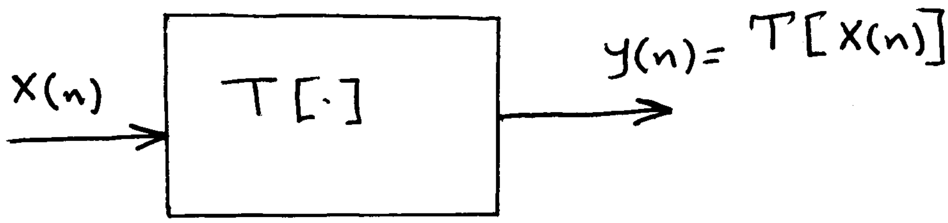
Discrete Time Systems

A discrete time system is a mathematical operator or mapping that transforms one signal (input) into another signal (output) by means of a fixed set of rules or operation



"T" is used to represent a general system as shown above in which an input signal $x(n)$ is transformed into an output signal $y(n)$ through the transformation $T[\cdot]$

System Properties :

Discrete time system may be classified in term of the properties that they possess

* Memory and Memory less :

A system is said to be memory less if the output depends on only the input at the same time. otherwise, the system is said to have memory.

EX

1- $$y(n) = x^2(n)$$

is memory less because $y(n_o)$ depends only on the value of $x(n)$ at time n_o

2- $$y(n) = x(n) + x(n-1)$$

is memory because the output at time n_o depends on the value of the input at time n_o and $n_o - 1$

* Linear system :

is the system that both additive and homogenous.

<u>Additive</u> :- is one for the response to asum of inputs is equal to the sum of input individually Thus,

$$T[X_1(n) + X_2(n)] = T[X_1(n)] + T[X_2(n)]$$

<u>Homogenetity</u> :- Asystem is said to be homogenous if scaling the input by aconstant result in ascaling of the output by the same constant

$$T[c\,X(n)] = c\,T[X(n)]$$

for any constant c and for any input sequeance X(n)

EX check the Linearity of

* $y = x^2(n)$ [non linear]

* $y = \cos x(n)$ [non Linear]

* Time Invariand and Time Varying:

asystem is called time-invariant if atime shift (delay or advance) in the input Signal causes the same time shift in the output signal. for continous-time system

$$T[x(t-\tau)] = y(t-\tau)$$

for discrete-time system

$$T[x[n-k]] = y[n-k]$$

* stable system :

A system is bounded-input / bounded output (BIBO) stable if for any bounded input x defined by

$$|K(n)| \leq Bx < \infty \quad \text{for all } n$$

* Causality :

A system is causal if for every choice of n_0, the output sequence value at the index $n = n_0$ depends on the input sequence value for $n \leq n_0$

$$y(n) = x(n+1) - x(n) \quad \text{(not causal)}$$

$$y(n) = x(n) - x(n-1) \quad \text{(causal)}$$

Ex/

$X(n)$ and $y(n)$ are the input and output of a DSP system. Determine which of the following properties are possessed by systems

- Linearity
- Time invariance
- causality
- stability
- invertiblity
- memory

* $y(n) = 3\,X(n) - 4\,X(n-1)$

- Linear: The output is weighted sum of present and previous inputs
- Time invariance :- properties don't vary as time progress
- Causal :- the output depend only on present and previous input
- Stable :- output is bounded if input is bounded
- invertable :- input is unambiguously related to the output
- memory :- out put depends on previous input.

* $y(n) = 2y(n-1) + x(n+2)$

- Linear
- time invariance
- not causal :- The present output depends on afuature input
- not stable :- if the input signal ceases The out put goes on rising with out limit. Since each out put value is twice the previous one.
- invertible
- Memory

* $Y(n) = n\,x(n)$

- Linear
- Time Variant :- out put depends on the independent Variablen.

- causal
- stable
- invertible
- Memory less :- The out put depends on the present input only

* $y(n) = \cos[x(n)]$

- nonlinear :- if we double $x(n)$ we don't double $y(n)$

- time invariant
- Causal
- Stable
- not invertible :- cosine is Periodic different values of $x(n)$ Produce the same value $y(n)$

- memory less

* $y(n) = T\,x(n) = x(n-1)$

1) Causal [future]
2) Linear [additive + homogeneous]
3) time invariant
4) Stable

EX system has the input-output relation given by

$$y = T[x(n)] = x^2(n)$$

Show that this system is nonlinear

Sol

$$T[x_1(n) + x_2(n)] = (x_1(n) + x_2(n))^2$$

$$= x_1^2(n) + x_2^2(n) + 2x_1x_2$$

$$\neq T\,x_1(n) + T\,x_2(n) = x_1^2(n) + x_2^2(n)$$

Thus the system is non linear

H.W

5- i- Express $u(n)$ in terms of $\delta(n)$

ii- Express $\delta(n)$ in terms of $u(n)$

iii- Express $r(n)$ in terms of $\delta(n)$