Discrete Fourier Transform

Introduction:- For finite-length sequences there is another representation called the Discrete Fourier transform (DFT). The DFT is a mathematical procedure used to determine the harmonic or freq. content of a discrete signal sequence unlike the DTFT. which is a continuous function of a continuous variable w, the DFT is a sequence that corresponds to samples of the DTFT. such a representation is very useful for digital computations and for digital hard ware implementations.

The DFT is an important decomposition for sequences that are in length. whereas the DTFT is a mapping from a sequence to a function of a continous variable w

$$x(n) \overset{DTFT}{\Longleftrightarrow} X(e^{jw})$$

the DFT is a mapping from a sequence $x(n)$

to another sequence $X(K)$

$$x(n) \xleftrightarrow{DFT} X(K)$$

The sequence $X(K)$ is called the N-Point DFT

of $x(n)$. These coéfficients are related to

$x(n)$ as follows

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-J2\pi Kn/N} \qquad 0 \leq K \leq N$$

Alternatively, the DFT cofficients correspond

to N samples of $X(Z)$ that are taken at

N equally spaced points around the unit

circle

$$X(K) = X(Z)\Big|_{Z = \exp[J2\pi K/N]}$$

Ex Evaluate the DFT of the sequence

$$x(n) = [1, 0, 0, 1] \quad n \geq 0$$

Sol:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-J 2\pi k n / N}$$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-J 2\pi k n / 4}$$

$$X(0) = \sum_{n=0}^{3} x(n) = x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 0 + 0 + 1$$

$$= 2$$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-J \frac{\pi}{2} n} = 1 + e^{-J \frac{3\pi}{2}}$$

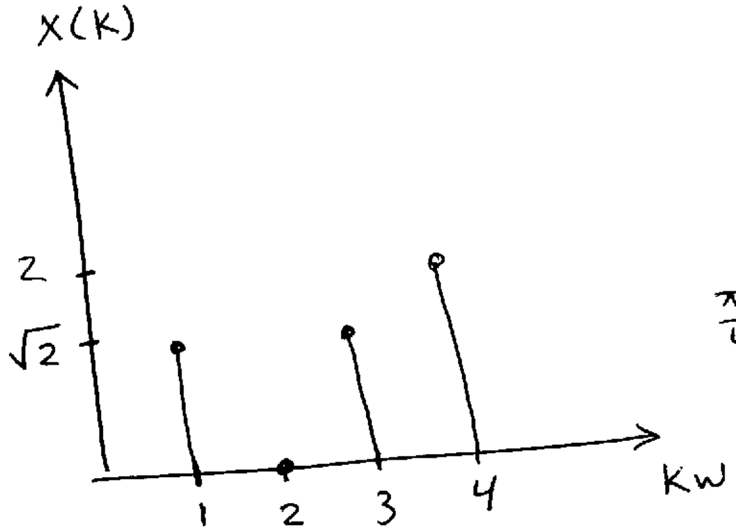
$$= 1 + J$$

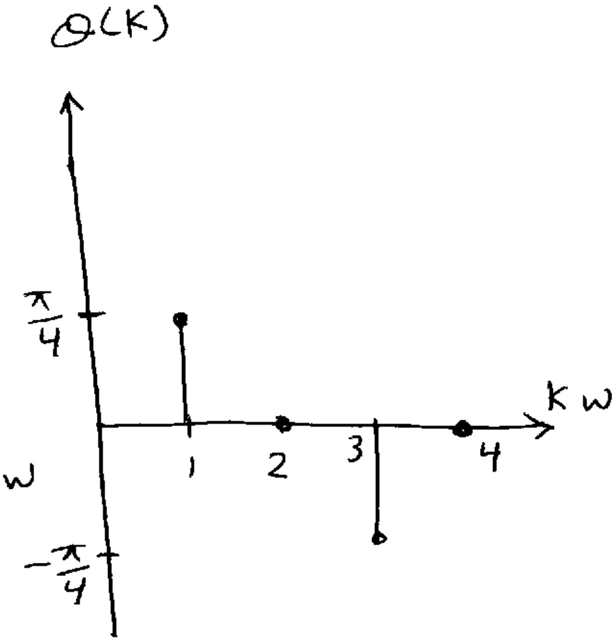
$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-J \pi n} = 1 + e^{-J 3\pi} = 0$$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-J 6\pi n/4} = 1 + e^{-J \frac{4}{2} \pi}$$

$$= 1 - J$$

$X(0) = 0$, $X(1) = \sqrt{2} \angle 45^\circ$, $X(2) = 0$, $X(3) = \sqrt{2} \angle -45$


X(K)
2
√2
1
2
3
4
KW


θ(K)
π/4
-π/4
1
2
3
4
KW

Inverse Discrete Fourier Transform (IDFT)

Inverse Discrete fourier transform can be obtained by:-

$$X(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{J2\pi kn/N}$$

Example Evalnate the IDFT of the sequence

$$X[K] = [\,2\,,\,1+J\,,\,0\,,\,1-J\,]$$

SoL

$$X(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{J2\pi Kn/N}$$

$$= \frac{1}{N} \sum_{K=0}^{3} X(K)\, e^{J2\pi Kn/4}$$

$$X(0) = \frac{1}{4} \sum_{K=0}^{3} X(K)\, e^{J2\pi K0/4}$$

$$= \frac{1}{4}\left[2+(1+J)+0+(1-J)\right] = 1$$

$$X(1) = \frac{1}{4} \sum_{k=0}^{3} X(k)\, e^{J2\pi k/4}$$

$$= \frac{1}{4}\left[2 + (1+J)e^{J\frac{\pi}{2}} + 0 + (1-J)e^{J\frac{3\pi}{2}}\right]$$

$$= \frac{1}{4}\left[2 + J - 1 - J - 1\right] = 0$$

$$X(2) = \frac{1}{4} \sum_{k=0}^{3} X(k)\, e^{J\pi k}$$

$$= \frac{1}{4}\left[2 + (1+J)e^{J\pi} + 0 + (1-J)e^{J3\pi}\right.$$

$$= \frac{1}{4}\left[2 - (1+J) - (1-J)\right] = 0$$

$$X(3) = \frac{1}{4} \sum_{k=0}^{3} X(k)\, e^{J\frac{3\pi k}{2}}$$

$$= \frac{1}{4}\left[2 + (1+J)e^{J\frac{3\pi}{2}} + 0 + (1-J)e^{J\frac{9\pi}{2}}\right]$$

$$= \frac{1}{4}\left[2 + (1+J)(-J) + (1-J)J\right] = 1$$

Note

The Formula of the DFT and IDFT may be expressed as:-

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_n^{KN} \qquad K=0,\ 1,\ \cdots N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(K)\, W_N^{-KN} \qquad n=0,1,\ \cdots N-1$$

where:-

$$W_N = e^{-J2\pi/N}$$