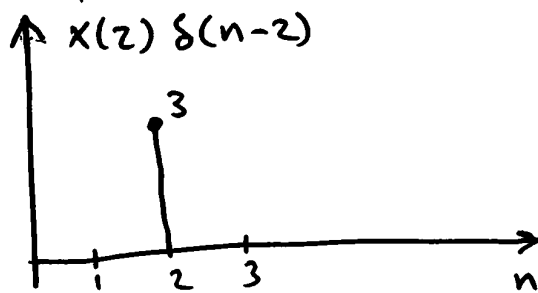
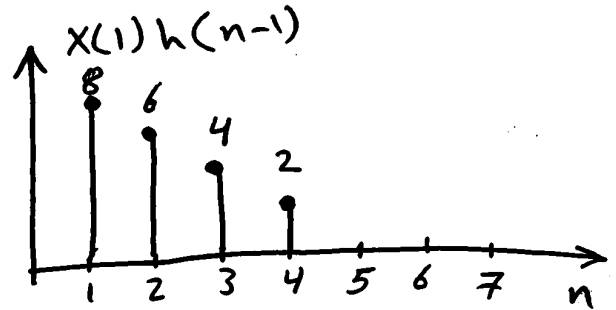
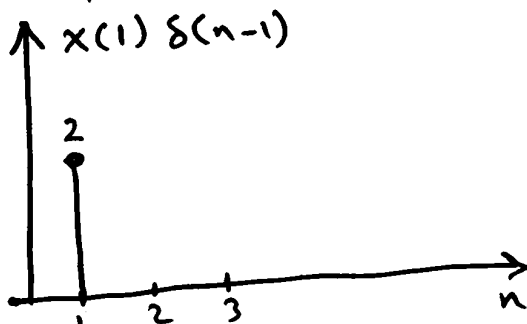
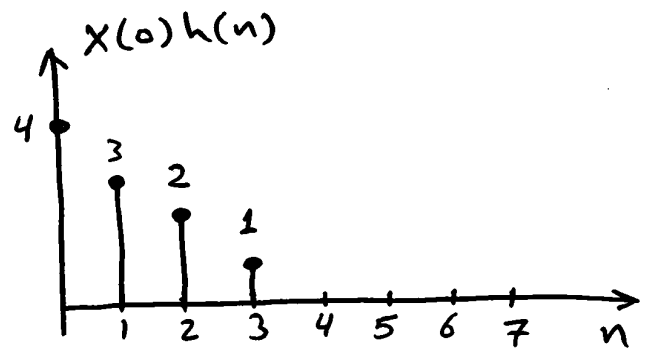
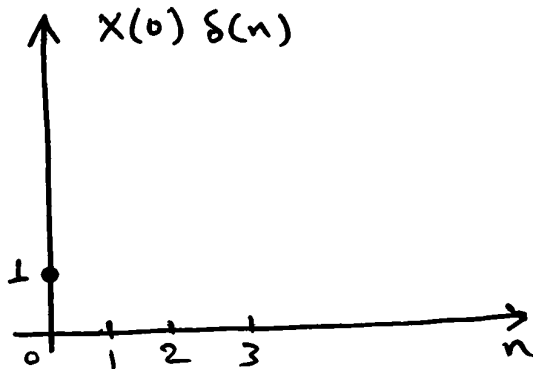
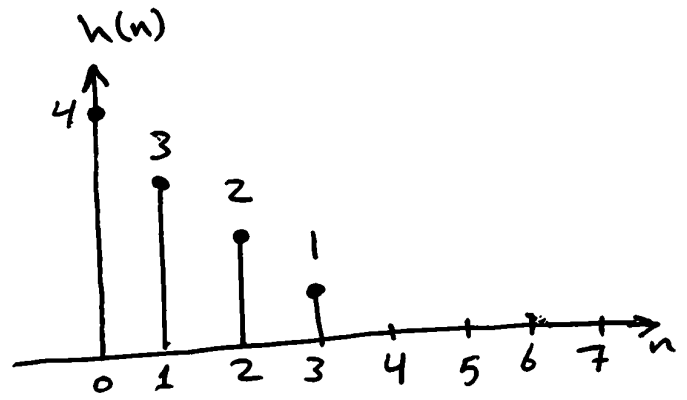
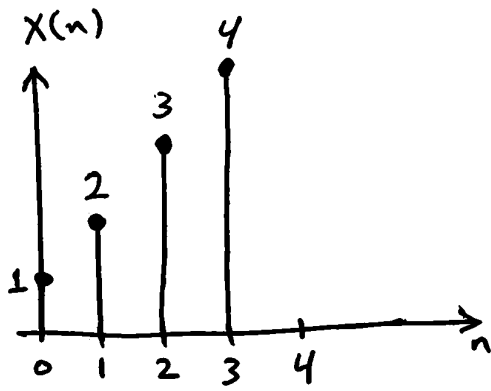


Fig (a)





Digital Convolution (method 3)

$$x(n) = 1, 2, 3, 4 \quad n \geq 0$$

$$h(n) = 4, 3, 2, 1 \quad n \geq 0$$

	1	2	3	4	
$y(0)$	1	2	3	4	$\leftarrow x(n)$
$y(1)$	—	1	2	3	\leftarrow Reversed $h(n)$
$y(2)$	—	—	1	2	
$y(3)$	—	—	—	1	
$y(4)$	—	—	—	—	
$y(5)$	—	—	—	—	
$y(6)$	—	—	—	—	
$y(7)$	—	—	—	—	

$$y(0) = 4 \times 1 = 4$$

$$y(1) = 3 \times 1 + 4 \times 2 = 11$$

$$y(2) = 2 \times 1 + 3 \times 2 + 4 \times 3 = 20$$

$$y(3) = 1 \times 1 + 2 \times 2 + 3 \times 3 + 4 \times 4 = 30$$

$$y(4) = 1 \times 2 + 2 \times 3 + 3 \times 4 = 20$$

$$y(5) = 1 \times 3 + 2 \times 4 = 11$$

$$y(6) = 1 \times 4 = 4$$

$$y(7) = 1 \times 0 = 0$$

$$y(n) = 4, 11, 20, 30, 20, 11, 4, 0$$