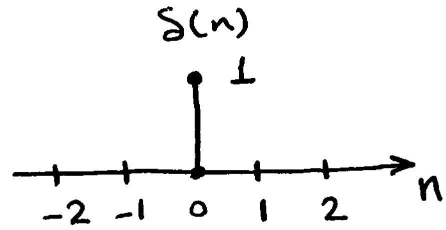
Basic Sequences

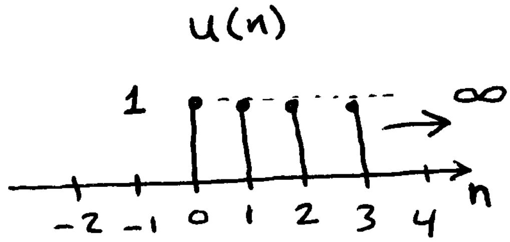
* Unit sample sequence (Delta function):

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$



* Unit step Function

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

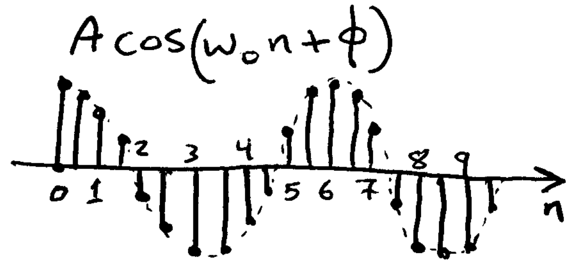


Note: $u[n] = \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta(n-k)$

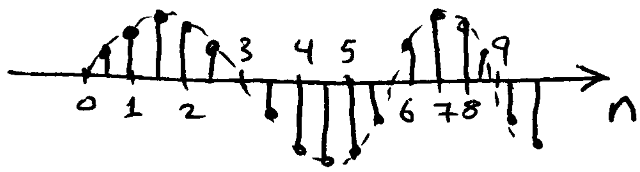
$$\delta[n] = u[n] - u[n-1]$$

* Sinusoidal sequence

- $x[n] = A\cos(w_0 n + \phi)$



- $x[n] = A\sin(w_0 n + \phi)$



* Complex exponential sequance

$$x[n] = A e^{j(w_0 n + \phi)}$$

$$= A\cos(w_0 n + \phi) + jA\sin(w_0 n + \phi)$$

Saif Asaad

Periodic and Aperiodic Sequences

* A sinsoidal sequance is always periodic (with period 2π) with respect to the frequency. That is for any integer r

$$A\cos(w_0 n+\phi)=A\cos[(w_0+2\pi r)n+\phi]$$

Therefore we need only consider frequencies in an interval of length 2π such as $-\pi < w_0 \leq \pi$ or $0 \leq w_0 < 2\pi$

* Aperiodic sequence is asequence for which

$$x[n]=x[n+N]$$

For the sinusoidal sequence it implies

$$A\cos(w_0 n+\phi)=A\cos(w_0 n+w_0 N+\phi$$

which requiers $w_0 N = 2\pi k$ For integers N and k there for the period N is not necessaly equal to $\frac{2\pi}{w_0}$

* if $X_1(n)$ is asequence that is Periodic with aperiod N_1 and $X_2(n)$ is another sequance that is Periodic with a Period N_2 the sum

$$X(n) = X_1(n) + X_2(n)$$

$$N = \frac{N_1 N_2}{gcd(N_1, N_2)}$$

Transformation of the independent Variable:

The most common transformations include shifting, Reversal, and scaling which are defined below:-

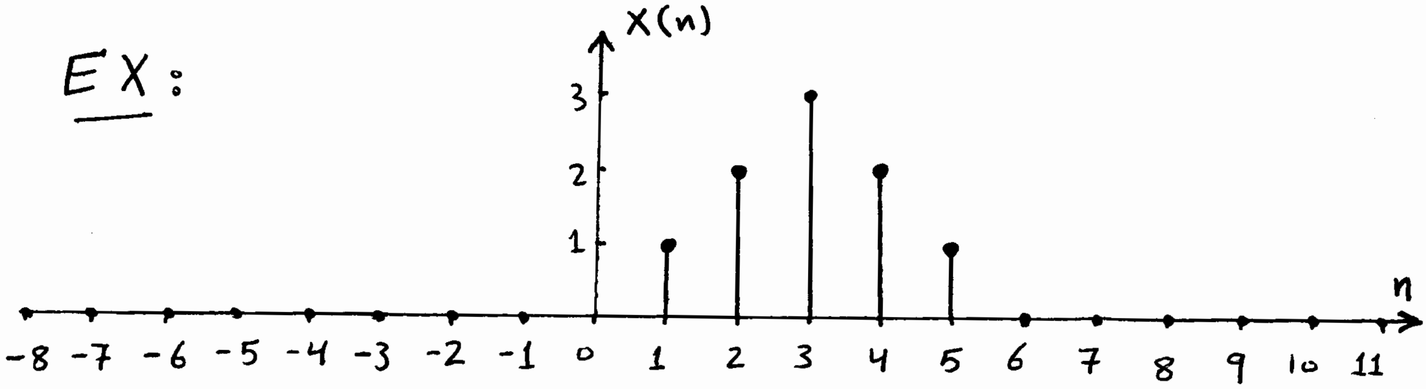
* **shifting**:- this is the transformation defined by $f(n) = n - n_o$ if $y(n) = x(n - n_o)$, $x(n)$ is shifted to the right by n_o samples if n_o is Positive (this refered to as adelay) and its shifted to the left by n_o samples if n_o is negative (refered to as an advance).

* **Reversal**:- This transformation is given by $f(n) = -n$ and Simply involves "flipping" the Signal $x(n)$ with respect to the index n.

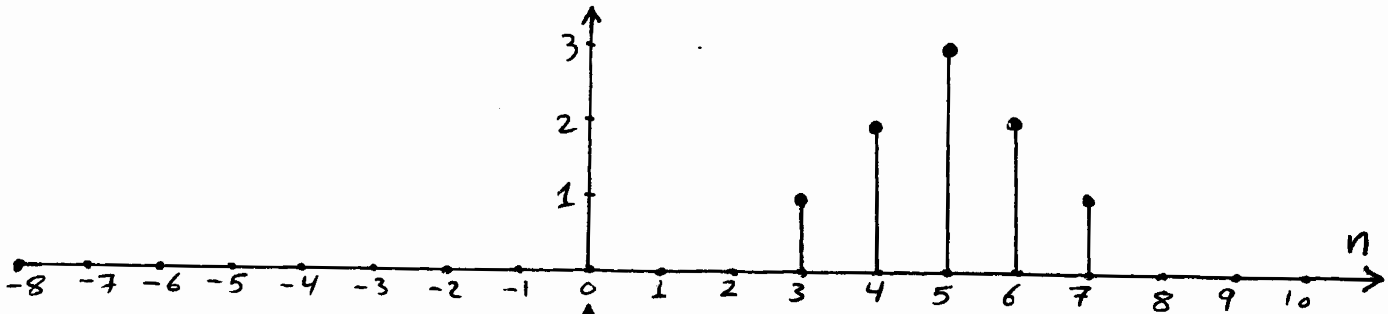
* Time Scaling :- this transformation is defined by $f(n) = Mn$ or $f(n) = \frac{n}{N}$ where M and N are Positive integers. in this case of $f(n) = Mn$, the sequence $x(Mn)$ is Formed by taking every M th sample of $x(n)$ [this operation is Known as down sampling] with $f(n) = \frac{n}{N}$ the sequence $y(n) = x(f(n))$ is defined as follows :-

$$y(n) = \begin{cases} x\left(\frac{n}{N}\right) & n = 0, \pm N, \pm 2N, \ldots \\ 0 & \text{other wise} \end{cases}$$

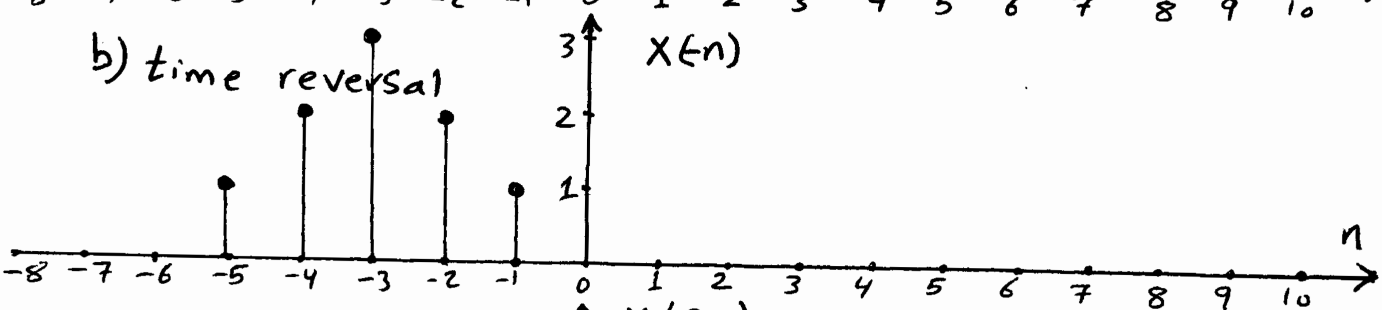
EX:



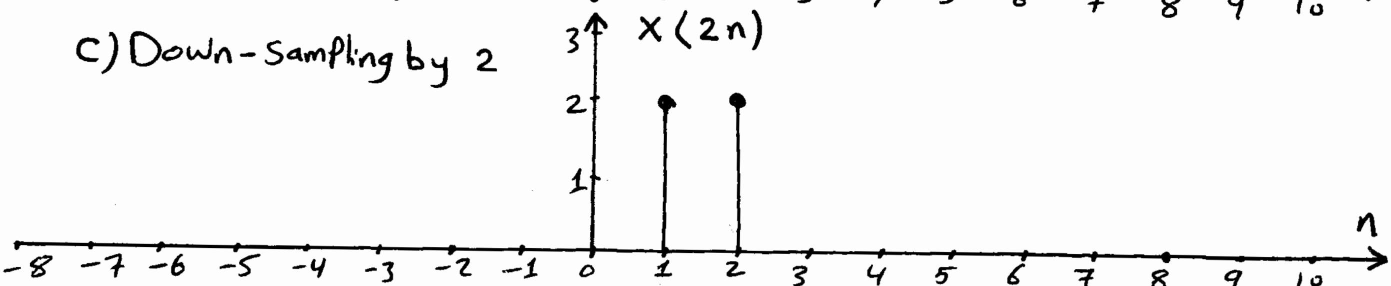
a) adelay by no=2 [x(n-2)]



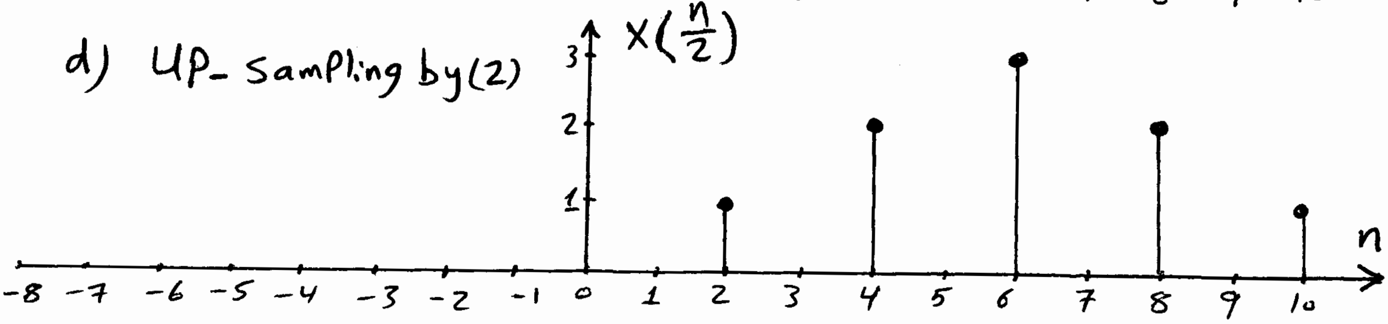
b) time reversal



c) Down-sampling by 2



d) UP-sampling by(2)



Examples :-

Express the sequence
as a sum of scaled and
shifted unit steps ?

$$x(n) = \begin{cases} 1 & n=0 \\ 2 & n=1 \\ 3 & n=2 \\ 0 & \text{otherwise} \end{cases}$$

Sol

$$X[n] = \delta(n) + 2\delta(n-1) + 3\delta(n-2)$$

$$\because \quad \delta(n) = u(n) - u(n-1)$$

$$\delta(n-1) = u(n-1) - u(n-2)$$

$$\delta(n-2) = u(n-2) - u(n-3)$$

$$\therefore \quad X(n) = u(n) - u(n-1) + 2[u(n-1) - u(n-2)] + 3[u(n-2) - u(n-3)]$$

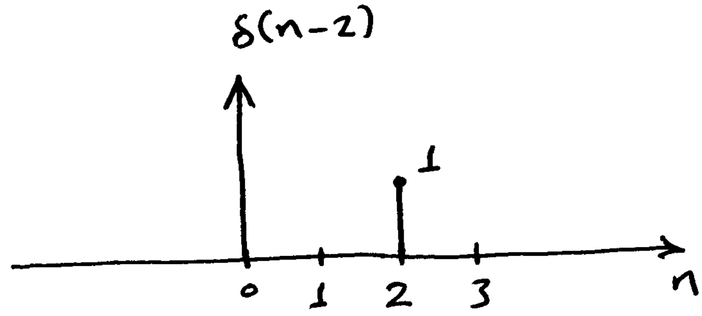
$$= u(n) - u(n-1) + 2u(n-1) - 2u(n-2) + 3u(n-2) - 3u(n-3)$$

$$= u(n) + u(n-1) + u(n-2) - 3u(n-3)$$

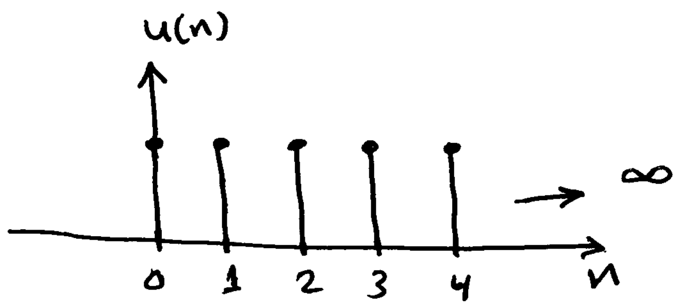
EX/

Draw the following sequence

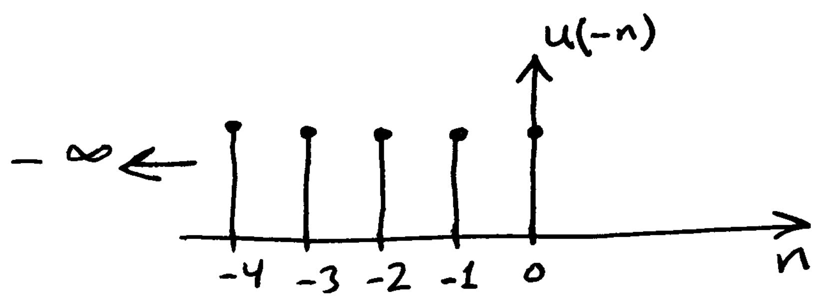
* $\delta(n-2)$



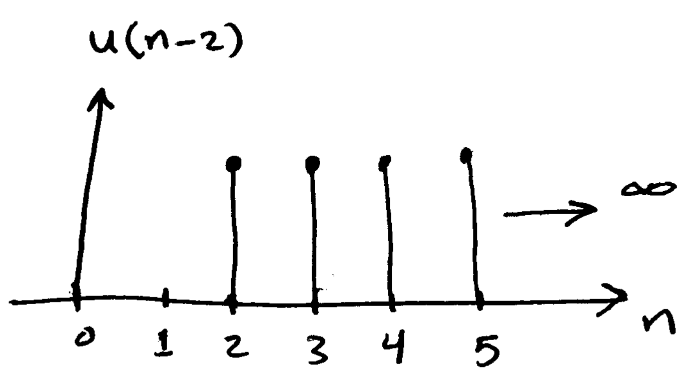
* $u(n)$



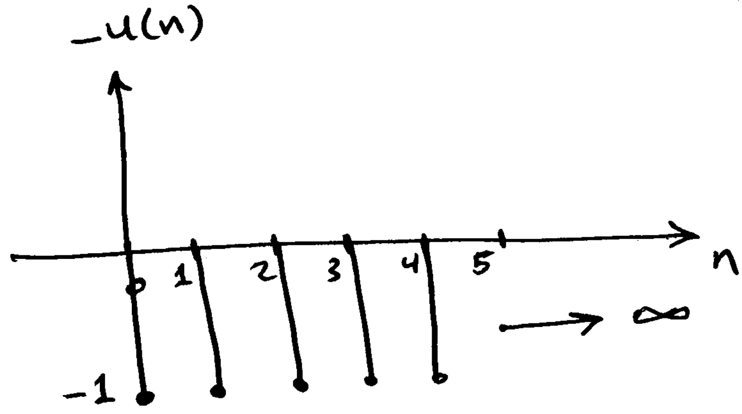
* $u(-n)$



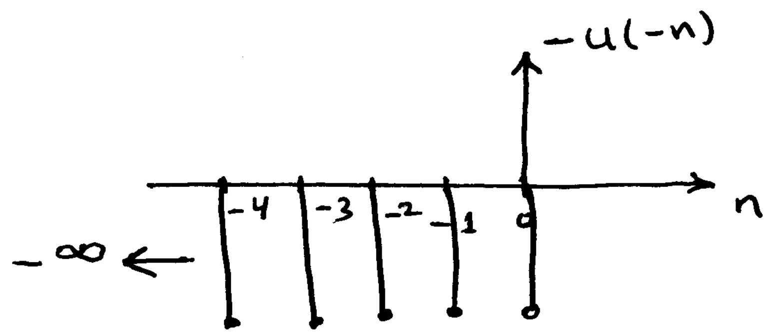
* $u(n-2)$



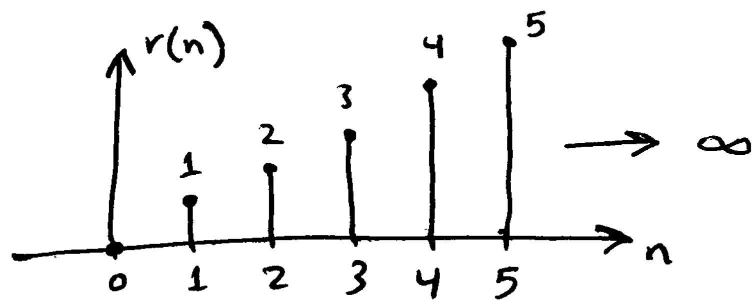
* $-u(n)$



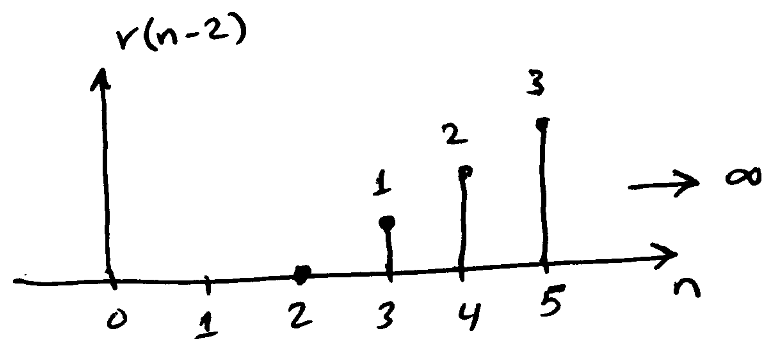
* $-u(-n)$



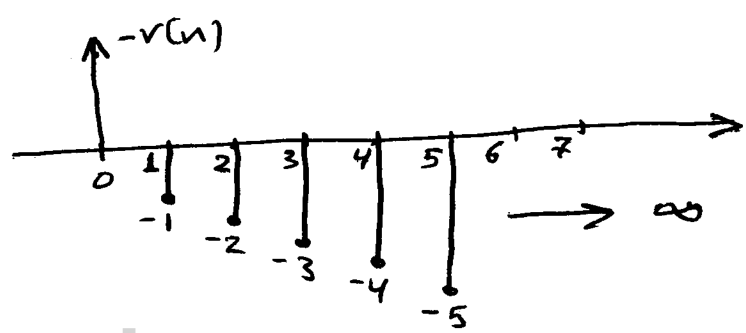
* $r(n)$



* $r(n-2)$



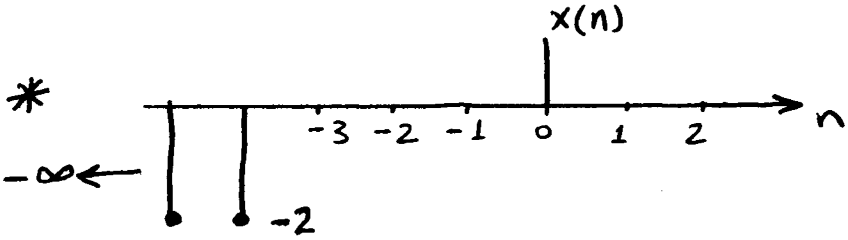
* $-r(n)$



EX/

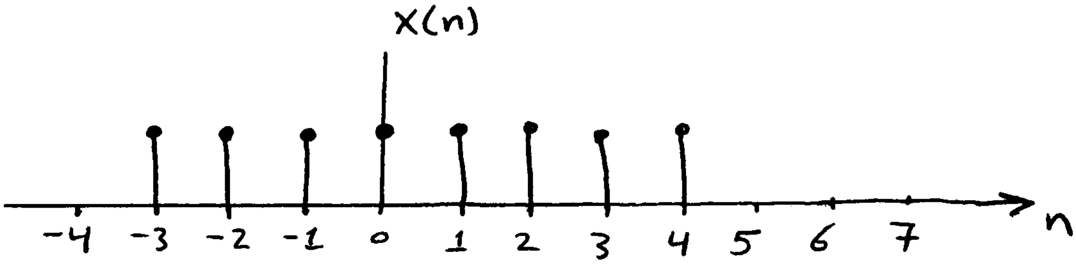
Find Expressions for various signals below

*



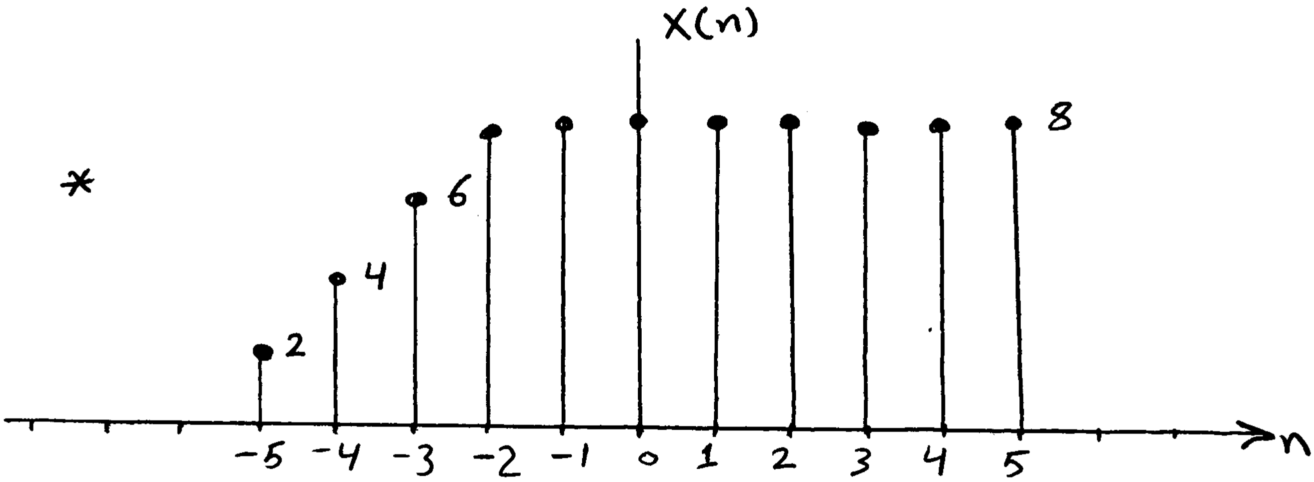
$$x(n) = -2u(-n-4)$$

*



$$x(n) = u(n+3) - u(n-5)$$

*



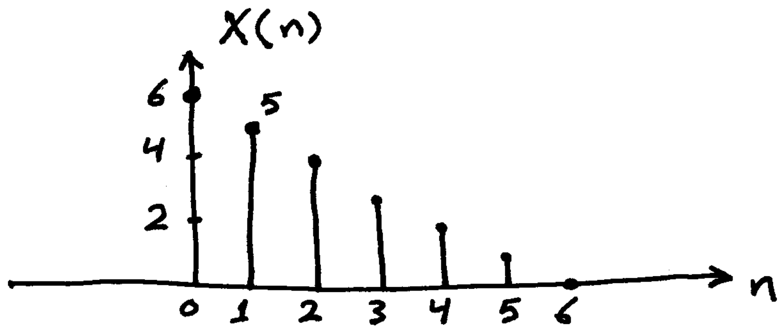
$$x(n) = 2r(n+6) - 2r(n+2)$$

EX/

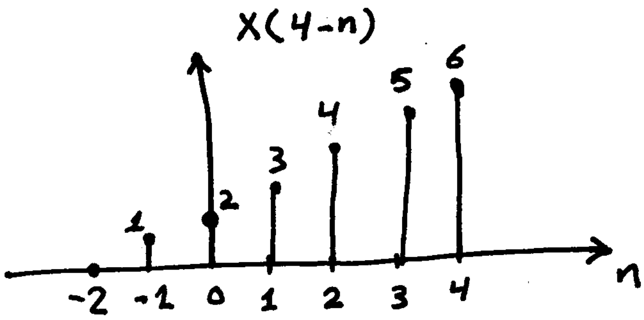
Given the sequence $x(n) = (6-n)[u(n) - u(n-6)]$

sketch of

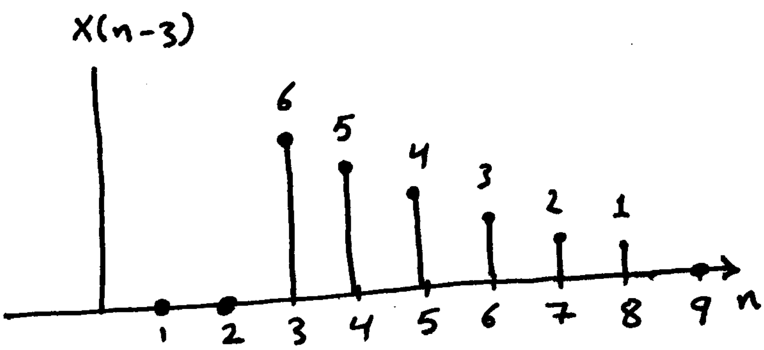
a) $x(n)$

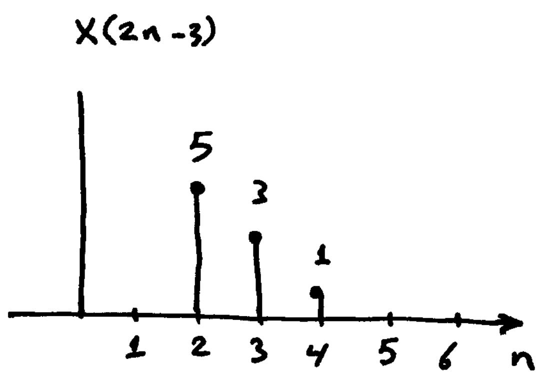


b) $y(n) = x(4-n)$

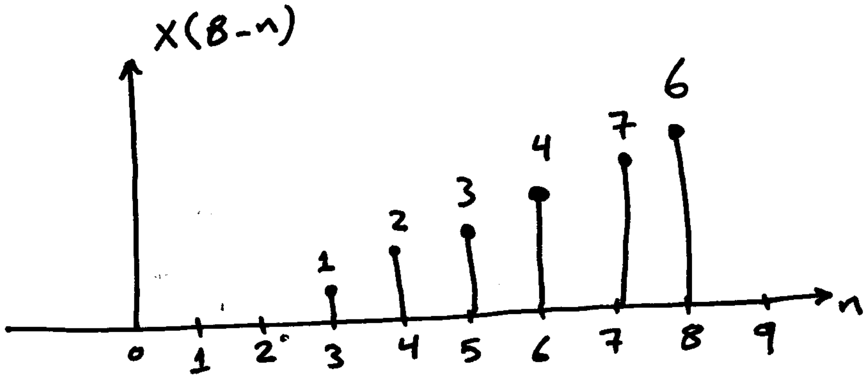


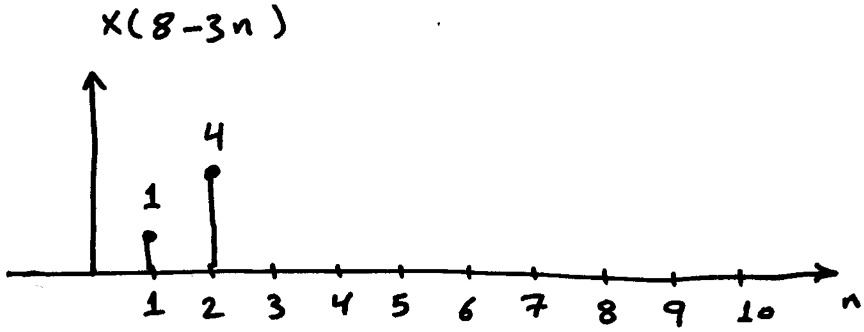
c) $y(n) = x(2n-3)$





d) $y(n) = x(8-3n)$





e) $y(n) = x(n^2 - 2n + 1)$