Signals and Systems

Introduction :

* Signal :- is a function of independent Variables such as time, frequency, Position.

* System :- is an interconnection of components with terminals that allow the exchange of energy, matters or information.

* Signal Processing :- is concerned with the mathmatical representation of the signal and the algorithmic operation carried out on it to extract the information Present.

Discrete time signals :

Discrete time signals are often derived by sampling a continuos time signal such as speech with an analog to digital converter (A/D)

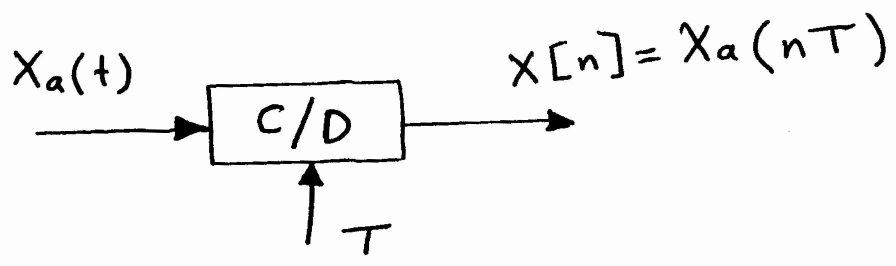
* the typical method of obtaining a discrete-time representation of a continuous-time signal is through Period sampling

$$X[n] = X_a(nT) \qquad -\infty < n < \infty$$

where T is called the sampling Period its reciprocal $f_s = \frac{1}{T}$ is the sampling frequency

we also express the sampling frequency as

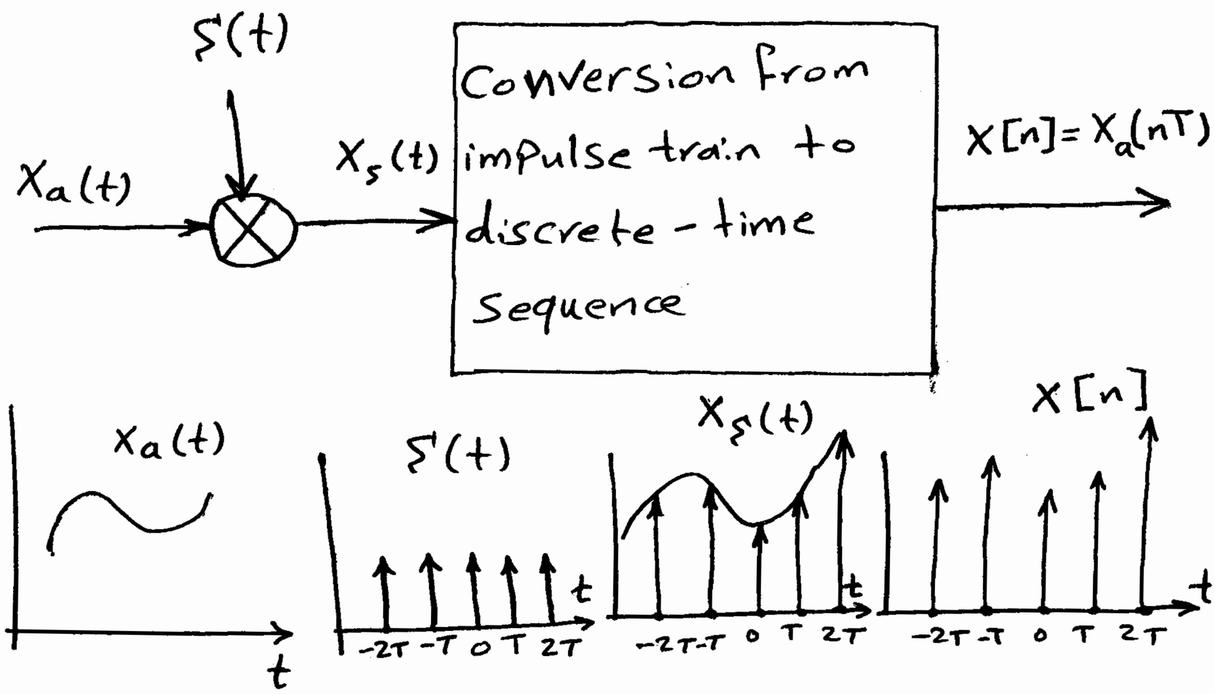
$$W_s = 2\pi f = \frac{2\pi}{T} \quad \text{rad/sec}$$



Continuous to Discrete time Converter
(C/D) converter

* in Practical setting the operation of sampling is implemented by an analog-to-digital (A/D) converter. Important consideration in the implementation or choice of an A/D converter include quantization of the output samples, linearity of quantization steps and limitations on the sampling rate

* we use the following two stage mathematical representation of the sampling process



Frequency Domain Representation Of Sampling

* We are going to derive the frequency-domain relation between the input and output of an ideal C/D converter

* Consider the relationship between $X_s(j\omega)$ and $X_a(j\omega)$

* Consider the conversion of $x_a(t)$ to $x_s(t)$ through modulation of the Periodic impulsive train

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

where $\delta(t)$ is the unit impulse function or (Dirac delta function)

* We modulate $s(t)$ with $x_a(t)$ obtaining

$$x_s(t) = x_a(t)\, s(t) = x_a(t) \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$= \sum_{n=-\infty}^{\infty} x_a(nT)\, \delta(t-nT)$$

* The fourier transform of a periodic impulse train is a periodic impulse train

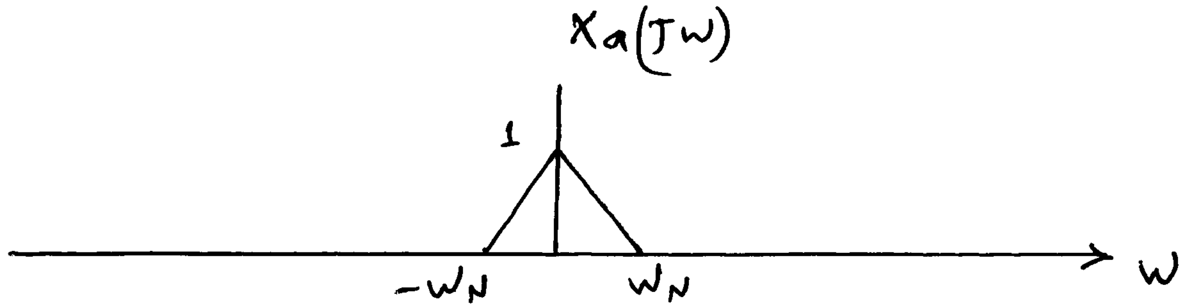
$$S(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

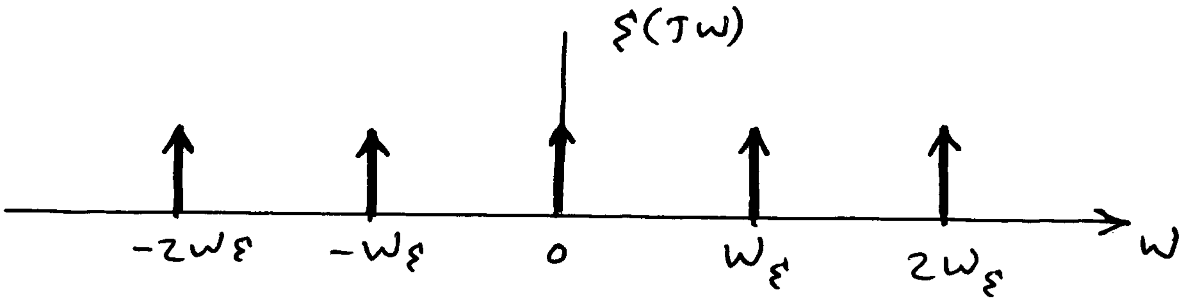
where

$$\omega_s = \frac{2\pi}{T}$$

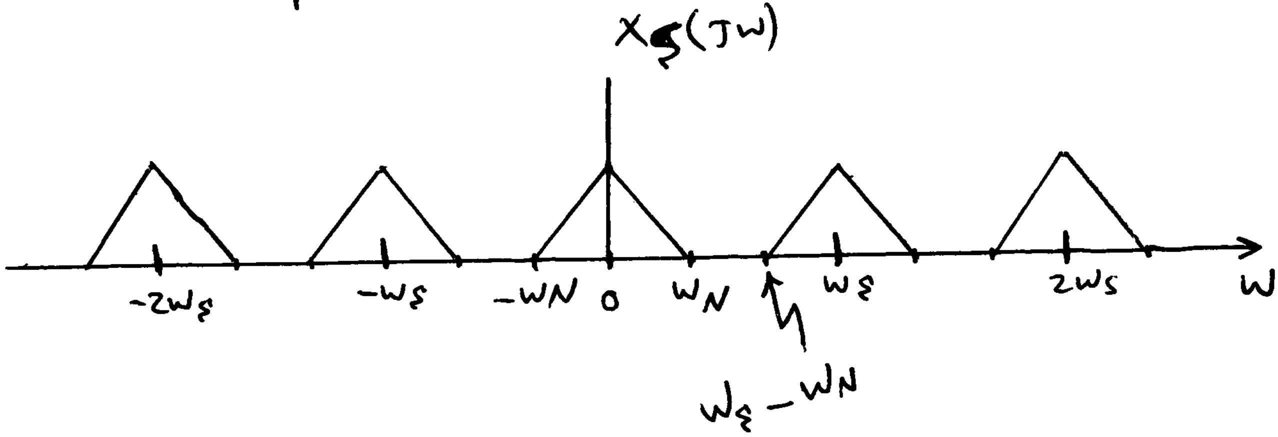
$$X_s(j\omega) = \frac{1}{2\pi} X_a(j\omega) * S(j\omega)$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a(j(\omega - k\omega_s))$$

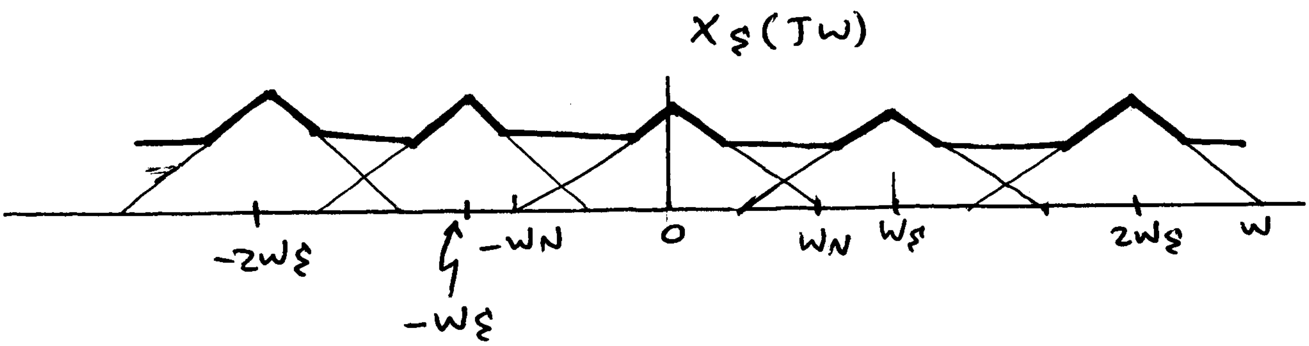




* if $W_s > 2W_N$ the replicas of $X_a(jW)$ do not overlap



* if $W_s < 2W_N$, the replicas of $X_a(jW)$ is no longer recoverable



* The frequency W_N is commonly referred as the Nyquist frequency
* The frequency $2W_N$ that must be exceeded by the sampling frequency is called the Nyquist rate