كلية المنصور الجامعة

قسم
هندسة اتصالات
المرحلة الاولى

Electrical Fundamentals

Lectuers

2017 – 2018

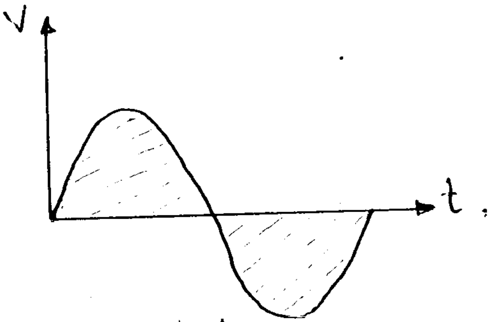
الاسس الكهربائية

Second semester

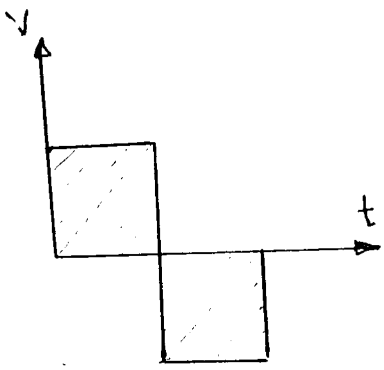
Lec. Salah Y. H.

Alternating Current (A-C) circuits

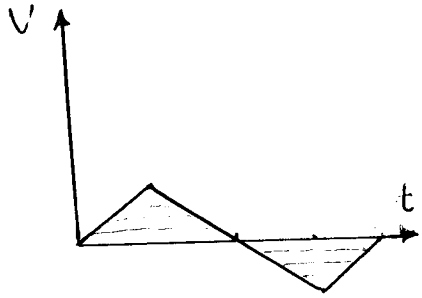
The analysis of networks in which the magnitude of the Source of e.m.f Varies in a set manner



Sinusoidal-wave



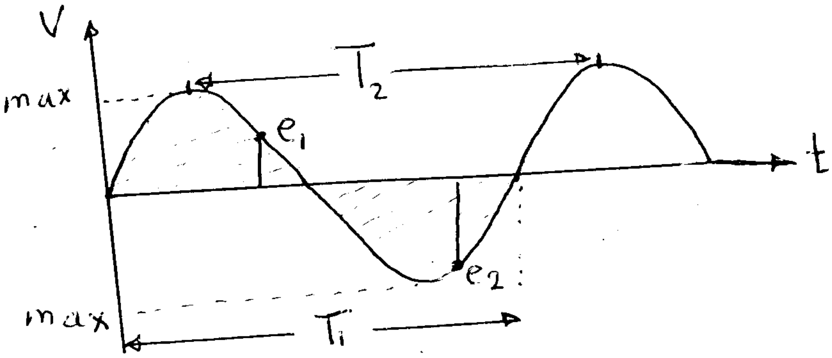
Square-wave



Trangular-wave

The term alternating indicates only that the wave-form alternates between two prescribed levels, the term Sinusoidal, Square, trangular must also be applied. The pattern of particular interest is the Sinusoidal a-c Voltage.

** Definitions :-



* **Waveform**: The path traced by a quantity, such as the e.m.f in fig-above. plotted as a function of some variable such as time (above), degree, radian, and so on.

* **Instantanous value**: The magnitude of awave form at any instant of time, denoted by lower-case letters (e, v, e_2).

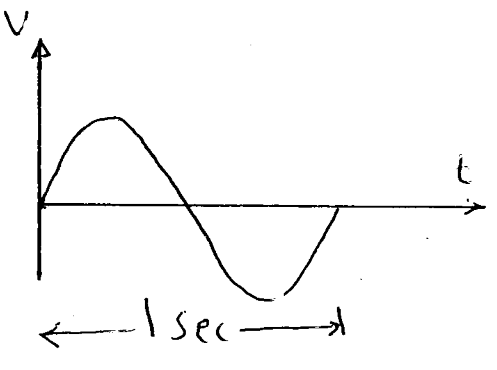
* **Amplitude or peak value**: The maximum value of awave-form, denoted by upper-case letters (max).

* **periodic waveform**: A wave form that continually repeats it self after the same time interval.

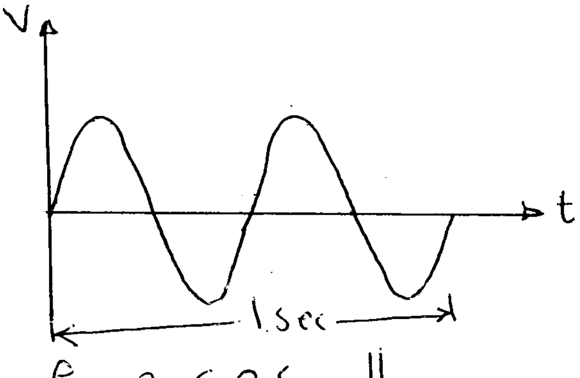
* **Period (T)**: The time interval between two successive repetitions of aperiodic wave form ($T_1 = T_2$)

* **Cycle**: The portion of a wave form contained in one period of time.

* **Frequency**: The number of cycles that occur in 1 sec. The unit of frequency is cycle/sec (cps) or Hertz.



$f_1 = 1$ cps. $f = 1$ Hz. || $T = 1$ sec

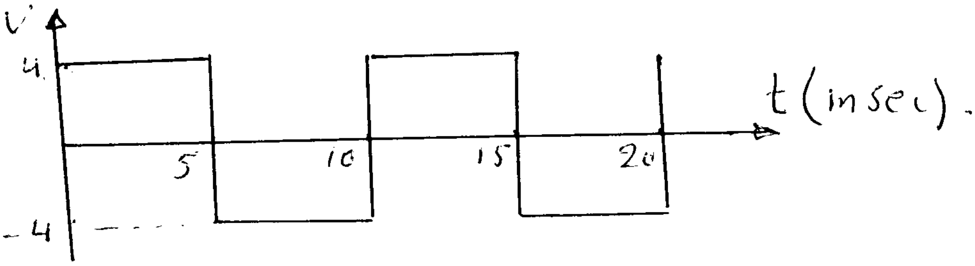


$f = 2$ cps. $f = 2$ Hz. || $T = \frac{1}{2}$ sec.

** Since the frequency is inversly propotional to the period. The two can be related by the following equation.

$$\boxed{f = \frac{1}{T}}$$

Ex: A wave form as shown, find the value of period and the frequency.



$T = 10$ msec.

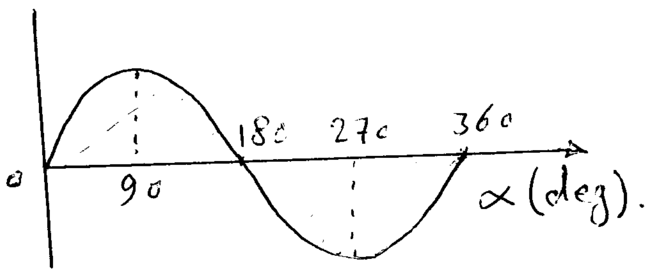
$\therefore f = \frac{1}{T} = 100\,Hz$.

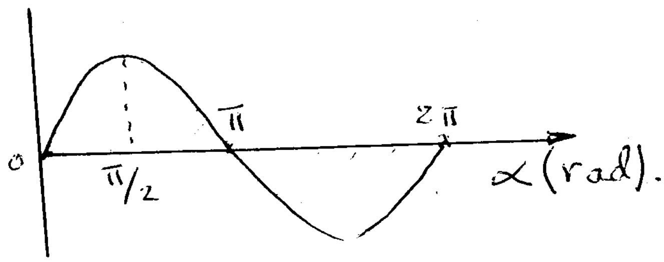
The Sine Wave

The basic mathematical format for sinusoidal wave form is «A Sin α».

where: A is the max. or peak value

α is the unit of measurement of horizontal axis, it may be in degree or radian.





$2\pi\ (rad) = 360$

or

$Rad = \frac{\pi}{180} \cdot Deg.$

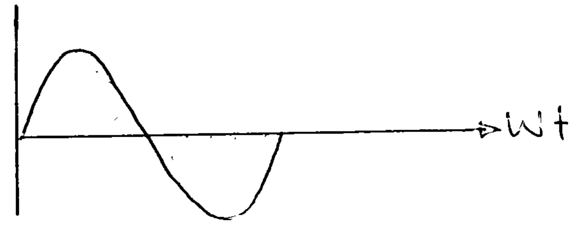
$Deg = \frac{180}{\pi} \cdot Rad.$

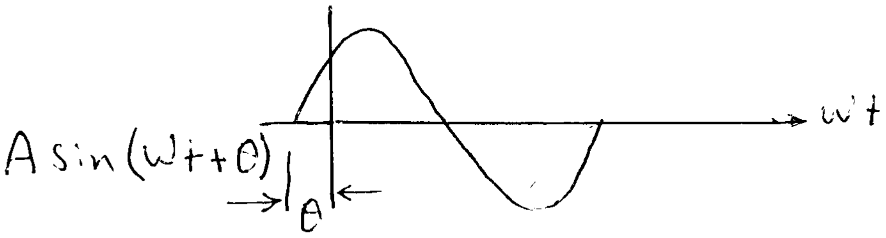
phase relations

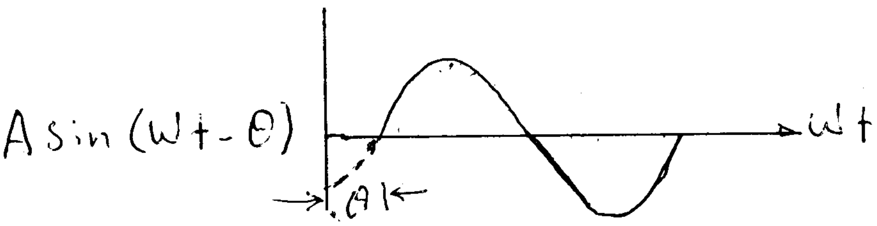
If the wave form is shifted to the right or left of Zero, the expression becomes:

$A \sin(\omega t \mp \theta)$.

A $\sin \omega t$.

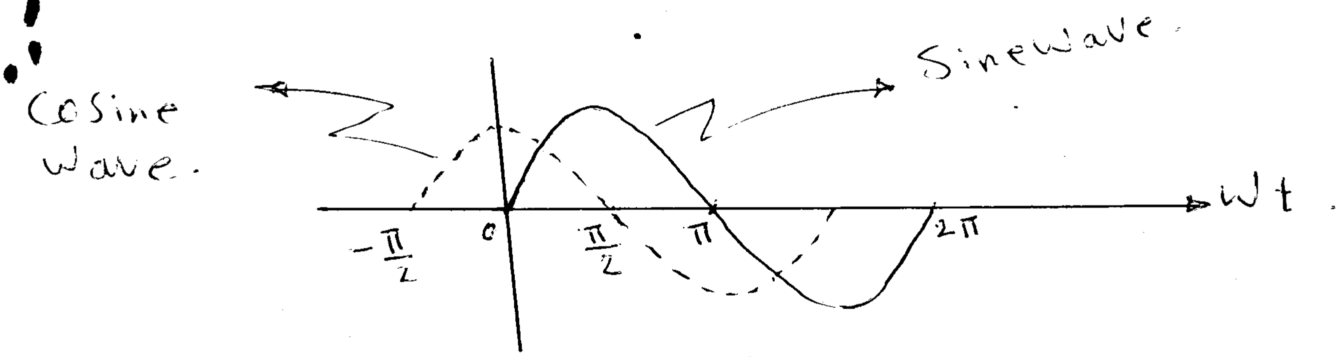






The cosine wave is said to lead the sine wave by 90°, and the sine wave is said to lag the cosine wave by 90°.

Note: lead and lag indicate the relationship between two sine wave of same frequency.

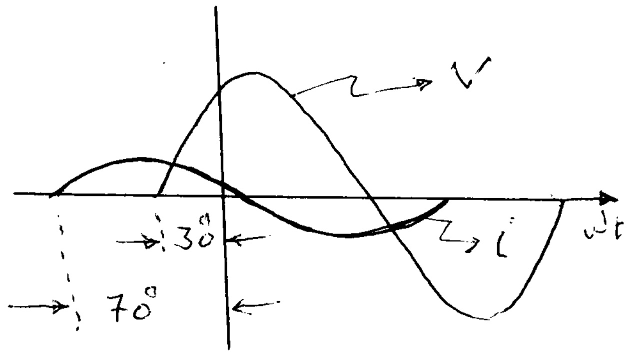


Ex: What is the phase relationship between V & i

(37)

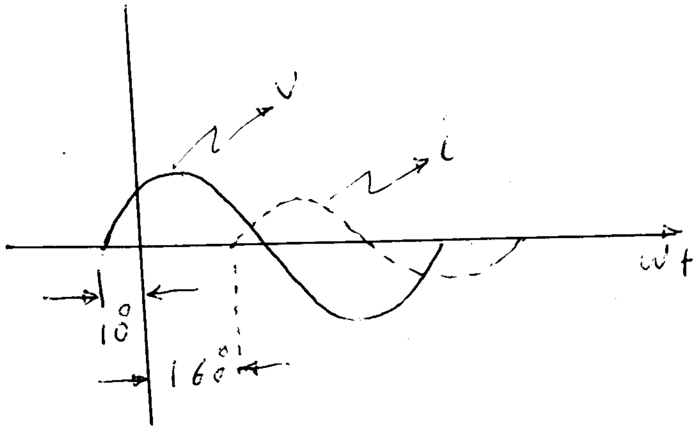
$V = 15 \sin(\omega t + 30)$.
$i = 4 \sin(\omega t + 70)$.

$\therefore$ i lead V by 40°
or V lags i by 40°



Ex: $i = -2 \sin(\omega t + 20^\circ)$.
$V = 5 \sin(\omega t + 10^\circ)$.

$i = -2 \sin(\omega t + 20^\circ)$.
$= 2 \sin(\omega t + 20 - 180^\circ)$.
$= 2 \sin(\omega t - 160^\circ)$.



$\therefore$ V lead i by 170°.
or i lag V by 170°

** Average (mean) Value

The average value of any current or voltage is the value indicated on a d.c meter.

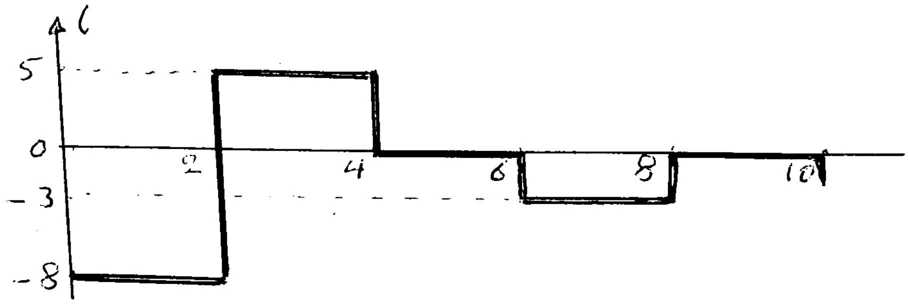
$$I \text{ or } V \text{ (average value)} = \frac{\text{algebriac Sum of areas}}{\text{length of curve}}$$

OR

$$\boxed{I_{(average)} = \frac{1}{T}\int_0^T i\,dt}$$

$$\boxed{V_{(average)} = \frac{1}{T}\int_0^T v\,dt}$$

Ex: Find the average value of the following wave form over one full cycle.



$$I_{(av)} = \frac{(-8\times 2)+(5\times 2)+(-3\times 2)}{10} = \frac{-12}{10} = -1.2 \text{ Amp.}$$

OR

$$I_{(av)} = \frac{1}{T}\int_0^T i\,dt = \frac{1}{10}\left[\int_0^2 -8\,dt + \int_2^4 5\,dt + \int_4^6 0\,dt + \int_6^8 -3\,dt + \int_8^{10} 0\,dt\right]$$

$$= \frac{1}{10}\left[-8\times 2 + 5\times 2 + (-3\times 2)\right]$$

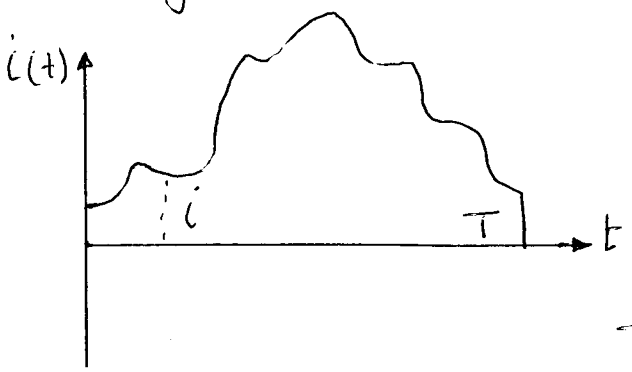
$$= \frac{-12}{10} = -1.2 \text{ Amp.}$$

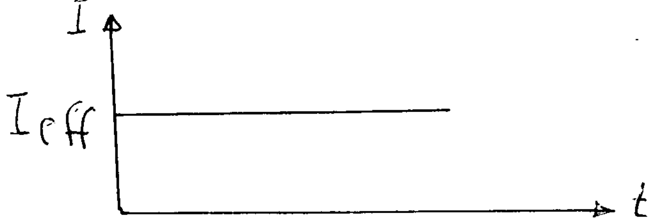
** Effective Value (r.m.s).

The effective value of alternating current is measured in terms of the direct constants current that produced the same heating effect in the same resistance for the same period of time.

∴ Heat generated by current (I) for time (t) in a resistance (R) is :

Heat generated $= I^2 \cdot R \cdot t$ (Joules).





- Heat generated by a.c $= \int_0^T i^2 \cdot R \cdot dt$. (J).

$$= R \int_0^T i^2 \cdot dt \quad (J).$$

- Heat generated by constant current $= I_{eff}^2 \cdot R \cdot T$.

$$\therefore I_{eff}^2 \cdot R \cdot T = \int_0^T i^2 \cdot R \cdot dt.$$

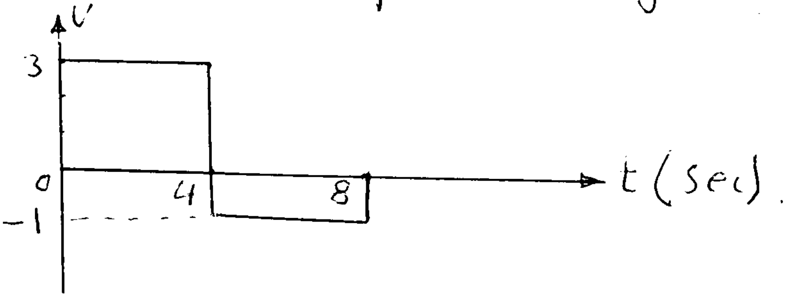
$$\therefore I_{eff}^2 = \frac{1}{T} \int_0^T i^2 dt.$$

square.

OR : $\boxed{I_{eff} = I_{r.m.s} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}.}$

root. mean

Ex.: Find the effective value (r.m.s) of the voltage shown

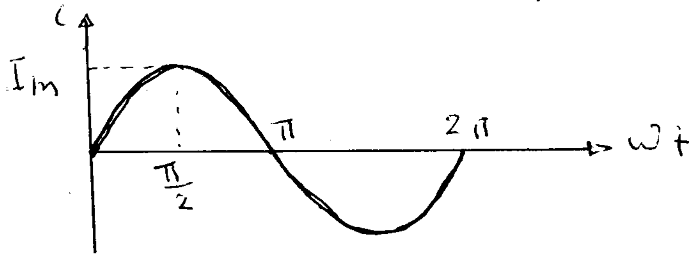


$$V_{eff} = V_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}\,.$$

$$= \sqrt{\frac{1}{8}\left[\int_0^4 3^2\,dt + \int_4^8 (-1)^2\,dt\right]} = \sqrt{5} = 2.23 \text{ Volt}.$$

Ex.: Show that the effective value or (r.m.s) of sine wave current with peak current I_m is

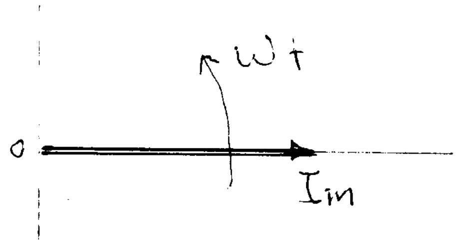
$$I_{r.m.s} = \frac{I_m}{\sqrt{2}}$$



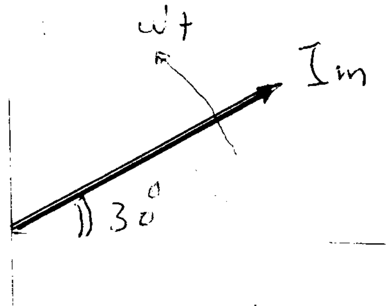
* phasors: A rotating radius vector having a constant magnitude (length) with one end fixed at origin, during its rotation it produces the Sine wave.

we can express the instantaneous value of current (i)

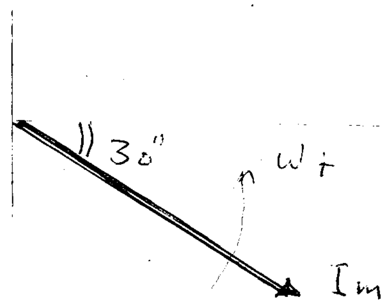
* $i = I_m \sin \omega t$.



* $i = I_m \sin(\omega t + 30^\circ)$



* $i = I_m \sin(\omega t - 30^\circ)$.



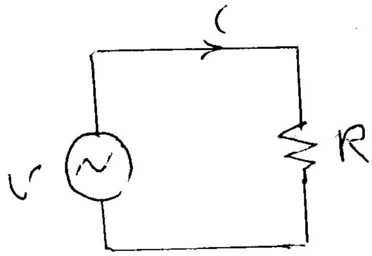
ω = Angular Velocity = $\dfrac{\text{distance (Deg or Rad)}}{\text{Time}}$

$= \dfrac{\alpha}{t}$.

$= \dfrac{2\pi}{T} = 2\pi f$ (rad/sec).

* Response of basic R, L and C elements to a Sinusoidal Voltage or current.

To find the response we can use the ohm's law and the basic equations for C, L & R.



① The Resistor (R)

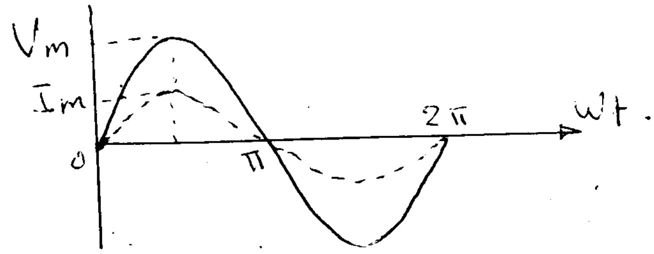
let $v = V_m \sin\omega t$.

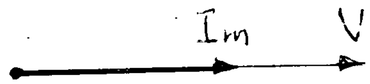
then $i = \frac{v}{R} = \frac{V_m}{R} \sin\omega t$.

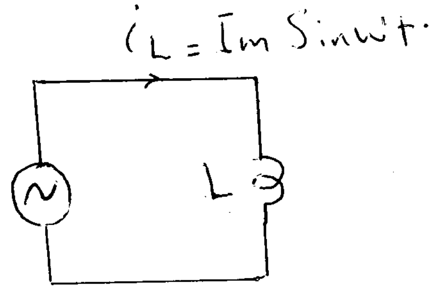
$= I_m \sin\omega t$.

where: $I_m = \frac{V_m}{R}$.

∴ The voltage and current are in phase.







② The Inductor (L)

For the inductor we know that.

$$v_L = L \frac{di}{dt}$$

let $i_L = I_m \sin\omega t$.

$$\therefore v_L = L \frac{d}{dt}(I_m \sin\omega t).$$

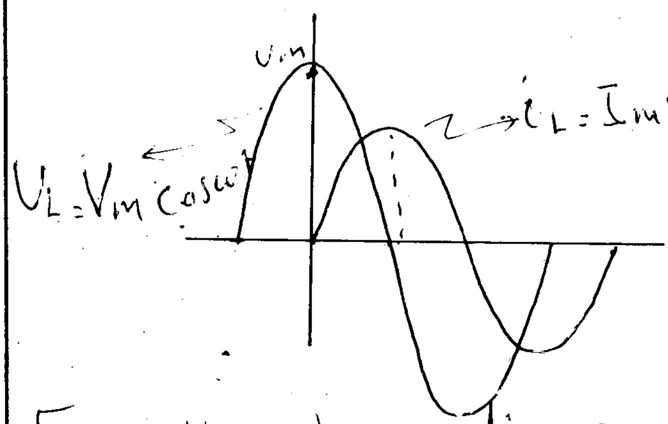
$V_L = L(\omega I_m \cos\omega t) = \omega L I_m \cos\omega t.$

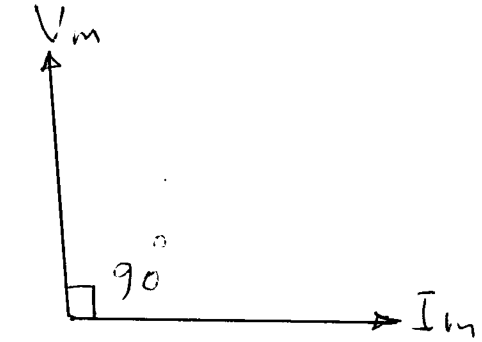
$\therefore V_L = V_m \cos\omega t = V_m \sin(\omega t + 90^\circ).$

Where $V_m = \omega L I_m$.

$$\therefore \frac{V_m}{I_m} = \omega L = 2\pi f L = X_L \quad (\Omega).$$

* The quantity ωL, called the reactance of an inductor and measured in ohm, It is symbolically represented by X_l $\quad \therefore X_L = \omega L$





From the phasor diagram, Shows that V_L is lead i_L by 90° or i_L is lags V_L by 90°.

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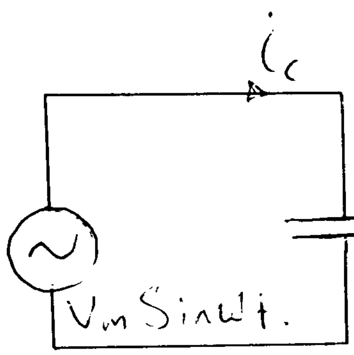
3 The Capacitor (C)

For the capacitor we know that



$$i_c = C\frac{dV_c}{dt}.$$

let $V_c = V_m \sin\omega t$.

$$\therefore i_c = C\frac{d(V_m \sin\omega t)}{dt}.$$

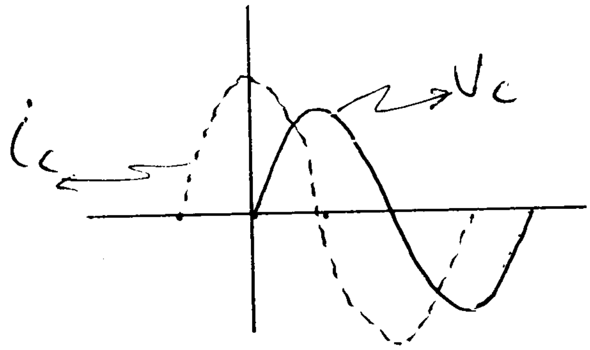
$i_c = C(\omega V_m \cos\omega t) = \omega c V_m \cos\omega t$.   (44)

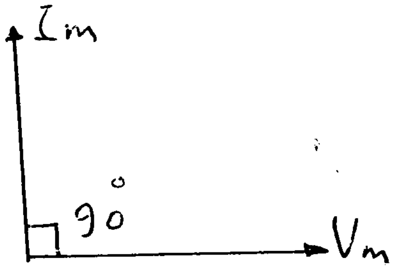
$\therefore i_c = I_m \cos\omega t = I_m \sin(\omega t + 90^\circ)$.

Where $I_m = \omega c V_m$

$$\therefore \frac{V_m}{I_m} = \frac{1}{\omega c} = \frac{1}{2\pi f c} = X_c \quad (\Omega).$$

* The quantity $\left(\frac{1}{\omega c}\right)$, called the reactance of Capacitance and symbolically represented by X_c and measured in ohm.



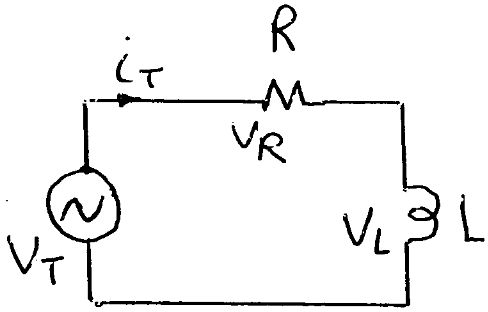


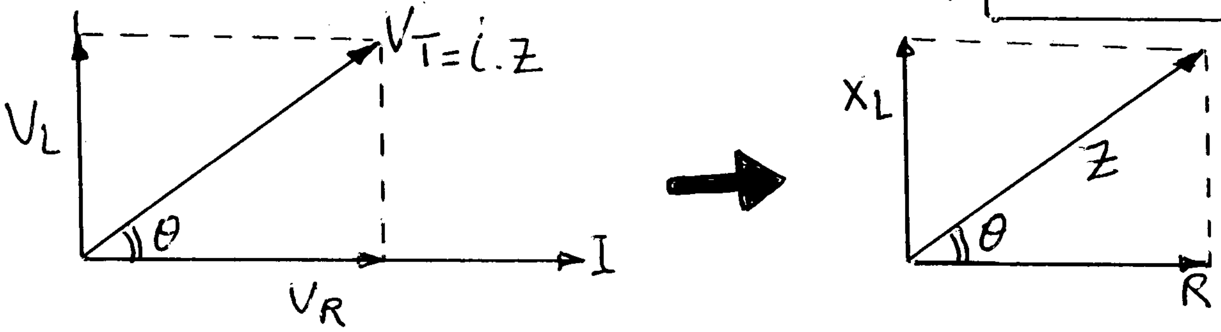
From the phasor diagram, shows that i_c is lead V_c by 90°, or V_c is lags i_c by 90°.

R_L and C Connections

① R_L in Series:

Let $i_T = I_m \sin \omega t$.





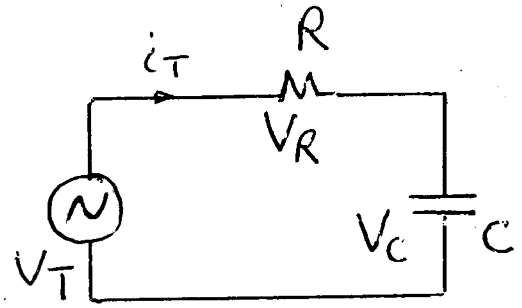
$\therefore V_T = \sqrt{V_R^2 + V_L^2}$ and $\theta = \tan^{-1} \frac{V_L}{V_R}$.

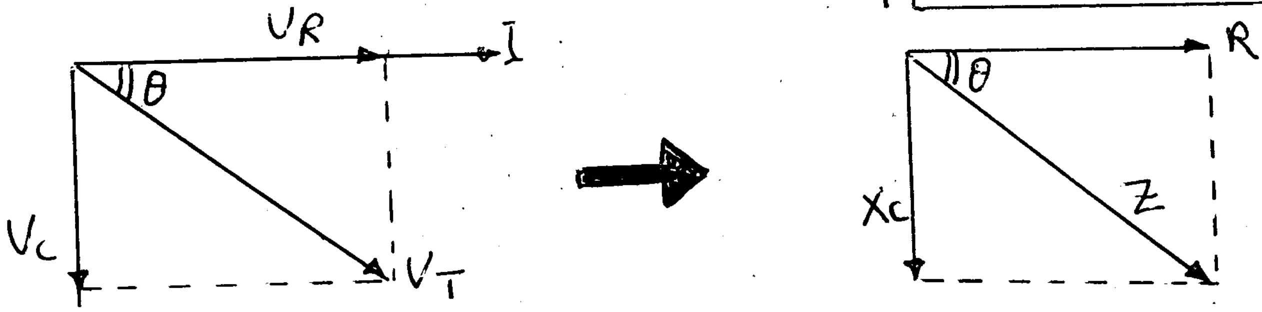
and $Z = \sqrt{R^2 + X_L^2}$. or $\theta = \tan^{-1} \frac{X_L}{R}$.

V_T lead i_T by angle θ, or i_T lags V_T by θ.

② R_C in Series:

Let $i_T = I_m \sin \omega t$.





$V_T = \sqrt{V_R^2 + V_C^2}$

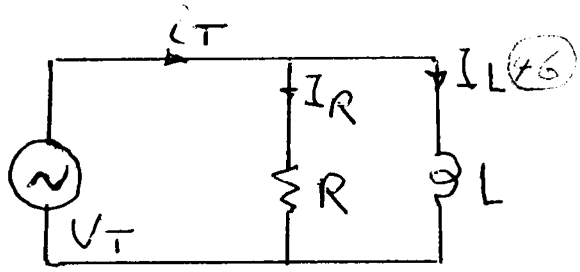
and $Z = \sqrt{R^2 + X_C^2}$ and $\theta = \tan^{-1} \frac{V_C}{V_R} = \tan^{-1} \frac{X_C}{R}$.

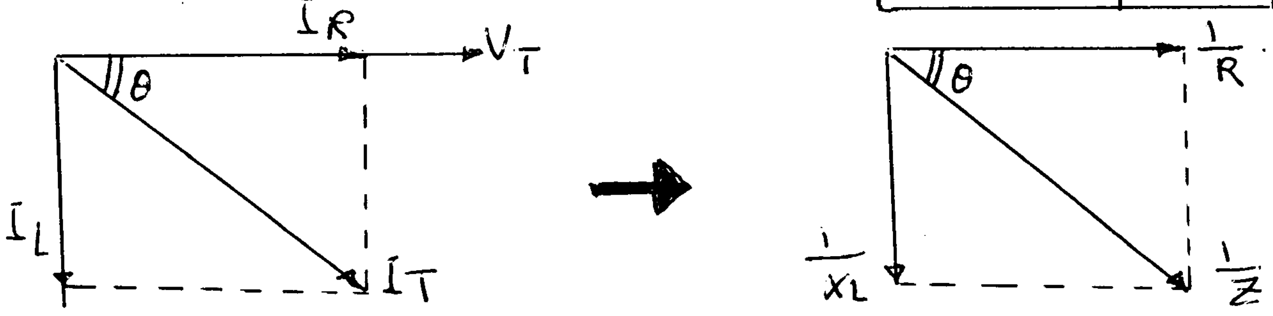
V_T lags i_T by angle θ, or i_T lead V_T by θ.

Note: Z is the total impedance, and its Unit is (Ω).

③ R_L in parallel;

Let $V_T = V_m \sin \omega t$.





$\therefore i_T = \sqrt{I_R^2 + I_L^2}$

and $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}$. and $\theta = \tan^{-1} \frac{I_L}{I_R} = \tan^{-1} \frac{R}{X_L}$.

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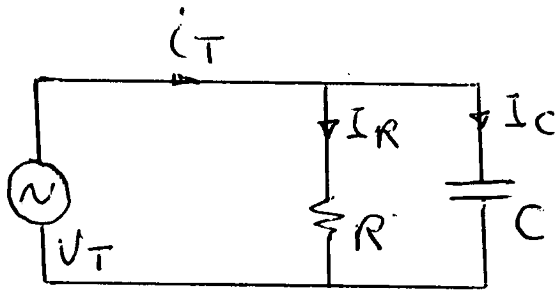
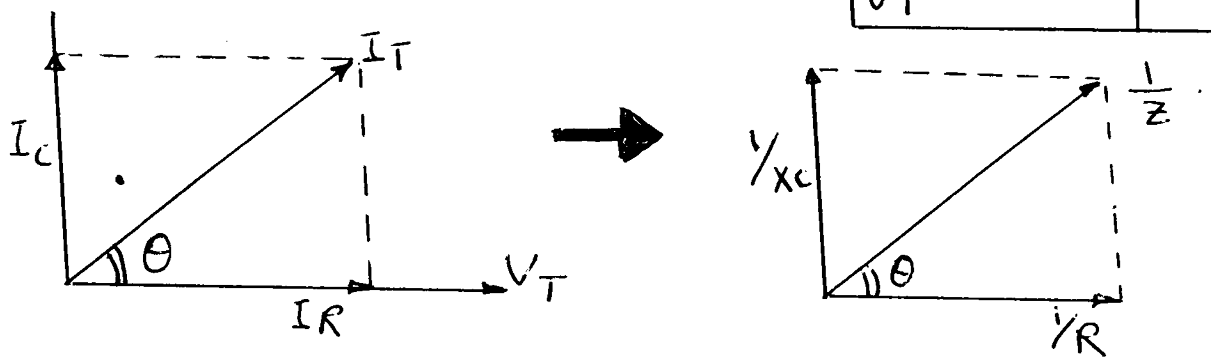
④ R_C in parallel;

Let $V_T = V_m \sin \omega t$.

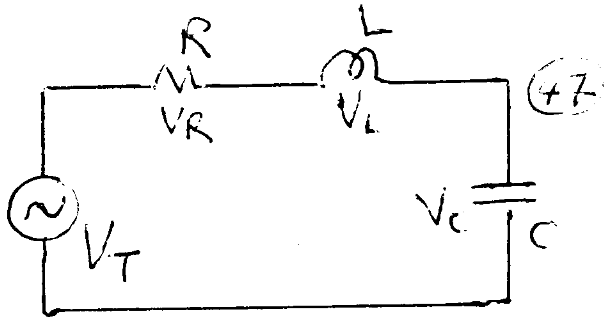




$\therefore i_T = \sqrt{I_R^2 + I_C^2}$.

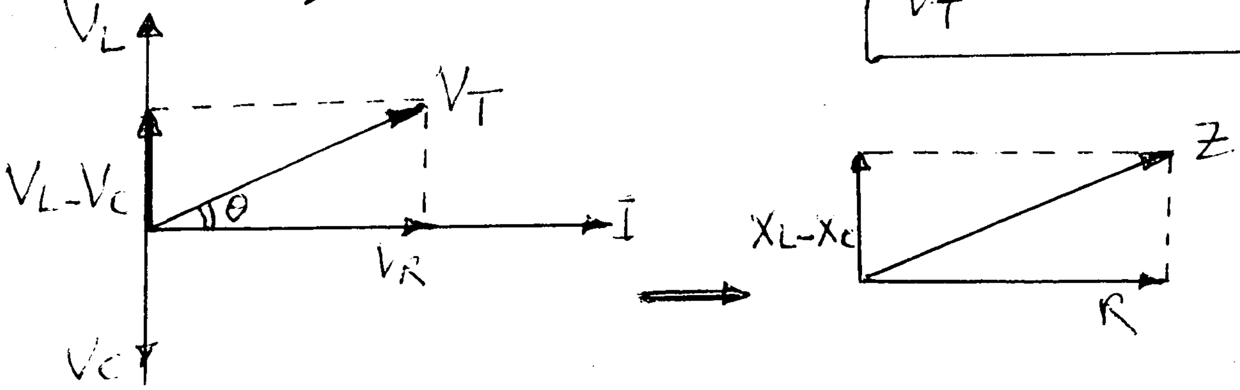
and $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}}$ and $\theta = \tan^{-1} \frac{I_C}{I_R} = \tan^{-1} \frac{R}{X_C}$.

⑤ R-L and C in series:



Let $i_T = I_m \sin \omega t$.

If $X_L > X_C$



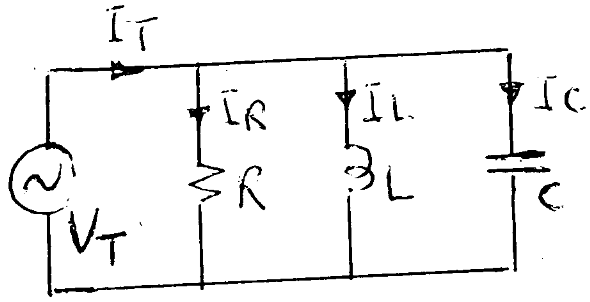
$$\therefore V_T = \sqrt{V_R^2 + (V_L - V_c)^2}$$

and $Z = \sqrt{R^2 + (X_L - X_c)^2}$. and $\theta = \tan^{-1} \frac{V_L - V_c}{V_R} = \tan^{-1} \frac{X_L - X_c}{R}$

** If $X_L > X_c$ $\therefore V_T$ lead i_T by θ.

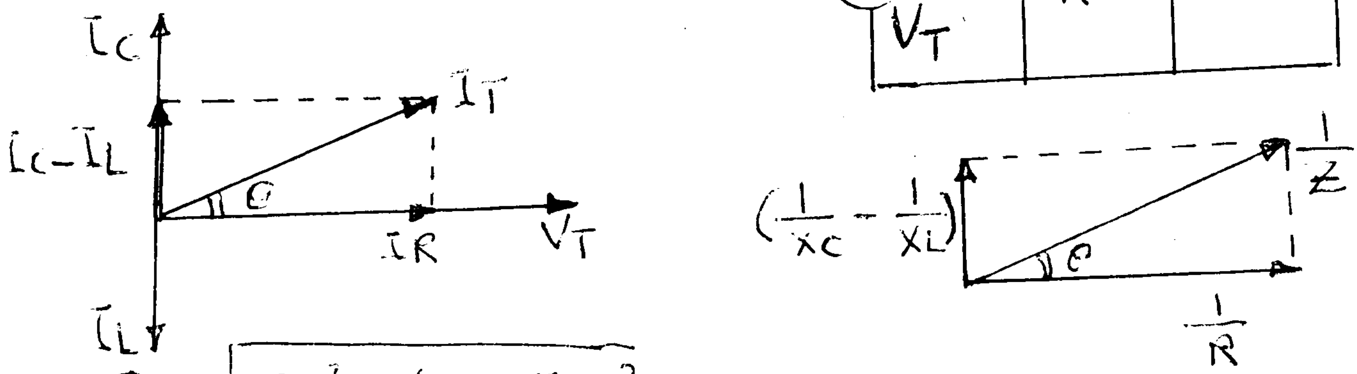
$X_L < X_c$ $\therefore V_T$ lag i_T by θ.

⑥ R-L and C in parallel



Let $V_T = V_m \sin \omega t$.

if $X_L > X_c$



$$\therefore I_T = \sqrt{I_R^2 + (I_c - I_L)^2}.$$

and $\frac{1}{Z} = \sqrt{(\frac{1}{R})^2 + (\frac{1}{X_c} - \frac{1}{X_L})^2}$. and $\theta = \tan^{-1} \frac{I_c - I_L}{I_R}$

** If $X_L > X_c$ $\therefore V_T$ lag I_T by θ.

$X_L < X_c$ $\therefore V_T$ lead I_T by θ.

Complex Number

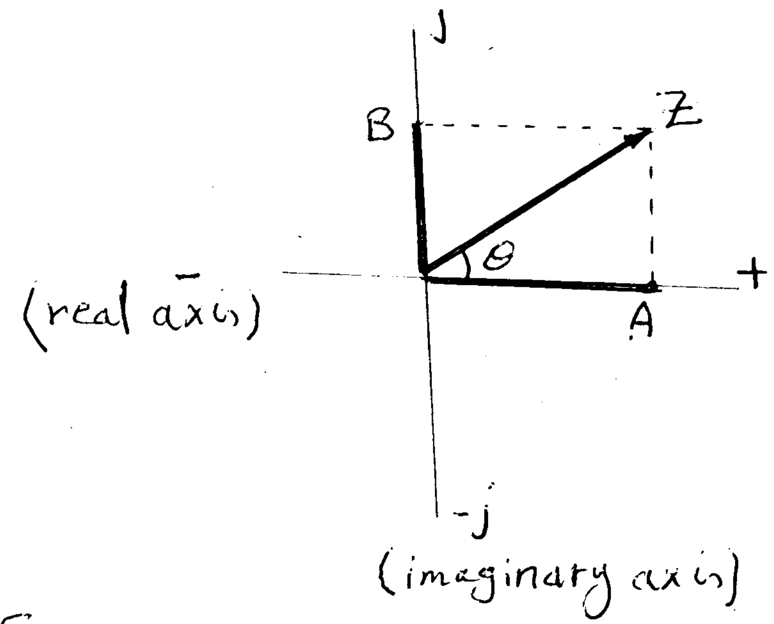
There are two form used to represent a complex number (C), the rectangular form, and polar form

* Rectangular form:

$C = A + jB$.

* polar form:

$C = Z\angle\theta$.



Conversion between Forms

① Rectangular ⟶ polar.

If $C = A + jB$

$\therefore Z = \sqrt{A^2 + B^2}$ $\qquad \theta = \tan^{-1}\frac{B}{A}$.

$\therefore C = Z\angle\theta = \sqrt{A^2+B^2}\angle\tan^{-1}\frac{B}{A}$.

$j = \angle 90^\circ = \sqrt{-1}$
$j \times j = -1$
$j \cdot j \cdot j = -j$
$j \cdot j \cdot j \cdot j = j^4 = 1$

② polar ⟶ Rectangular.

If $C = Z\angle\theta$.

$A = Z\cos\theta$.
$B = Z\sin\theta$.

$\therefore C = A + jB = Z(\cos\theta + j\sin\theta)$.

Mathematical operations with Complex number

If $C_1 = A_1 + jB_1 = Z_1 \angle \theta_1$
$C_2 = A_2 + jB_2 = Z_2 \angle \theta_1$

① **Addition**:

$$C = C_1 + C_2 = (A_1 + A_2) + j(B_1 + B_2).$$

② **Subtraction**:

$$C = C_1 - C_2 = (A_1 - A_2) + j(B_1 - B_2).$$

③ **Multiplication**:

$$C = C_1 \cdot C_2$$

* In polar form: $C = C_1 \cdot C_2 = Z_1 \cdot Z_2 \angle \theta_1 + \theta_2$
* In rectangular form: $C = C_1 \cdot C_2 = (A_1 + jB_1)(A_2 + jB_2)$

$$= A_1 A_2 + jB_1 A_2 + jA_1 B_2 - B_1 B_2.$$
$$= (A_1 A_2 - B_1 B_2) + j(B_1 A_2 + A_1 B_2).$$

④ **Division**:

$$C = \frac{C_1}{C_2}$$

* In polar form: $C = \frac{C_1}{C_2} = \frac{Z_1}{Z_2} \angle \theta_1 - \theta_2.$
* In regtongular form: $C = \frac{C_1}{C_2} = \frac{A_1 + jB_1}{A_2 + jB_2}.$

$$= \frac{A_1 + jB_1}{A_2 + jB_2} \times \left(\frac{A_2 - jB_2}{A_2 - jB_2}\right)$$
$$= \frac{(A_1 + jB_1)(A_2 - jB_2)}{A_2^2 + B_2^2}$$

Where: $(A_2 - jB_2)$ is the conjugate of the $(A_2 + jB_2)$.

Ex: If $C_1 = 6+j8 = 10\angle 53.13$. (50)

$C_2 = 3+j4 = 5\angle 53.13$. Find:-

① Addition: $C = C_1 + C_2 = 9 + j12$

② Subtraction: $C = C_1 - C_2 = (6-3) + j(8-4) = 3+j4$.

③ Multiplication: $C = C_1 \cdot C_2$

* In polar: $C = 10 \times 5 \angle 53.13 + 53.13 = 50\angle 106.26$.

* In regtangular: $C = (3\times 6 - 8\times 4) + j(8\times 3 + 4\times 6) = -14 + j48$.

Check: Same result.

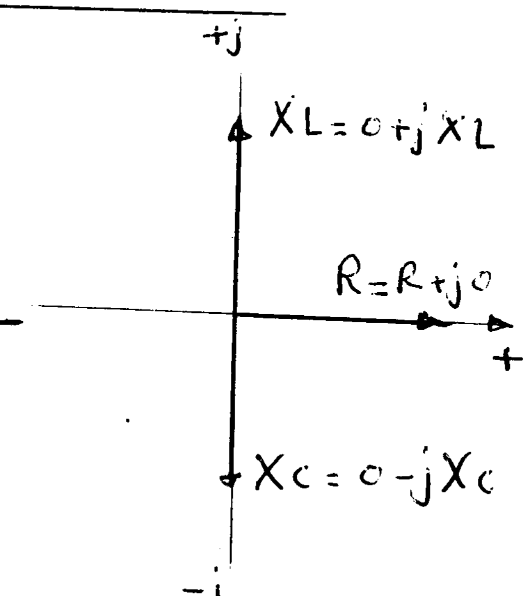
④ Division: $C = \frac{C_1}{C_2}$.

* In polar: $C = \frac{10}{5}\angle 53.13 - 53.13 = 2\angle 0 = 2 + j0$

* In rectangular $= \frac{(6+j8)(3-j4)}{3^2+4^2} = \frac{18 - \cancel{j24} + \cancel{j24} + 32}{25}$

$= 50/25 = 2$.

In General:



* Horizontal axis called Real axis or (resistance axis).

* Vertical axis called imaginary axis or (reactance axis).

∴ The resistance (R) represented as

$\boxed{R = R + j0}$

The Inductance (XL) represented as

$\boxed{X_L = 0 + jX_L}$

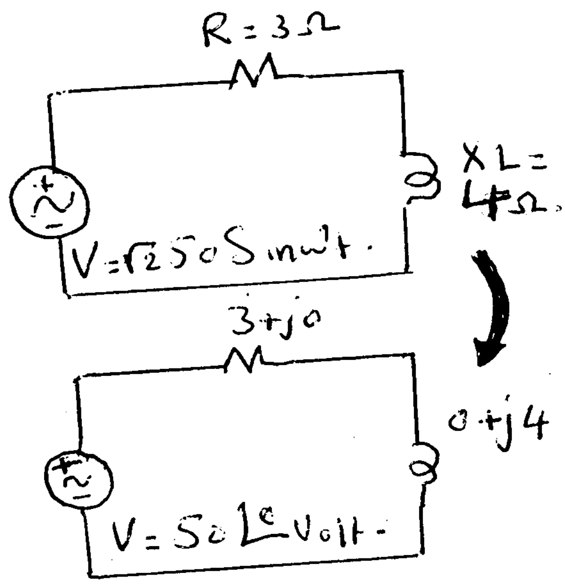
The Capacitor reactor (X_c) represented as

$\boxed{X_c = 0 - jX_c}$

Conversion between Time to phasor Domain

Time Domain	phasor Domain
$\sqrt{2}\,50\,\sin\omega t$	$50\angle 0$
$45\cos\omega t = 45\sin(\omega t + 90)$	$\frac{45}{\sqrt{2}}\angle 90$
$\sqrt{2}\,100\,\sin(\omega t + 30)$	$100\angle 30^\circ$
$\sqrt{2}\,10\,\sin(\omega t + 70)$	$10\angle 70$
$\sqrt{2}\,120\,\sin(\omega t - 80)$	$120\angle -80$
⋮	⋮

Ex.: Find the total impedance and the current.



Since R in series with XL.

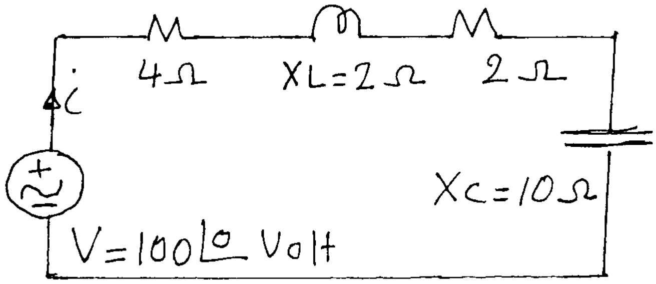
$$\therefore Z = (3 + j0) + (0 + j4)$$
$$= (3 + j4)\,\Omega$$
$$= 5\angle 53.13\,\Omega.$$

$$I = \frac{V_T}{Z} = \frac{50\angle 0}{5\angle 53.13} = 10\angle -53.13 \text{ Amp.}$$

and the current is $(\sqrt{2}\cdot 10\,\sin(\omega t - 53.13))$.
Amp.

Ex: Find the current in the Circuit.

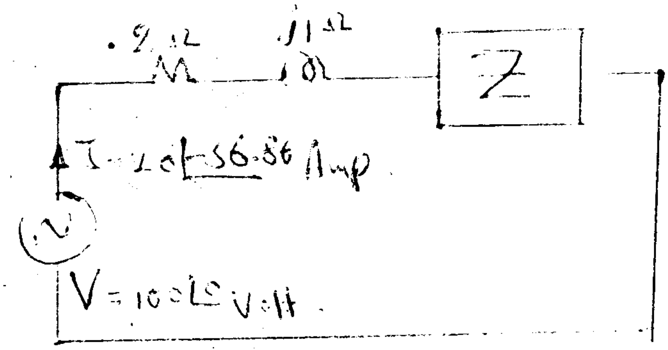
$Z = (6 - j8)\,\Omega = 10\angle{-53.13}\,\Omega.$



$\therefore i = \dfrac{V}{Z} = \dfrac{100\angle 0}{10\angle{-53.13}}.$

$= 10\angle{53.13}$ Amp.

Ex: Find the Value of (Z) for the Circuit.
If $I = 20\angle{-36.86}$ Amp.



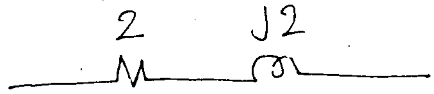
Total impedance $= \dfrac{V}{I}.$

$= \dfrac{100\angle 0}{20\angle{-36.86}} = 5\angle{36.86}\,\Omega.$

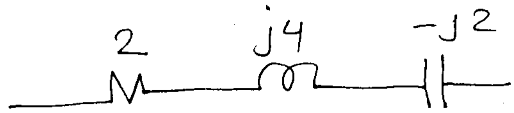
$= (4 + j3)\,\Omega.$

$\therefore$ total $R = 4\,\Omega$, total inductor $= 3\,\Omega.$

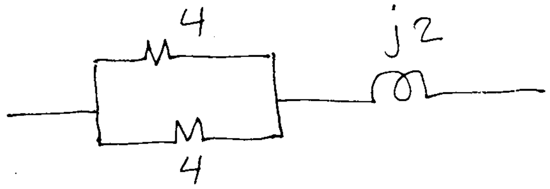
$\therefore$ Z =



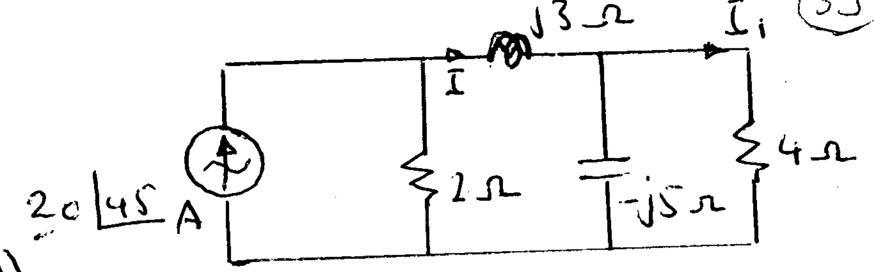
OR Z =



OR Z =



Ex: For the circuit shown, find (I) & (I_1)?



$$Z = 2 \,||\, (j3 + (-j5 \,||\, 4\Omega))\ \Omega.$$

$$= 2 \,||\, \left(j3 + \frac{4(-j5)}{4-j5}\right) = 2 \,||\, (j3 + 3.12\angle{-38.7})\ \Omega.$$

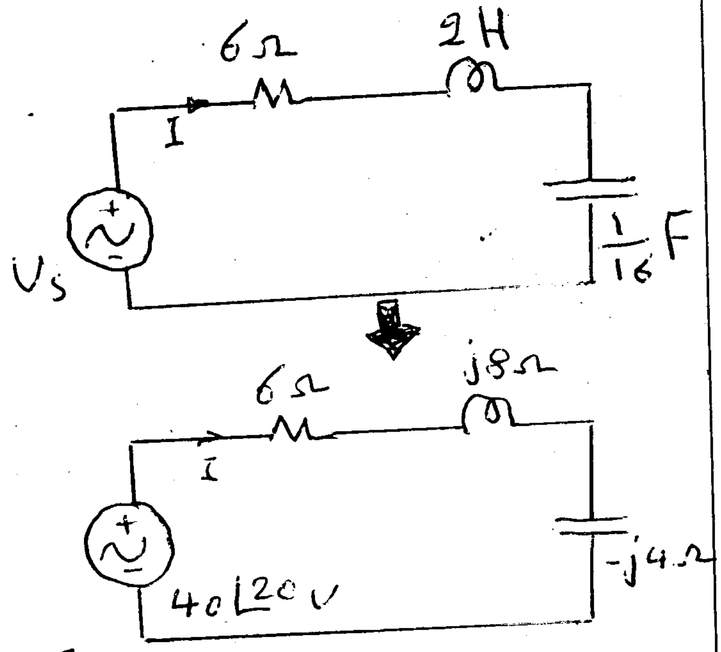
$$= 2 \,||\, 2.65\angle{23.3}\ \Omega.$$

$$I = 20\angle{45}\ \frac{2}{2 + 2.65\angle{23.3}} = 8.77\angle{31.7}\ A \quad \text{(By current divider Rule)}.$$

&

$$I_1 = 8.77\angle{31.7}\ \frac{-j5}{4-j5} = 6.85\angle{-7}\ A \quad (\text{'' '' ''}).$$

Ex: Find (I) in time domain?



$V_s = \sqrt{2} \times 40 \sin(4t + 20)$. Volt.

$\therefore V_s = 40\angle{20}$ Volt.

$X_L = \omega L = 4 \times 2 = 8\ \Omega.$

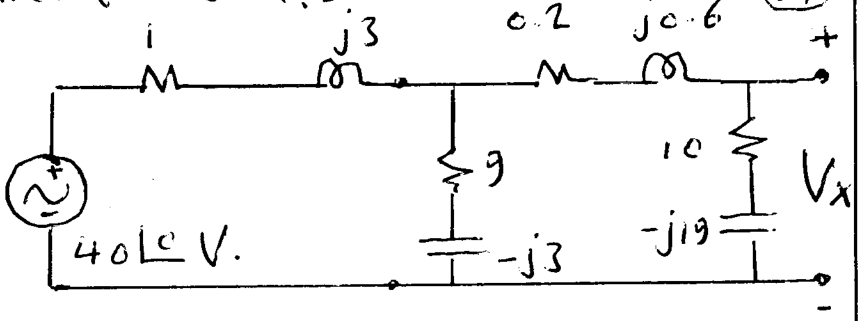
$X_c = \frac{1}{\omega c} = \frac{1}{4 \times 1/16} = 4\ \Omega.$

$$\therefore I = \frac{V}{Z} = \frac{40\angle{20}}{6 + j4}$$

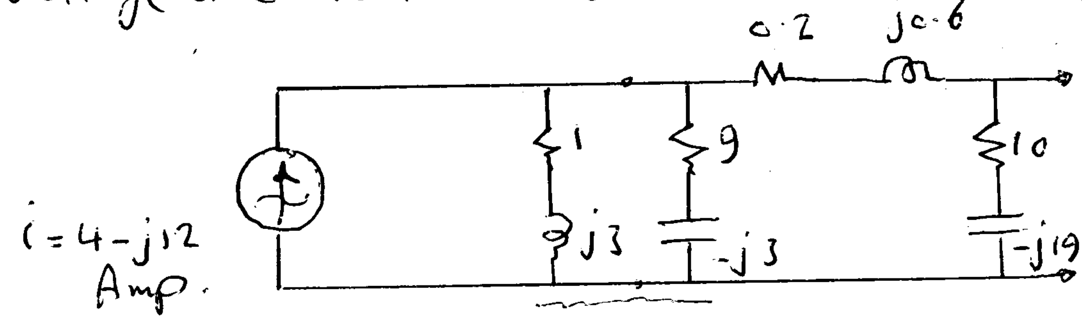
$$= \frac{40\angle{20}}{7.211\angle{33.7}} = 5.54\angle{-13.7}\ A.$$

$\therefore I = \sqrt{2} \times 5.54 \sin(4t - 13.7)$ A.

Ex: Find (V_x) for the Circuit Shown:-



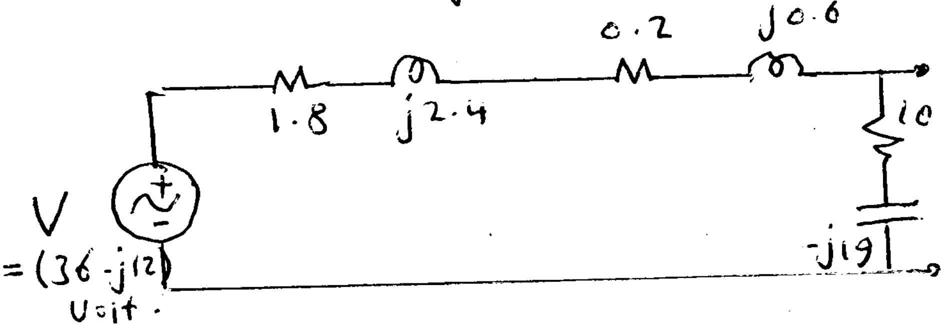
a) Convert the Voltage to Current Source:-



$$i = \frac{40\angle 0°}{1+j3} = \frac{40}{1+j3} = \frac{40}{10}(1-j3) = 4-j12 \text{ Amp.}$$

$$(1+j3) \parallel (9-j3) = \frac{(1+j3)(9-j3)}{10} = (1.8+j\,2.4)\,\Omega.$$

b) Convert the current Source to Voltage Source:

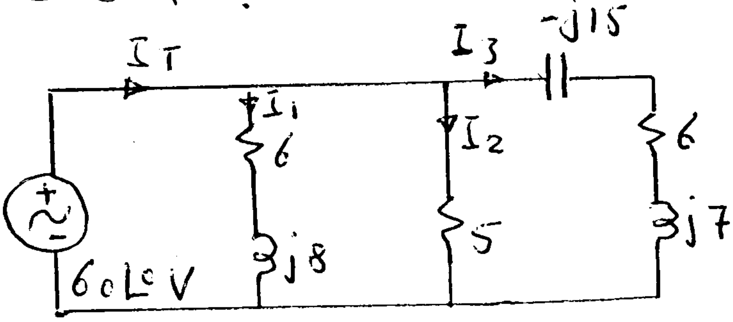


$$V = I \cdot Z = (4-j12)(1.8+j2.4)$$
$$= (36-j12) \text{ Volt.}$$

$$\therefore I_T = \frac{V}{Z_T} = \frac{36-j12}{12-j16} = (1.56+j1.08) \text{ Amp.}$$

$$\therefore V_x = I_T(10-j19) = (1.56+j1.08)(10-j19)$$
$$= (36.12 - j18.84) \text{ Volt.}$$

Ex: Find the (I_T) of the Circuit ?



$$I_1 = \frac{60\angle 0}{6+j8} = \frac{60\angle 0}{10\angle 53.1}$$

$$= 6\angle -53.1$$

$$= 3.6 - j4.8 \text{ Amp.}$$

$$I_2 = \frac{60\angle 0}{5} = 12 \text{ Amp.}$$

$$I_3 = \frac{60\angle 0}{6-j8} = \frac{60\angle 0}{10\angle -53.1} = 6\angle 53.1 = 3.6 + j4.8 \text{ Amp.}$$

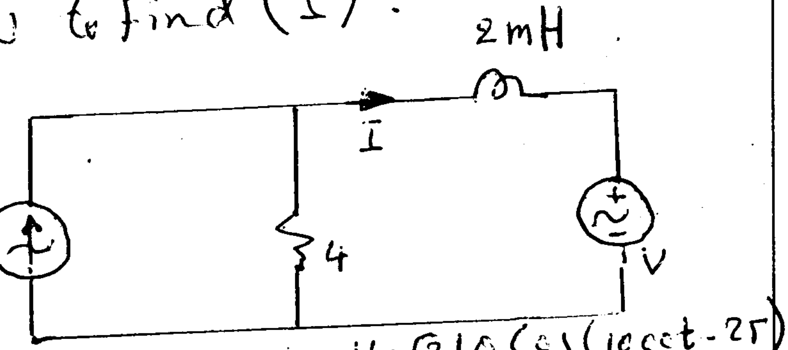
$$\therefore I_T = I_1 + I_2 + I_3 = 19.2\angle 0 \text{ Amp.}$$

OR: Find the total impedance (Z).

$$Z = (6+j8) \| 5 \| (6-j8) \text{ then.}$$

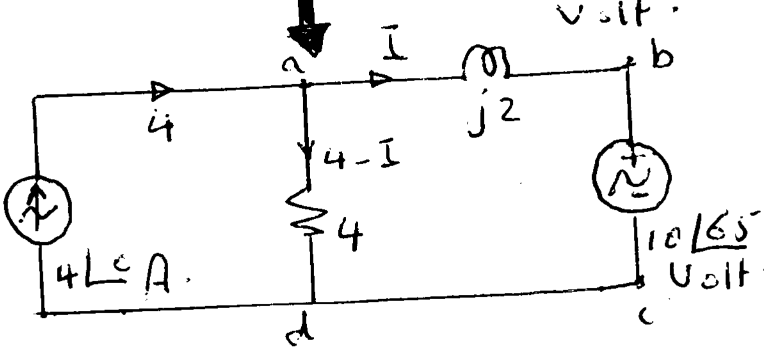
$$I = \frac{60\angle 0}{Z} = 19.2\angle 0 \text{ Amp}$$

Ex: Using Kirchhoff's Law to find (I) ?



$4\sqrt{2}\, \text{Sin}1000t$ Amp.

$V = \sqrt{2}\, 10 \,\text{Cos}(1000t - 25)$ Volt.



from abcd

$$4(4-I) = j2I + 10\angle 65.$$

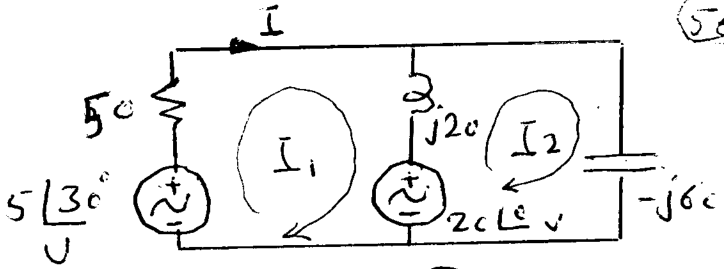
$$16 - 4I = j2I + 10\angle 65$$

$$16 - 10\angle 65 = I(4+j2).$$

$$\therefore I = \frac{16 - 10\angle 65}{4+j2} = 3.32\angle -64.2 \text{ Amp.}$$

Ex: Use loop current method (mesh) to find (I) ?

$I = I_1$



loop(1):

$5\angle 30° - 20\angle 0° = (50 + j20) I_1 - j20 I_2$ ——— (1).

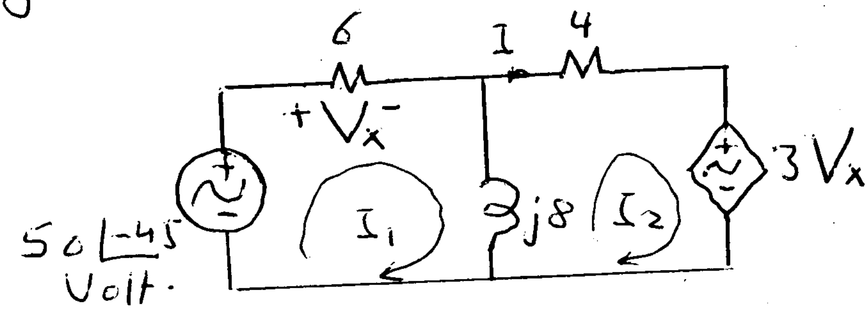
loop(2):

$20\angle 0° = -j40 I_2 - j20 I_1$ ——— (2).

Solving: $I = I_1 = 0.442\angle 144°$ Amp.

Ex: Find (I) by using loop method?

$I = I_2$.



loop(1):

$50\angle -45 = (6 + j8) I_1 - j8 I_2$ ——— (1).

loop(2):

$-3V_x = (4 + j8) I_2 - j8 I_1$ ——— (2).

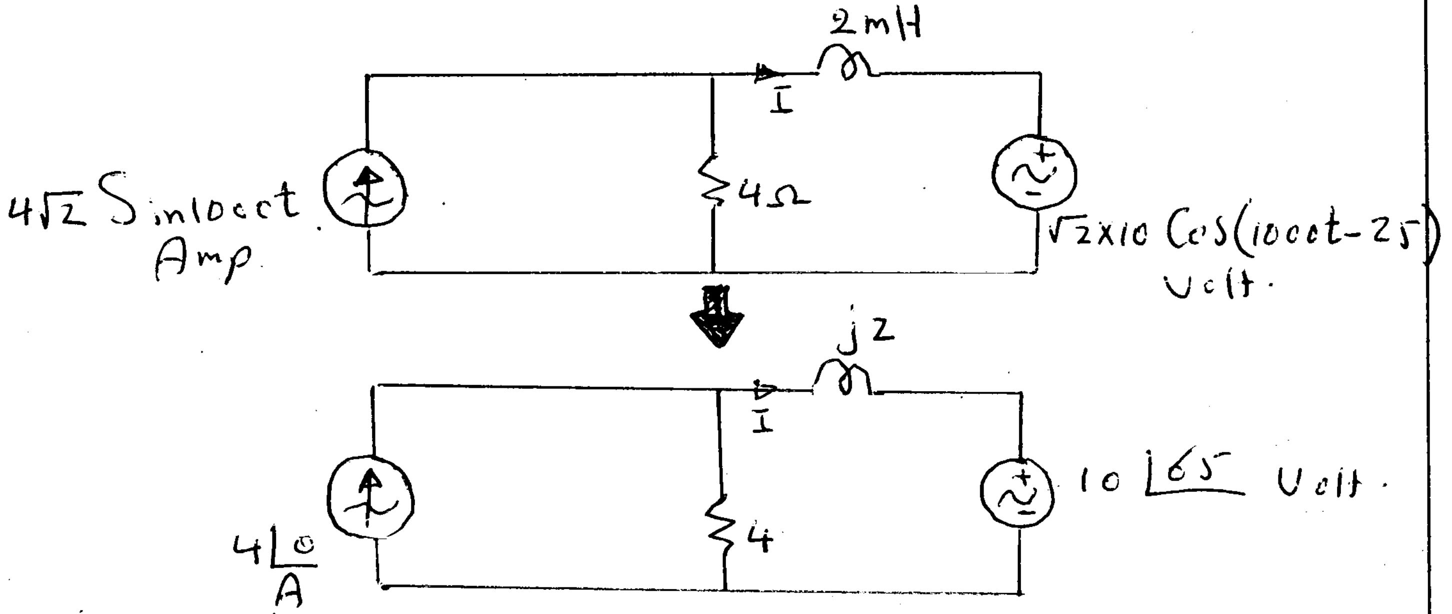
But $V_x = 6 I_1$.

$\therefore -18 I_1 = (4 + j8) I_2 - j8 I_1$ ——— (2).

Solving:

$I = I_2 = 4.37\angle 27.2$ Amp.

Ex: Use the Superposition method to find (I)? (57)



$X_L = \omega L = 1000 \times 2\times 10^{-3} = 2\ \Omega$.

* By effect of $4\angle 0$ Amp $\longrightarrow 10\angle 65$ (S/C):

$$I_1 = 4\angle 0 \ \frac{4}{4+j2} = \frac{16\angle 0}{4+j2} = 3.58\angle -26.6 \text{ Amp}.$$

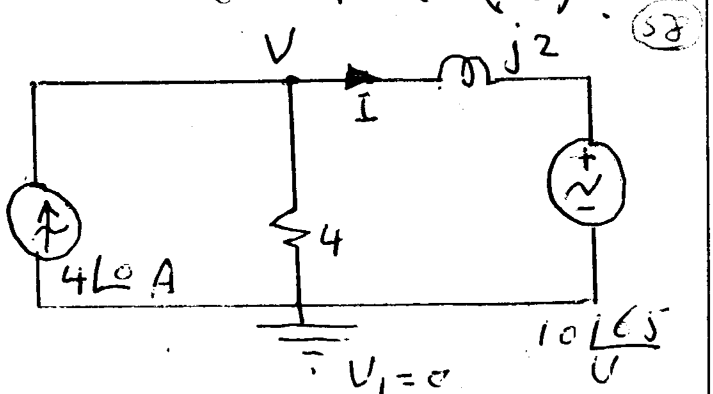
* By effect of $10\angle 65$ V $\longrightarrow 4\angle 0$ Am (O/C):

$$I_2 = \frac{10\angle 65}{4+j2} = 2.24\angle 38.4 \text{ Amp}.$$

$\therefore I = I_1 - I_2 = 3.32\angle -64.2$ Amp.

in Time domain $I = \sqrt{2}\times 3.32 \sin(1000t - 64.2)$ Amp.

Ex: Use the Nodal Voltage method to find (I)?



$$I = \frac{V - 10\angle 65}{j2}$$

Node (V):

$$\left(\frac{1}{4} + \frac{1}{j2}\right)V - \frac{10\angle 65}{j2} = 4\angle 0$$

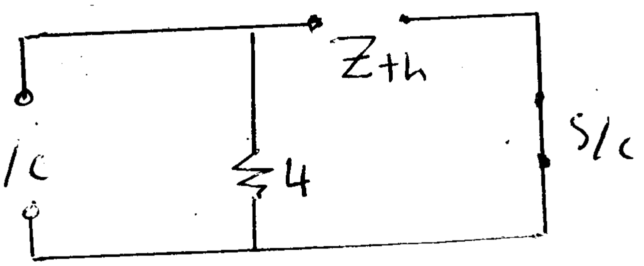
$$(0.25 - j0.5)V - 5\angle -25 = 4\angle 0$$

$$(0.25 - j0.5)V = 4\angle 0 + 5\angle -25 = 8.53 - j2.11.$$

$$\therefore V = \frac{8.53 - j2.11}{0.25 - j0.5} = 15.72\angle 49.54 \text{ Volt.}$$

$$\therefore I = \frac{15.72\angle 49.54 - 10\angle 65}{j2} = \frac{5.98 + j2.9}{j2} = 3.32\angle -64.2 \text{ Amp}$$

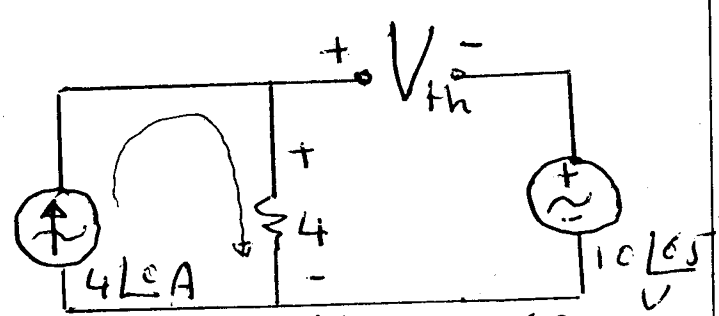
Ex: Solve the above question by Thevenin's Theory



$$I = \frac{V_{th}}{R_{th} + j2}$$

* $R_{th} = Z_{th} = 4\Omega.$

* $V_{th} = 16 - 10\angle 65$ Volt.

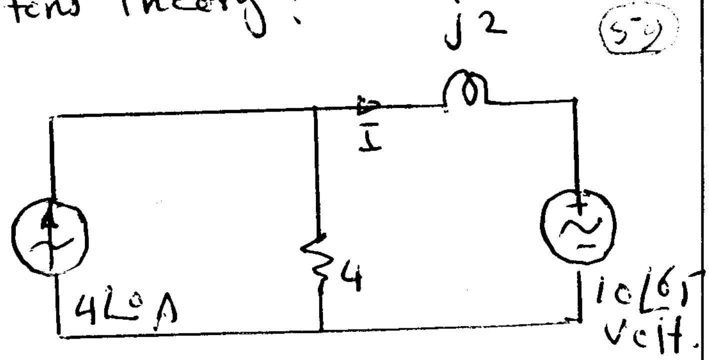


$$16 = V_{th} + 10\angle 65$$

$$\therefore I = \frac{16 - 10\angle 65}{4 + j2}$$

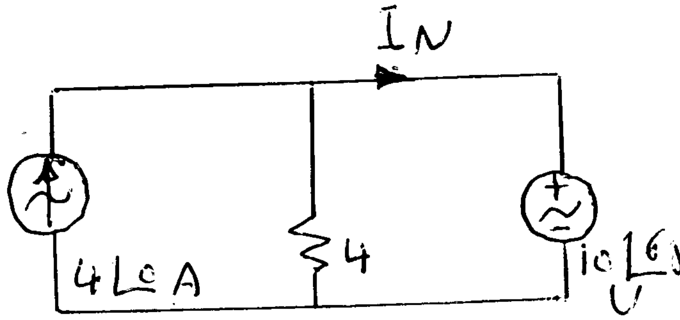
$$= 3.32\angle -64.2 \text{ Amp}$$

Ex; Find (I) by using Norton's Theory?

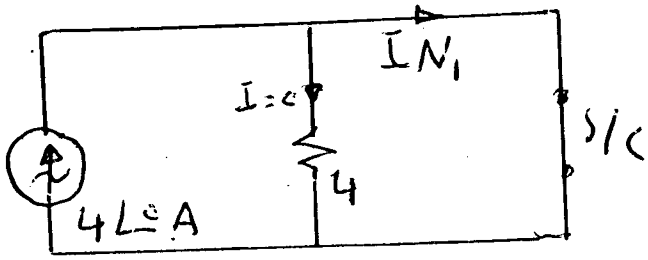


$$I = I_N \frac{Z_N}{Z_N + j2}$$

$Z_N = Z_{th} = 4\ \Omega$.

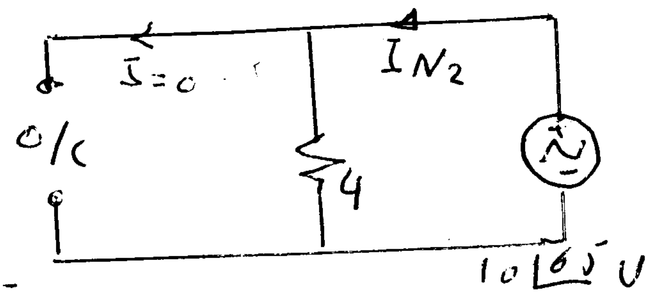


* $I_N \longrightarrow$



$I_{N_1} \rightarrow$ by effect of $4\angle 0$ A.

$\therefore I_{N_1} = 4\angle 0$ Amp



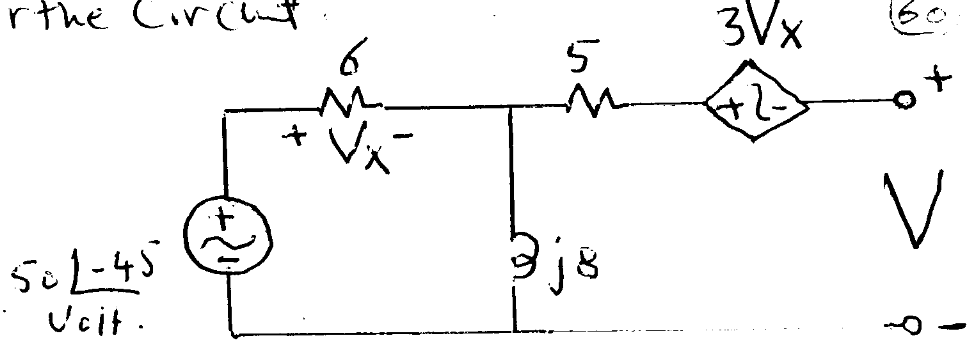
$I_{N_2} \rightarrow$ by effect of $10\angle 65$ V.

$$I_{N_2} = \frac{10\angle 65}{4} = 2.5\angle 65 \text{ Amp.}$$

$$I_N = I_{N_1} - I_{N_2} = 4\angle 0 - 2.5\angle 65$$
$$= 3.71\angle -37.57 \text{ Amp}$$

$$\therefore I = 3.71\angle -37.57 \; \frac{4}{4 + j2} = 3.32\angle -64.2 \text{ Amp.}$$

Ex: Find (V) for the Circuit

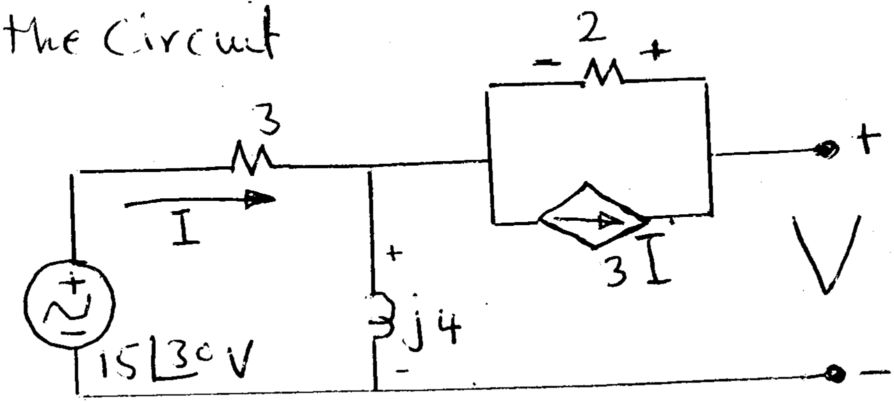


$$V_x = \frac{50\angle -45}{6+j8} \times (6) = 30\angle -98.1 \text{ Volt.}$$

$$\therefore V = V_{j8} - 3V_x$$

$$= \frac{50\angle -45}{6+j8} \times (j8) - 3\left(30\angle -98.1\right) = \underline{98.49\angle 57.93} \text{ Volt.}$$

Ex: Find (V) for the Circuit



$$V = V_{j4\Omega} + V_{2\Omega}$$

$$I = \frac{15\angle 30}{3+j4} = 3\angle -23.1 \text{ Amp}$$

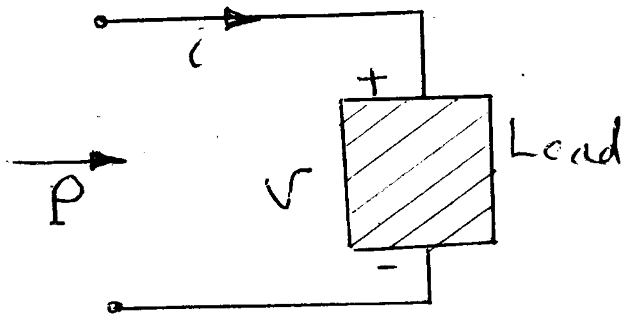
$$\therefore V = I \times j4 + 3I \times 2$$

$$= I(6+j4)$$

$$= 3\angle -23.1 \times (6+j4) = \underline{21.6\angle 10.6} \text{ Volt.}$$

A-C power Calculation

The average power absorbed by a two-terminal a-c Circuit can be derived from the instantaneous power absorbed.



* let the applied Voltage

** $V = V_m \sin(\omega t + \theta)$

If the current

** $i = I_m \sin \omega t$.

Where θ is the phase angle by which V lead i.

∴ The instantaneous power absorbed by the circuit :-

$$P = V \cdot i = V_m \sin(\omega t + \theta) \cdot I_m \sin \omega t.$$

$$= V_m \cdot I_m \sin(\omega t + \theta) \cdot \sin \omega t.$$

But we have:

$$\sin X \cdot \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right].$$

$$\therefore \text{ If } x = \omega t + \theta.$$
$$y = \omega t.$$

$$\therefore P = \frac{V_m \cdot I_m}{2}\left[\cos\theta - \cos(2\omega t + \theta)\right].$$

$$= \frac{V_m \cdot I_m}{2}(\cos\theta - \cos\theta \cos 2\omega t + \sin\theta \sin 2\omega t).$$

$$= \frac{V_m \cdot I_m}{2}\left[\cos\theta(1 - \cos 2\omega t) + \sin\theta \sin 2\omega t\right].$$

$$= V \cdot I \cos\theta(1 - \cos 2\omega t) + V \cdot I \sin\theta \sin 2\omega t.$$

Where V & I are the effective value (r.m.s). (62)

∴ The average power absorbed by the circuit is:

$$\boxed{P = V.I.\cos\theta}$$

because the average power for the Sinusoidal has Zero Values.

It is important to remember that the angle (θ) of the above equation is the angle by which the input voltage leads the input current. For a circuit that does not contain any independent sources, this angle is the impedance angle of the circuit.

$$Z = Z \angle \theta$$

and ($\cos\theta$) is called the Power factor (P.f).

*** For a purely resistive circuit $\theta = 0$ and $\cos\theta = P.f = 1$ ∴ $P = V.I$ (watts).

*** For a purely inductive circuit $\theta = 90^\circ$ and $\cos\theta = P.f = 0$ ∴ $P = 0$ (watts).

*** For a purely capacitive circuit $\theta = -90^\circ$ and $\cos\theta = P.f = 0$ ∴ $P = 0$ (watts).

That means a purely inductive and capacitive circuits absorbs Zero average power.

** For inductive circuit is called (lagging P.f circuit) (i lag v by θ).

** For Capacitive circuit is called (leading P.f Circuits) (i leads v by θ).

Apparent power (S)

The power delivered to the load (Power absorbed) from the analysis of D-C networks and in resistance circuit of A-C are: $P = V . I$ (watts).

But for other networks not always give power delivered. In an A-C network contains a number of electrical Components and systems, the power is Called (Apparent power) (S).

Where: $S = V . I$ (voltamper) or (VA).

$= I^2 . Z$ (VA)

$= \frac{V^2}{Z}$ (VA).

But we have

P = average power $= V . I . \cos\theta$. (watts).

$\therefore P = S \cos\theta$ (watts).

or $P.f = \cos\theta = \frac{P}{S}$

** For purely resistive Circuit.

$$P = S = V.I$$

$\therefore P.f = 1 = \frac{P}{S}$ (Unity power factor).

In General electrical equipment is rated in Volt-amper (VA) or in (KVA) not in watts. The power factor (P.f) is varying with load.

① Resistive Circuit

In resistive circuit v & i are in phase $(\theta = 0)$

$$\therefore P = V.I \cos(0)\left[1 - \cos 2\omega t\right] + V.I \sin(0) \sin 2\omega t.$$

$$= V.I - V.I \cos 2\omega t.$$

$$\therefore P_{av} = P = V.I = \frac{V_m . I_m}{2} = I^2 R = \frac{V^2}{R} \quad \text{(watts)}.$$

② Inductive Circuit

For a purely inductive circuit, we have v leads i by 90° $(\theta = 90^\circ)$.

$$\therefore P = V.I \cos(90)\left[1 - \cos 2\omega t\right] + V.I \sin(90) \sin 2\omega t.$$

$$= 0 + V.I \sin 2\omega t.$$

$$\therefore P_{av} = P = 0 \qquad \text{and } P.f = \cos 90 = 0$$

In General the reactive power associated with any circuit is given by:

$$Q = V.I \sin\theta \qquad \text{(Volt-amper reactive) (VAR)}.$$

For purely inductor

$Q_L = V.I.\sin\theta = V.I.\sin 90 = V.I = S$ (VAR)

But $V = I.X_L$

$\therefore Q_L = V.I = I^2.X_L = \frac{V^2}{X_L}$ (VAR)

③ Capacitive Circuit:

$P = V.I\cos(-90)[1-\cos 2\omega t] + V.I\sin(-90)\sin 2\omega t.$

$= 0 - V.I\sin 2\omega t.$

$\therefore P_{av} = P = 0$ $\quad$ $p.f = \cos\theta = 0.$

also for purely capacitive circuit

$Q_c = V.I.\sin\theta = V.I.\sin(-90) = V.I = S$ (VAR).

But $V = I.X_c.$

$\therefore Q_c = V.I = I^2.X_c = \frac{V^2}{X_c}$ (VAR).

The power Triangle :-

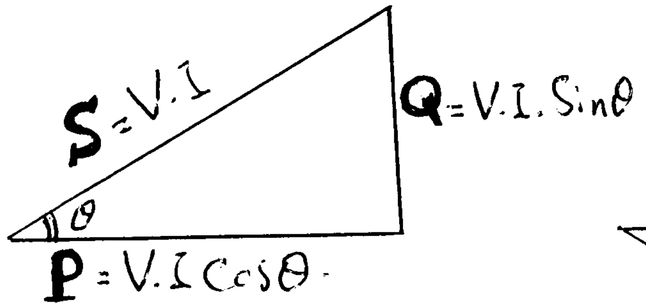
The three quantities:

** Average power (P) (watts).

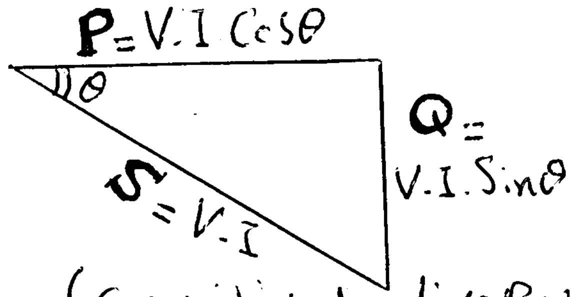
** Apparent power (S) (VA).

** Reactive power (Q) (VAR).

Can be related graphically by a power triangle as:



(Inductive reactive power)



(Capacitive ~~reactive~~ reactive Power)

By Pythagoream theorem

$$S^2 = P^2 + Q^2 \quad VA.$$

or $$S = \sqrt{P^2 + Q^2} \quad VA.$$

and $$P.f = \cos\theta = \frac{P}{S}.$$

The power quantities can be written in complex form

$$S = P + jQ = S \angle\theta \,.$$

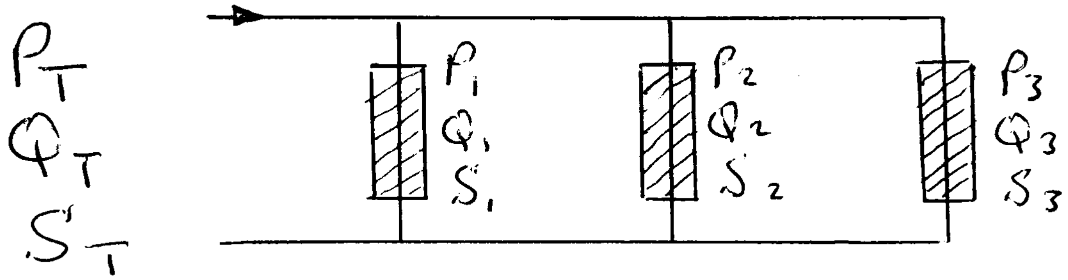
In General:

$$S = V.I = I^2.Z = \frac{V^2}{Z} \quad (VA).$$

$$P = V.I.\cos\theta = V_R.I = I^2.R = \frac{V_R^2}{R} \quad (\text{watts})$$

$$Q_L = V.I \sin\theta = V_L.I = I^2.X_L = \frac{V_L^2}{X_L} \quad (VAR).$$

$$Q_C = V.I.\sin\theta = V_C.I = I^2.X_C = \frac{V_C^2}{X_C} \quad (VAR).$$



① Total number of real power (active) P_T is equal to the sum of the average (active) power delivered to each branch.

$$\therefore \boxed{P_T = P_1 + P_2 + P_3} \qquad \text{(watts)}$$

② Total number of reactive power (Q_T) is equal to the sum of each power reactive (Difference between inductive loads and that of the capacitive loads).

$$\boxed{Q_T = Q_1 \mp Q_2 \mp Q_3} \qquad \text{(VAR)}$$

③ The above two points can not be applied to determine the total apparent power (S_T).

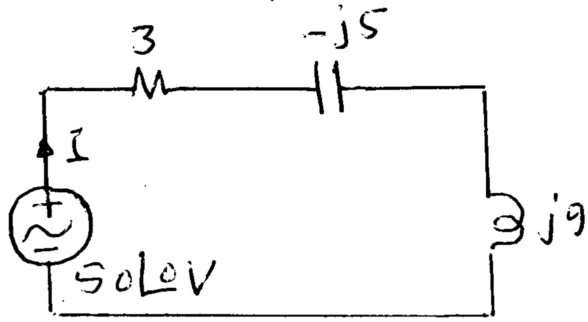
$S_T \neq$ Sum of each branch:

$$S_T \neq S_1 + S_2 + S_3 .$$

But $$\boxed{S_T = \sqrt{P_T^2 + Q_T^2}} \qquad \text{(VA)}$$

④ The total power factor $(Pf) = \dfrac{P_T}{S_T} = \cos\theta .$

<u>Ex</u>: For the circuit shown, find P, Q, S & P.F.



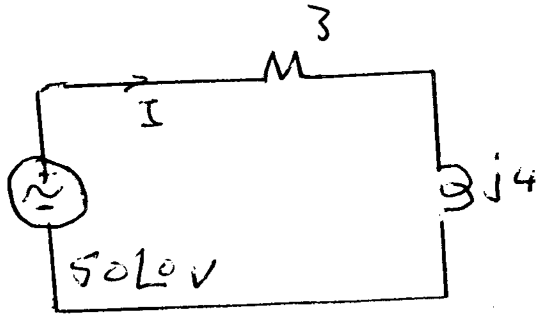
$$I = \frac{V}{Z} = \frac{50\angle 0}{3 - j5 + j9} = \frac{50\angle 0}{3 + j4} = \frac{50\angle 0}{5\angle 53.13} = 10\angle -53.13 \text{ Amp.}$$

* $P = V.I.\cos\theta = 50 \times 10 \times \cos(53.13) = 300$ Watts

OR $P = I^2.R = 10^2 \times 3 = 300$ Watts.

$P = V_R.I = (10 \times 3) \times 10 = 300$ Watts.

$P = \frac{V_R^2}{R} = \frac{(10 \times 3)^2}{3} = \frac{900}{3} = 300$ watts.



* $Q_L = V.I.\sin\theta$

$= 50 \times 10 \sin 53.13 = 400$ VAR.

OR $Q = I^2.X_L = (10)^2 \times 4 = 400$ VAR.

$Q = V_L.I = (10 \times 4) \times 10 = 400$ VAR.

$Q = \frac{(V_L)^2}{X_L} = \frac{(10 \times 4)^2}{4} = 400$ VAR.

* $S = P + jQ = 300 + j400 = 500$ VA.

OR $S = V.I = 50 \times 10 = 500$ VA.

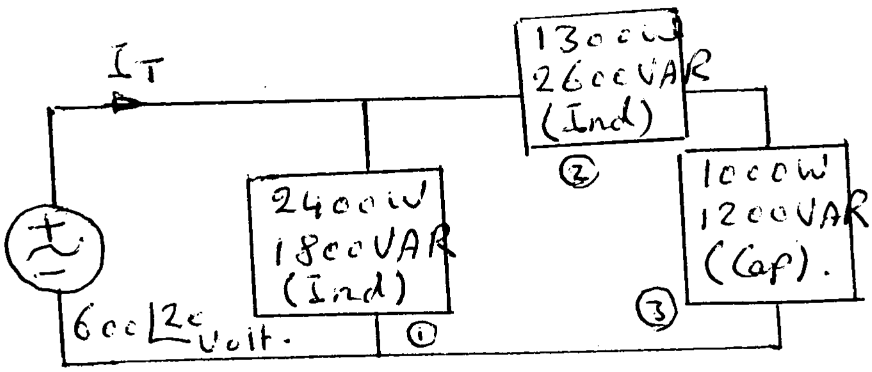
$S = I^2.Z = (10)^2 \times 5 = 500$ VA.

$S = \frac{V^2}{Z} = \frac{(50)^2}{5} = 500$ VA.

$\cos\theta = \frac{P}{S} = \frac{300}{500} = 0.6$ lagging

OR $\cos\theta = \frac{R}{Z} = \frac{3}{5} = 0.6$ lagging.

Ex :- Find P_T, Q_T, S_T, P.f and (I_T) for the circuit.



$$P_T = P_1 + P_2 + P_3$$

$$= 2400 + 1300 + 1000 = 4700 \text{ watts.}$$

$$Q_T = Q_1 + Q_2 - Q_3$$

$$= 1800 + 2600 - 1200 = 3200 \quad \text{VAR (Ind).}$$

$$S_T = P_T + jQ_T$$

$$= 4700 + j3200 = 5690 \quad \text{VA.}$$

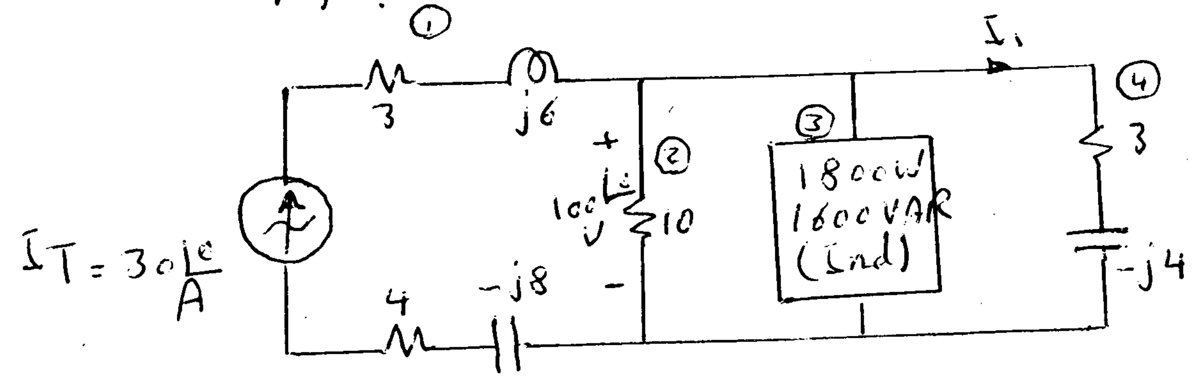
$$P.f = \frac{P_T}{S_T} = \frac{4700}{5690} = 0.826 \quad \text{(lagging).}$$

$$S_T = V_T \cdot I_T$$

$$\therefore I_T = \frac{S_T}{V_T} = \frac{5690}{600} = 9.48 \text{ Amp.}$$

Ex: Find P_T, Q_T, S_T, Pf_T, V_T and how you can make the circuit U.P.f ?

(70)



$$P_1 = (30)^2 \times 7 = 6300 \text{ W}.$$

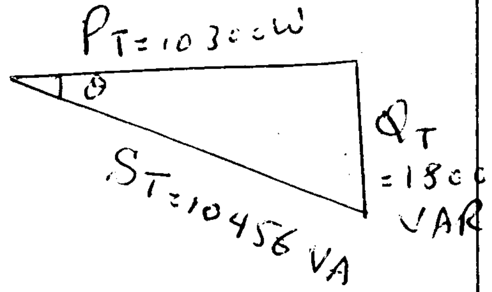
$$P_2 = \frac{V^2}{R} = \frac{(100)^2}{10} = 1000 \text{ W}.$$

$$P_3 = 1800 \text{ W}.$$

$$P_4 = I_1^2 \cdot R = I_1^2 \times 3 = \left(\frac{100}{5}\right)^2 \times 3 = (20)^2 \times 3 = 1200 \text{ W}.$$

$$\therefore P_T = P_1 + P_2 + P_3 + P_4 = 6300 + 1000 + 1800 + 1200 = 10300 \text{ Watt}$$

$$Q_1 = (30)^2 \times 2 = 1800 \text{ VAR (Cap)}.$$



$$Q_2 = 0$$

$$Q_3 = 1600 \text{ VAR (Ind)}.$$

$$Q_4 = (20)^2 \times 4 = 1600 \text{ VAR (Cap)}.$$

$$\therefore Q_T = 1800 \text{ VAR (Cap)}.$$

$$S_T = P_T + jQ_T$$

$$S_T = \sqrt{(10300)^2 + (1800)^2} = 10456 \text{ (VA)}$$

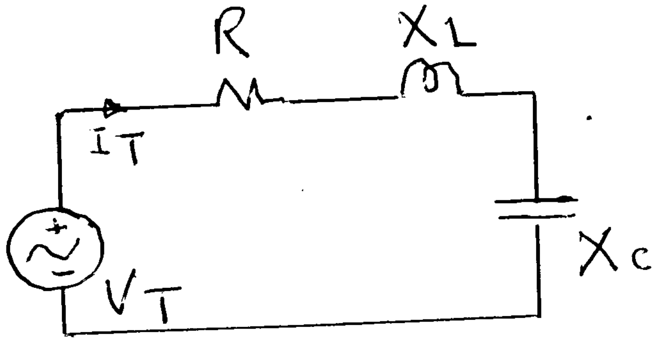
$$P.f_T = \frac{P_T}{S_T} = \frac{10300}{10456} = 0.985 \text{ (leading)}.$$

$$S_T = V_T \cdot I_T$$

$$\therefore V_T = \frac{S_T}{I_T} = 348.53 \text{ Volt}.$$

Resonance Circuit

① Series resonance :-



We have:

$Z_T = R + j(X_L - X_c)$.

$\therefore I_T = \dfrac{V_T}{Z_T}$.

But if $X_L = X_c$.

$\therefore Z_T = R$

and $I_T = \dfrac{V_T}{R}$ (maximum power transfer).

resonance takeplace when $\boxed{X_L = X_c}$

$X_L = 2\pi f L$, $X_c = \dfrac{1}{2\pi f c}$.

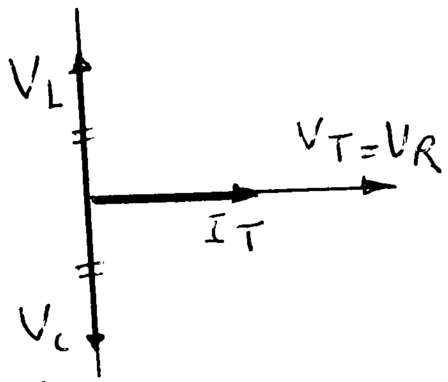
$\therefore$ at resonance: $2\pi f L = \dfrac{1}{2\pi f c}$.

OR: $wL = \dfrac{1}{wc}$.

$w^2 = \dfrac{1}{Lc}$ $\therefore w = \dfrac{1}{\sqrt{LC}}$

OR: $\boxed{f = \dfrac{1}{2\pi\sqrt{LC}}}$ → This frequency is called resonance frequency, its unit is (Hz) when (L) in Henry & (C) in Farad.

At resonance:-



$Z_T = R$

$V_c = V_L$

$V_T \equiv I_T$ ($\theta = 0$)

$\cos\theta = \dfrac{P}{S} = 1$ (unity power factor).

Quality factor (Q) :- is the ratio of the reactive power of either capacitor or inductor to the average power of the resistance at resonance.

$$\therefore \boxed{Q_s = \frac{\text{Reactive Power}}{\text{Average Power}}} = \frac{I^2 \cdot X_L}{I^2 \cdot R} = \frac{I^2 \cdot X_C}{I^2 \cdot R}$$

series

For Inductor

$$Q_s = \frac{X_L}{R} = \frac{WL}{R} = \frac{2\pi f_s L}{R}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore Q_s = \frac{2\pi \frac{1}{2\pi\sqrt{LC}} \cdot L}{R}$$

$$= \frac{L}{R\sqrt{LC}}$$

$$\therefore \boxed{Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

For Capacitor

$$Q_s = \frac{X_C}{R} = \frac{1}{WCR} = \frac{1}{2\pi f_s CR}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore Q_s = \frac{1}{2\pi \frac{1}{2\pi\sqrt{LC}} C \cdot R}$$

$$= \frac{\sqrt{LC}}{C \cdot R}$$

$$\therefore \boxed{Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

At resonance

* The Voltage across inductor:

$$V_{Ls} = \frac{V_T \times X_L}{Z_T} = \frac{V_T \times X_L}{R} = Q_s V_T.$$

* The Voltage across capacitor:

$$V_{Cs} = \frac{V_T \times X_C}{Z_T} = \frac{V_T \times X_C}{R} = Q_s V_T.$$

(Q) always greater than One.

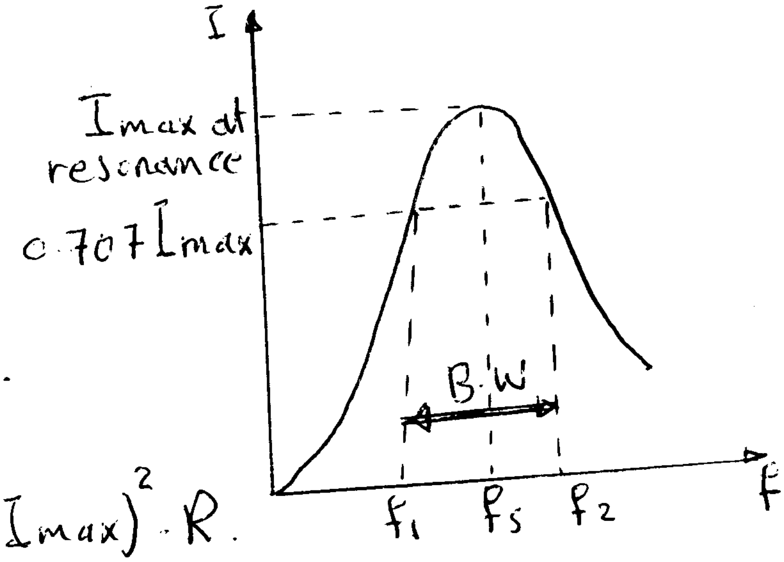
$\therefore$ The Voltage across Capacitor or inductor is greater than the total Voltage.

$$\therefore V_{LS} \ \& \ V_{CS} > V_T$$

Selectivity :- For different frequencies, the Current is near its maximum Value, When the impedance is at aminimum. These frequencies Corresponding to (0.707) of the maximum Current are called ((Band frequencies OR ~~Cutt~~ off frequencies)) OR (half-power frequencies (H.P.f)), they are indicated by (f_1) and (f_2).

(H.P.f) are those frequencies at Which the Power delivered is one-half that delivered at resonant frequency that is :-

$$\boxed{P_{H.P.f} = \frac{1}{2} P_{max}}$$



and $P_{max} = I_{max}^2 \cdot R$.

we have:

$$P_{H.P.f} = I^2 \cdot R = (0.707 I_{max})^2 \cdot R.$$

$$\therefore P_{H.P.f} = 0.5 \, I_{max}^2 \cdot R = \frac{1}{2} P_{max}.$$

The Band Width (BW) is the range frequency between f_1 & f_2 as shown: (74)

$$\boxed{BW = f_2 - f_1 = \frac{f_s}{Q_s} = \frac{R}{2\pi L}} \Longrightarrow \text{Prove}$$

We have $\frac{1}{Q_s} = \frac{BW}{f_s}$.

$\frac{1}{Q_s}$ are called fractional band width.

Prove: either $BW = \frac{f_s}{Q_s}$

$f_s = \frac{1}{2\pi\sqrt{LC}}$, $Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}$.

$$\therefore B.W = \frac{1/2\pi\sqrt{LC}}{\frac{1}{R}\sqrt{\frac{L}{C}}} = \frac{R}{2\pi\sqrt{LC}} \cdot \sqrt{\frac{C}{L}} = \frac{R}{2\pi L}.$$

OR

$W_s = 2\pi f_s$ $\therefore f_s = \frac{W_s}{2\pi}$

and $Q_s = \frac{W_s L}{R}$.

$$\therefore B.W = \frac{f_s}{Q_s} = \frac{W_s}{2\pi} \Big/ \frac{W_s L}{R} = \frac{R}{2\pi L}.$$

Ex. The band width (BW) of a series resonance circuit is (400 Hz) and the resonance frequency (f_s) is (4000 Hz), find the value of (Q_s), f_1, f_2. If $R = 10\,\Omega$, find the value of X_L at resonance and then find L & C.

$$BW = \frac{f_s}{Q_s} \longrightarrow \therefore Q_s = \frac{f_s}{BW} = \frac{4000}{400} = 10 .$$

$$f_1 = 4000 - 200 = 3800\ Hz \quad , \quad f_2 = 4000 + 200 = 4200\ Hz .$$

$$Q_s = \frac{X_L}{R} \longrightarrow \therefore X_L = Q_s \cdot R = 10 \times 10 = 100\,\Omega$$

$$X_L = 2\pi f_s L \longrightarrow \therefore L = \frac{X_L}{2\pi f_s} = \frac{100}{2\pi \times 4000} = 3.98\ mH .$$

$$X_C = \frac{1}{2\pi f_s C} \longrightarrow \therefore C = \frac{1}{2\pi \times 4000 \times 100} = 0.398\ \mu F$$

Ex. For a series R, L & C circuit has a series resonance frequency (f_s) of (24000 Hz), $R = 5\,\Omega$ and X_L at resonance is $600\,\Omega$, find the B.W & cut off frequencies (f_1 & f_2).

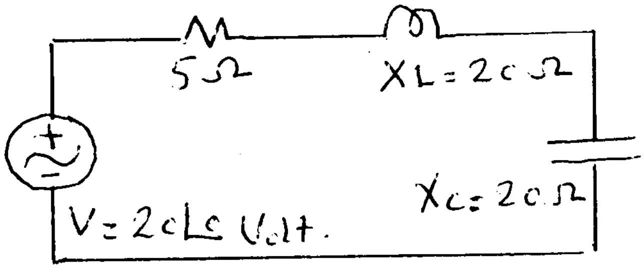
$$Q_s = \frac{X_L}{R} = \frac{600}{5} = 120 .$$

$$B.W = \frac{f_s}{Q_s} = \frac{24000}{120} = 200\ Hz$$

$$f_2 = f_s + \frac{B.W}{2} = 24000 + 100 = 24100\ Hz .$$

$$f_1 = f_s - \frac{BW}{2} = 24000 - 100 = 23900\ Hz$$

Ex. For the circuit shown, find Q_s, I_T, V_R, V_L, V_C, B.W and H.P.F ? If $f_s = 10\,KHz$.



$Z_T = R$ at resonance

$= 5\,\Omega .$

$$i_T = \frac{V_T}{Z} = \frac{20\angle 0}{5} = \underline{4\angle 0} \text{ Amp.}$$

$$V_R = 4 \times 5 = \underline{20\angle 0} \text{ volt.}$$

$$V_L = 4 \times 20\angle 90 = \underline{80\angle 90} \text{ volt.}$$

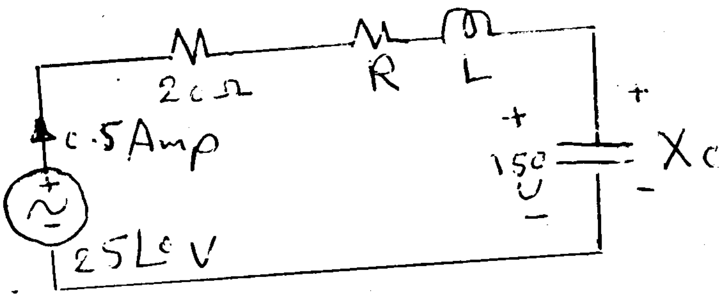
$$V_C = 4 \times 20\angle -90 = \underline{80\angle -90} \text{ volt.}$$

$$Q_s = \frac{X_L}{R} = \frac{X_C}{R} = \frac{20}{5} = \underline{4}.$$

$$B.W = \frac{f_s}{Q_s} = \frac{10000}{4} = \underline{2500 \text{ Hz}}$$

$$P_{H.P.f} = \frac{1}{2} P_{max} = \frac{1}{2} I_{max}^2 \cdot R = \frac{1}{2}(4)^2 \times 5 = \underline{40 \text{ watts}}$$

Ex: For the circuit shown, find C, L & internal resistance (R).



$f_s = 400 \text{ Hz}$.

$V_c = I X_c$.

$$\therefore X_c = \frac{V_c}{I} = \frac{150}{0.5} = 300\,\Omega.$$

$$X_c = \frac{1}{2\pi f_s C} \longrightarrow \therefore C = \frac{1}{2\pi f_s X_c} = \underline{1.326\,\mu F}$$

at resonance; $X_L = X_c$.

$$\therefore X_L = 300\,\Omega = 2\pi f_s L$$

$$\therefore L = \underline{0.119 \text{ H}}$$

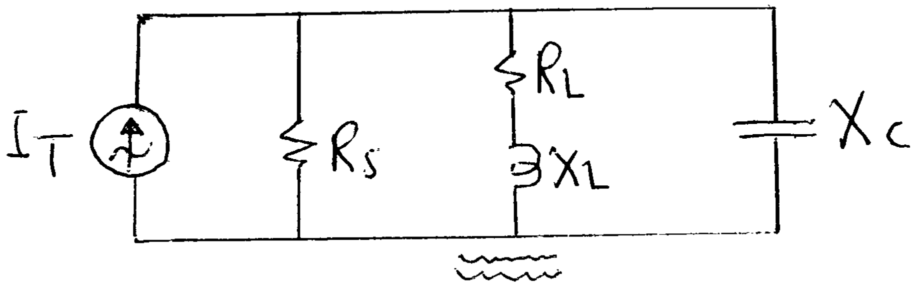
at resonance $Z_T = R_T = 20 + R$

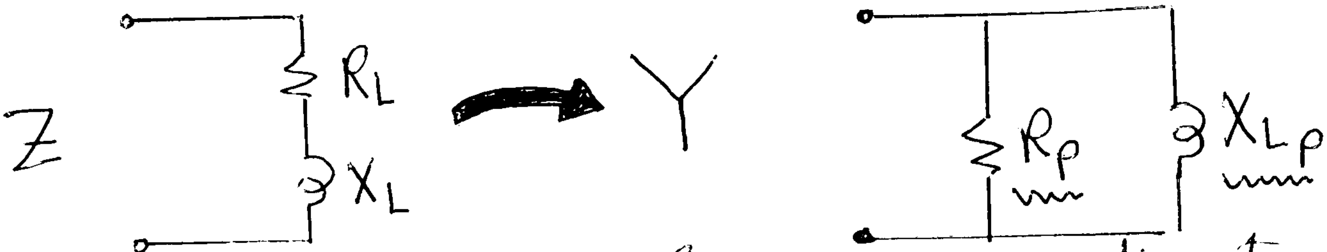
$$Z_T = \frac{V_T}{I} = \frac{25}{0.5} = 50\,\Omega.$$

$$50 = 20 + R \longrightarrow \therefore R = \underline{30\,\Omega}$$

② Parallel resonance :-

If the inductance have an internal resistance as shown





Change the impedance from Series connection to parallel connection, as shown.

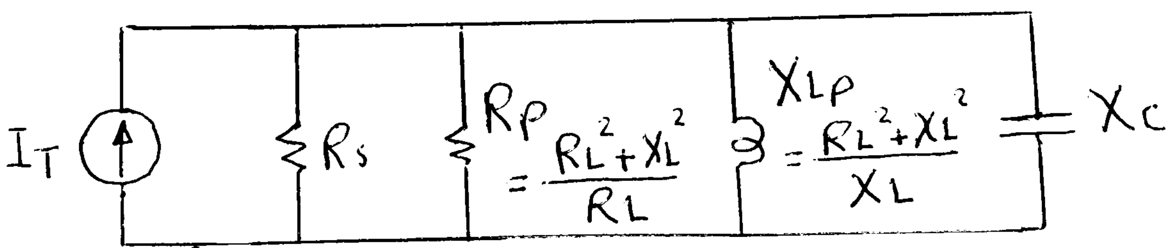
$$Z = R_L + jX_L$$

$$Y = \frac{1}{Z} = \frac{1}{R_L + jX_L} = \frac{1}{(R_L + jX_L)} \times \frac{(R_L - jX_L)}{(R_L - jX_L)}$$

$$= \frac{R_L - jX_L}{R_L^2 + X_L^2} = \frac{R_L}{R_L^2 + X_L^2} - j\frac{X_L}{R_L^2 + X_L^2}$$

$$\therefore R_p = \frac{R_L^2 + X_L^2}{R_L} \quad , \quad X_{Lp} = \frac{R_L^2 + X_L^2}{X_L}$$

then the circuit becomes:



Fig(1).

as in Series resonance.

$$\therefore X_{Lp} = X_C$$

$$\frac{R_L^2 + X_L^2}{X_L} = X_C$$

$$R_L^2 + X_L^2 = X_L X_C \qquad \text{But } X_L X_C = \frac{2\pi f L}{2\pi f C} = \frac{L}{C}.$$

$$\therefore X_L^2 = \frac{L}{C} - R_L^2$$

$$\therefore X_L = \sqrt{\frac{L}{C} - R_L^2}.$$

But $X_L = 2\pi f_p L$.

$$\therefore f_p = \frac{X_L}{2\pi L} = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_L^2}.$$

$$= \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - R_L^2\frac{C}{L}}.$$

But $f_s = \frac{1}{2\pi\sqrt{LC}}$.

$$\therefore \boxed{f_p = f_s\sqrt{1 - R_L^2\frac{C}{L}}}$$

where:
f_p = parallel resonance frequency.
f_s = Series resonance frequency.

Selectivity: as shown in Fig(1)

$$Z_{Tp} = R_s // R_p // X_{Lp} // X_C$$

$$\therefore Y_{Tp} = \frac{1}{R_S} + \frac{1}{R_p} - j\frac{1}{X_{Lp}} + j\frac{1}{X_C}$$

at resonance $(X_{Lp} = X_C)$ as in Series resonance

$$\therefore Y_{Tp} = \frac{1}{R_s} + \frac{1}{R_p}$$

and $Z_{Tp} = R_s // R_p$

we have $R_p = \frac{R_L^2 + X_L^2}{R_L}$ ------------ (1)

and $X_{Lp} = \frac{R_L^2 + X_L^2}{X_L} = X_C$ (at resonance).

$$\therefore R_L^2 + X_L^2 = X_L \cdot X_C = \frac{L}{C} \text{ ---------- (2)}$$

$$\therefore \boxed{R_p = \frac{L}{R_L \cdot C}}$$

and $\boxed{Z_{Tp} = R_s // R_p = R_s // \frac{L}{R_L \cdot C}}$

** $R_p = \frac{X_L^2 + R_L^2}{R_L} = R_L + \frac{X_L^2}{R_L}\left(\frac{R_L}{R_L}\right)$

$$\therefore \boxed{R_p = R_L + Q^2 R_L}$$ since $Q = \frac{X_L}{R_L}$

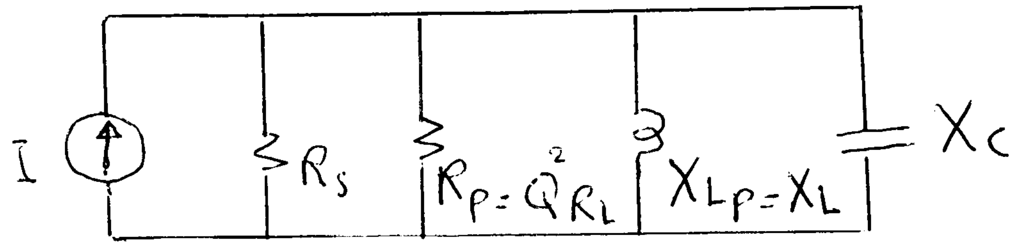
** $X_{Lp} = \frac{X_L^2 + R_L^2}{X_L} = X_L + \frac{R_L^2}{X_L}\left(\frac{X_L}{X_L}\right)$.

$$\therefore \boxed{X_{Lp} = X_L + \frac{X_L}{Q^2}}$$

*** But For Condition of $Q \geq 10$

and $\boxed{\begin{array}{l} R_p \simeq Q^2 R_L \\ X_{Lp} \simeq X_L \end{array}}$

and the circuit becomes:



at resonance $X_{Lp} = X_L = X_C$

$\therefore \boxed{f_p = \frac{1}{2\pi\sqrt{LC}}}$ for $Q \geq 10$

Quality factor (Q_p): is defined as the ratio of reactive power to the active (average) power.

let $R = R_s \| R_p$.

$$\therefore Q_p = \frac{\text{Reactive Power}}{\text{Average power}} = \frac{\frac{V_T^2}{X_{Lp}}}{\frac{V_T^2}{R}}$$

$$\therefore \boxed{Q_p = \frac{R}{X_{Lp}} = \frac{R}{X_C}}$$

$Q_p = \frac{R}{X_L}$ at $Q \geq 10$

Note

****: If R_s is very large (∞).

$$\therefore R = R_s \| R_p = R_p.$$

$$\therefore Q_p = \frac{R_p}{X_{Lp}} = \frac{R_L^2 + X_L^2 / R_L}{R_L^2 + X_L^2 / X_L} = \frac{X_L}{R_L} = Q$$

The band width (BW) is measured as in series resonance (8)

$$BW = f_2 - f_1 = \frac{f_P}{Q_P}.$$

Since $Z_{Tp} = R_s // R_p$

$$= R_s // Q^2 R_L \quad \text{at } Q \geqslant 10.$$

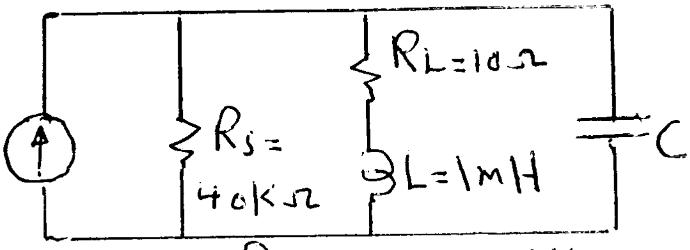
$$\therefore \boxed{f_P = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{1+Q^2}}} \Longrightarrow \text{Prove}$$

& $f_P = \frac{1}{2\pi\sqrt{LC}}$ also at $Q \geqslant 10$.

$I_L = Q I_T$
$I_C = Q I_T$

as in series resonance $V_L = Q V_T$ & $V_C = Q V_T$.

Ex: For the circuit shown, find Q, R_P, Z_{TP}, Q_P, BW, C at resonance.



$f_P = 0.04\,MHz$.

a) $Q = \dfrac{X_L}{R_L} = \dfrac{2\pi \times 0.04 \times 10^6 \times 1 \times 10^{-3}}{10} = \underline{25.12}$.

b) Since $Q \geq 10$

$\therefore R_P = Q^2 \times R_L = (25.12)^2 \times 10 = \underline{6.31}\,K\Omega$.

or $= R_L + Q^2 R = \underline{6.31}\,K\Omega$.

or $R_P = \dfrac{X_L^2 + R_L^2}{R_L} = \dfrac{10^2 + (2\pi \times 0.04 \times 10^6 \times 1 \times 10^{-3})^2}{10} = \underline{6.31}\,K\Omega$

c) $Z_{TP} = R_S \| R_P = 40 \times 10^3 \| 6.31 \times 10^3 = \underline{5.45}\,K\Omega$

d) Since $Q \geq 10$

$\therefore f_P = \dfrac{1}{2\pi\sqrt{LC}} = 0.04 \times 10^6\,Hz. = \dfrac{1}{2\pi\sqrt{LC}}\sqrt{\dfrac{Q^2}{1+Q^2}}$ check $\simeq 1$

$\therefore C = \dfrac{1}{L(2\pi f_P)^2} = \dfrac{1}{1 \times 10^{-3}(2\pi \times 0.04 \times 10^6)^2} = \underline{0.0159}\,\mu F$

e) Since $Q \geq 10$

$\therefore Q_P = \dfrac{R}{X_{LP}} = \dfrac{R}{X_L} = \dfrac{R_S \| R_P}{X_L}$

$= \dfrac{5.45 \times 10^3}{2\pi \times 0.04 \times 10^6 \times 1 \times 10^{-3}} = \underline{21.71}$

f) $BW = \dfrac{f_P}{Q_P} = \dfrac{0.04 \times 10^6}{21.71} = \underline{1.84}\,KHz$

Check: for example - $Q_P = Q$ if $R_S = \infty$

$\therefore Q_P = \dfrac{R_S \| R_P}{X_L} = \dfrac{R_P}{X_L} = \dfrac{6.31}{X_L} = 25.12$

Prove

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{1+Q^2}}\ .$$

** we have

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-R_L^2\frac{C}{L}}$$

$$= \frac{1}{2\pi\sqrt{LC}}\sqrt{1-R_L^2\frac{\omega C}{\omega L}}$$

$$= \frac{1}{2\pi\sqrt{LC}}\sqrt{1-R_L^2\frac{1}{X_L X_C}}\ .$$

we have $X_{LP} = \dfrac{R_L^2+X_L^2}{X_L} = X_C$ (at resonance).

$$\therefore X_L X_C = R_L^2 + X_L^2\ .$$

$$\therefore \sqrt{1-R_L^2\frac{1}{X_L X_C}} = \sqrt{1-\frac{R_L^2}{R_L^2+X_L^2}}\ .$$

$$= \sqrt{\frac{R_L^2+X_L^2-R_L^2}{R_L^2+X_L^2}} = \sqrt{\frac{X_L^2}{R_L^2+X_L^2}}$$

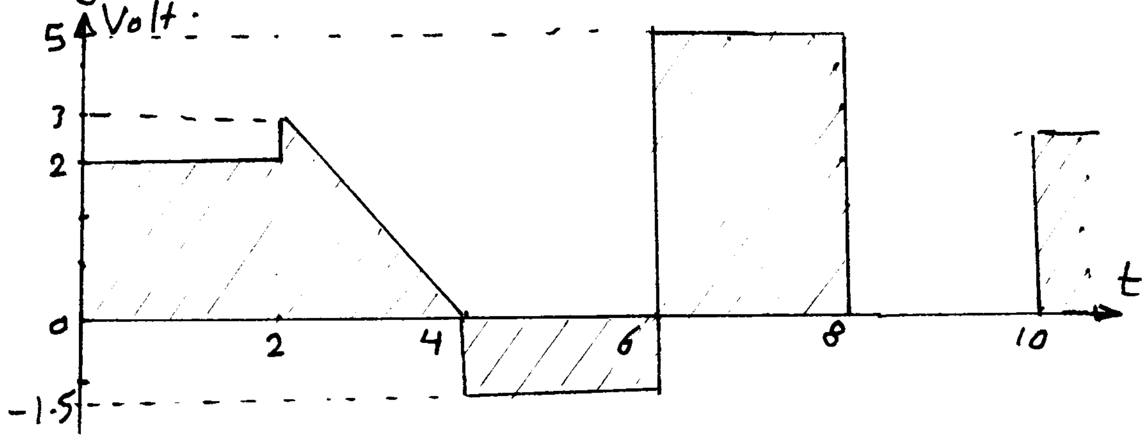
$$Q = \frac{X_L}{R_L} \longrightarrow \therefore X_L = Q R_L$$

$$\therefore \sqrt{\frac{X_L^2}{R_L^2+X_L^2}} = \sqrt{\frac{Q^2 R_L^2}{R_L^2+(Q^2 R_L^2)}} = \sqrt{\frac{Q^2}{1+Q^2}}\ .$$

$$\therefore f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-R_L^2\frac{C}{L}} = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{1+Q^2}}\ .$$

(A-C) Circuit Sheet No 2

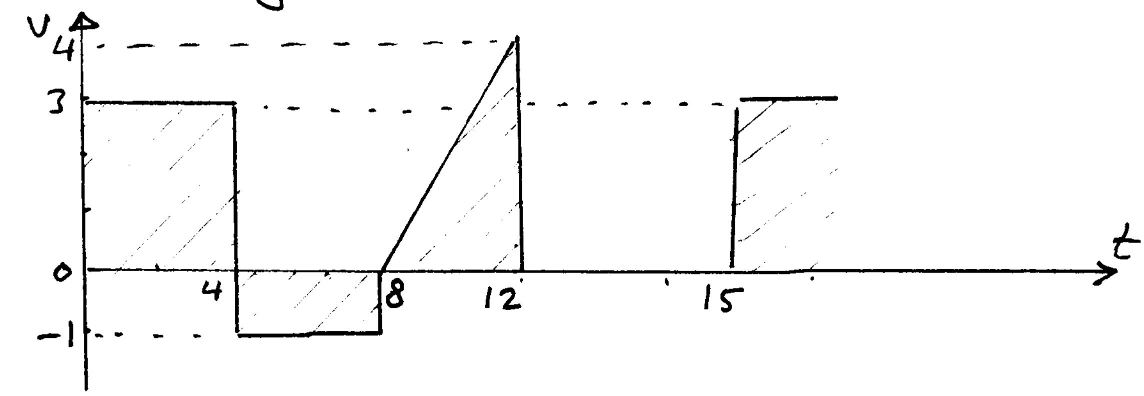
Q1 Find the average voltage value of a wave form over one full cycle?



$$V_{av} = \frac{2\times2 + \frac{1}{2}\times2\times3 - 2\times1.5 + 2\times5}{10}$$

$$= \frac{4+3-3+10}{10} = \frac{14}{10} = 1.4 \text{ Volt}.$$

Q2 Find the V average of a wave form over one full cycle?



$$V_{av} = \frac{3\times4 - 4\times1 + \frac{1}{2}4\times4}{15}$$

$$= \frac{12-4+8}{15} = \frac{16}{15} = 1.066 \text{ Volt}.$$

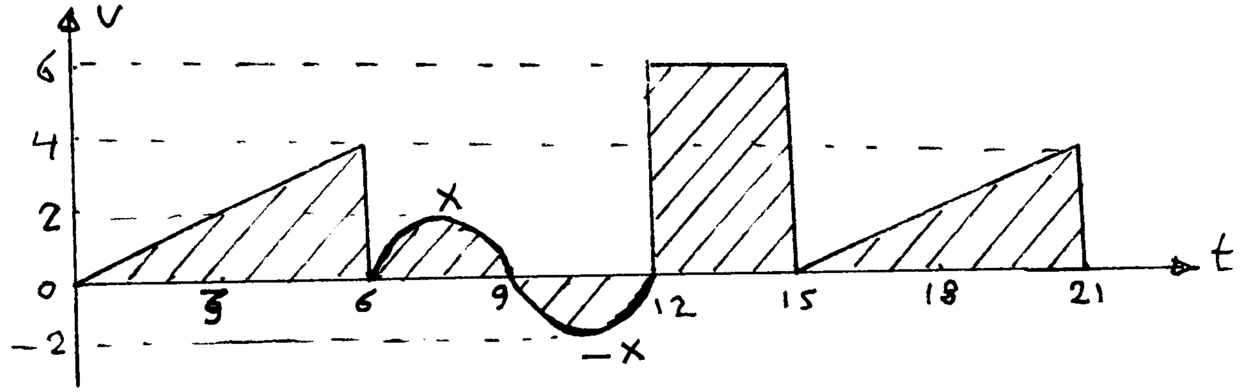
OR

$$V_{av} = \frac{1}{15}\left[\int_0^4 3\,dt + \int_4^8 -1\,dt + \int_8^{12} \frac{1}{2}4\,dt + \int_{12}^{15} 0\,dt\right].$$

$$= \frac{1}{15}\left[3(4-0) - 1(8-4) + 2(12-8) + 0\right].$$

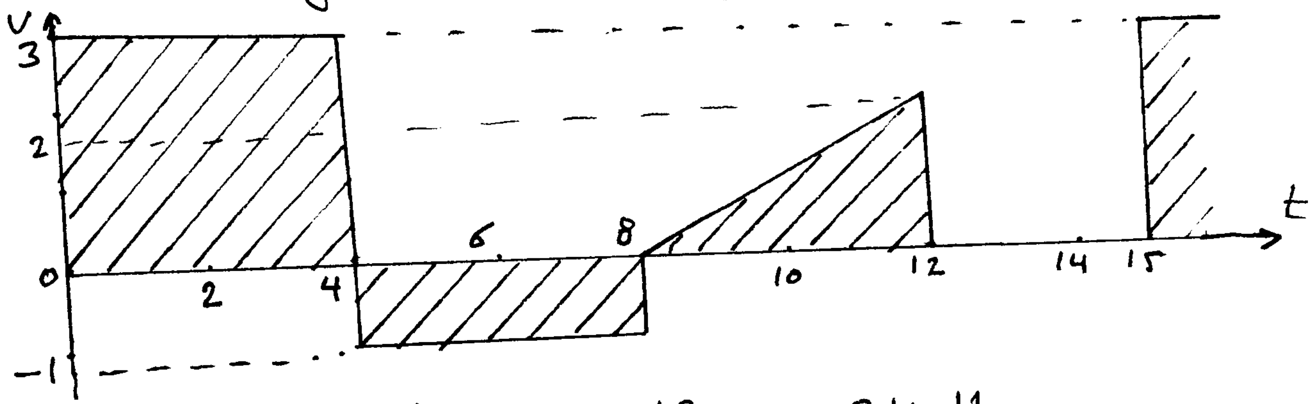
$$= 16/15 = 1.066$$

Q3] Find the average voltage value of a wave form over one full cycle?



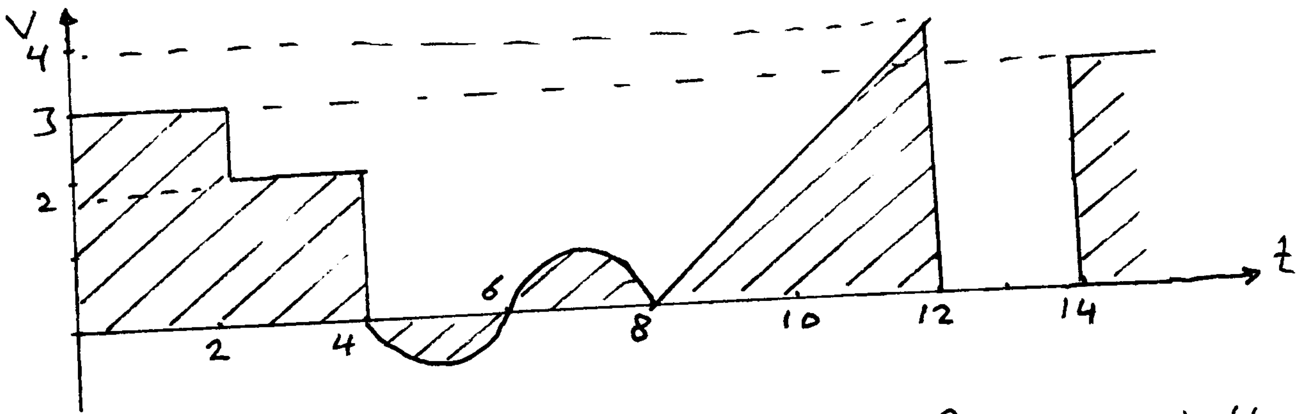
$$V_{av} = \frac{\frac{1}{2}6\times4 + X - X + 3\times6}{15} = \frac{30}{15} = 2 \text{ Volt.}$$

Q4] Find the average Value of voltage of a wave form?



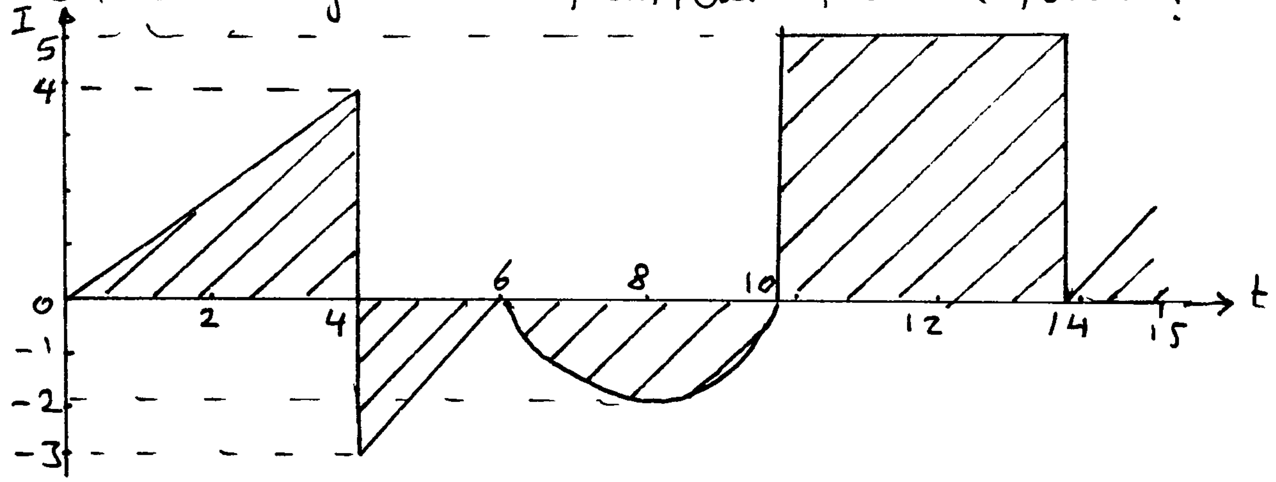
$$V_{av} = \frac{4\times3 - 4\times1 + \frac{1}{2}4\times2}{15} = \frac{12}{15} = 0.8 \text{ Volt.}$$

Q5] Find the average value of Voltage of a wave form?



$$V_{av} = \frac{3\times2 + 2\times2 + \frac{1}{2}\times4\times4}{14} = \frac{10+8}{14} = \frac{18}{14} = 1.285 \text{ Volt.}$$

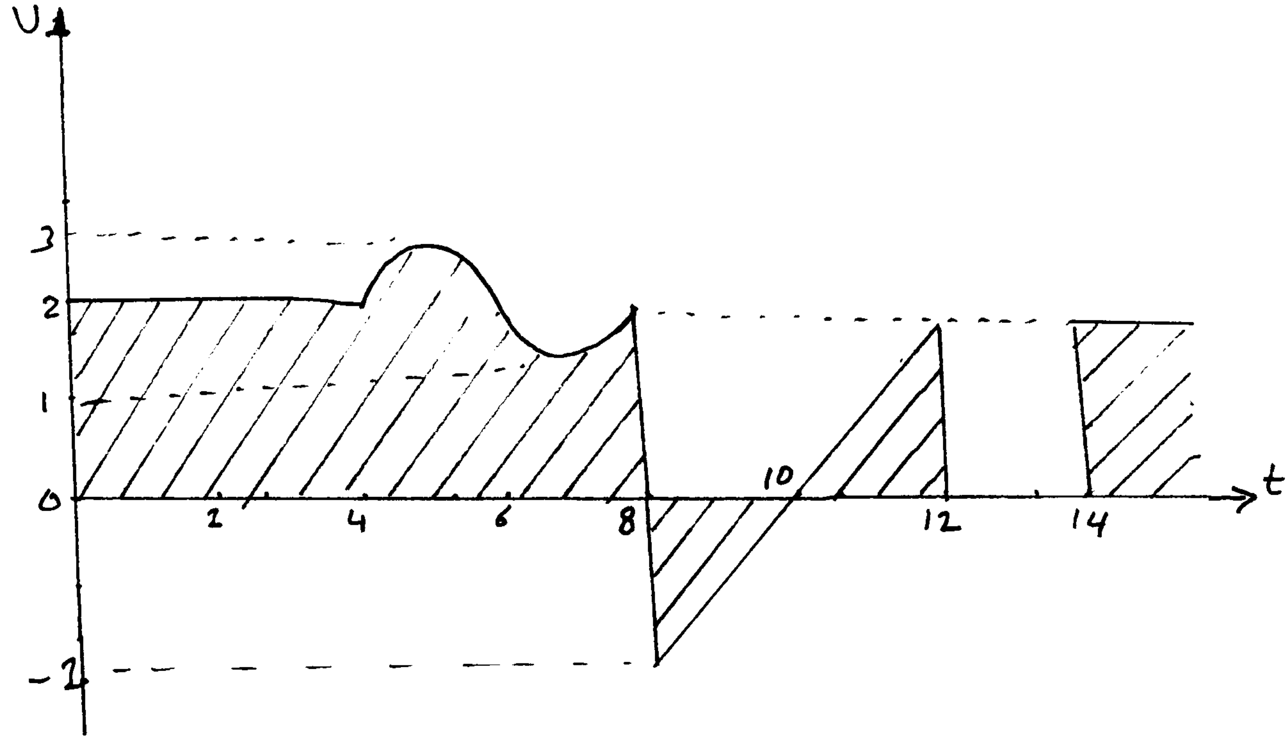
Q6) Find the average value of current of a wave form ?



$$I_{av} = \frac{\frac{1}{2}\times 4\times 4 - \frac{1}{2}\,2\times 3 - \frac{1}{2}(2)^2\times\pi + 4\times 5}{14} = \frac{8-3-6.28+5\times 4}{14}$$

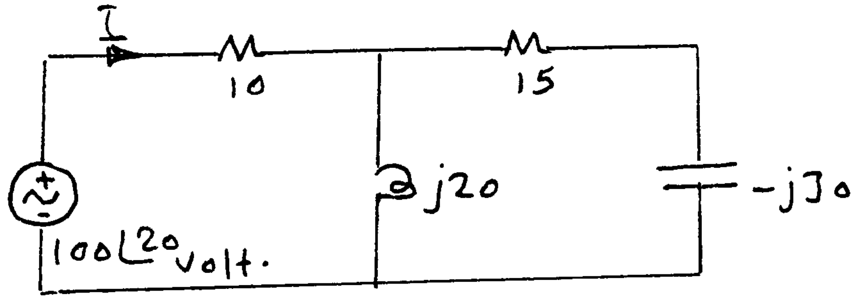
$= 1.336$ Amp.

Q7) Find the average voltage of a wave form ?



$$V_{av} = \frac{2\times 8 - \frac{1}{2}\,2\times 2 + \frac{1}{2}\,2\times 2}{14} = \frac{16}{14} = 1.142 \text{ Volt.}$$

Q8 | Find the current (I) for the circuit shown?



Solution:

$$I = \frac{V_T}{Z_T} = \frac{100\angle 20}{Z_T} \qquad Z_T = 10 + j20 // (15 - j30).$$

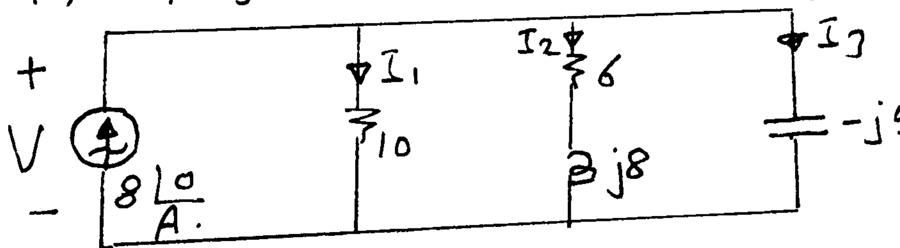
$$\therefore Z_T = 10 + \frac{j20(15-j30)}{15 - j30 + j20} = 10 + \frac{600 + j300}{15 - j10}$$

$$= \frac{150 - j100 + j300 + 600}{15 - j10} = \frac{750 + j200}{15 - j10} = \frac{776.2\angle 14.93}{18.027\angle -33.7}.$$

$$= 43.05\angle 48.6\ \Omega.$$

$$\therefore I = \frac{100\angle 20}{43.05\angle 48.6} = \underline{2.32\angle -28.6\ \text{Amp}}.$$

Q9 | Find I_1, I_2, I_3 & V for the circuit?



Solution:

$$I_1 = 8\angle 0 \times \frac{(-j5 // (6+j8))}{10 + (-j5 // (6+j8))} = 4\angle -36.87\ \text{Amp}.$$

$$I_2 = 8\angle 0 \times \frac{(10 // -j5)}{(6+j8) + (10 // -j5)} = 4\angle -90\ \text{Amp}.$$

$$I_3 = 8\angle 0 \times \frac{10 // (6+j8)}{-j5 + (10 // (6+j8))} = 8\angle 53.13\ \text{Amp}.$$

Current Divider Rule (C.D.R)

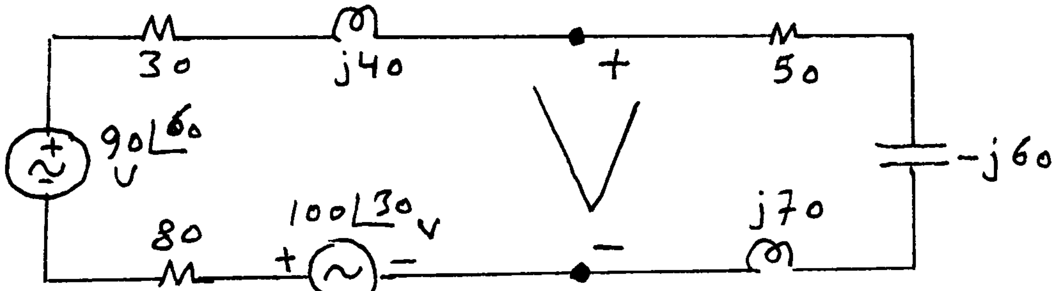
check $I_1 + I_2 + I_3 = 4\angle -36.87 + 4\angle -90 + 8\angle 53.13 = \underline{8\ \text{Amp}}$

$$V = V_{10} = V_{6+j8} = V_{-j5} = 10 I_1 = (6+j8) I_2 = -j5 I_3$$

$$= \underline{40\angle -36.87\ \text{Volt}}.$$

OR $V = I \cdot Z = 8\angle 0 \times (10 // (6+j8) // -j5)$

Q10 Find the voltage (V) for the circuit ?



Solution

$$I_T = \frac{V_T}{Z_T} = \frac{100\angle 30 + 90\angle 60}{30+j40+50-j60+j70+80} = \frac{100\angle 30 + 90\angle 60}{160+j50} \text{ Amp.}$$

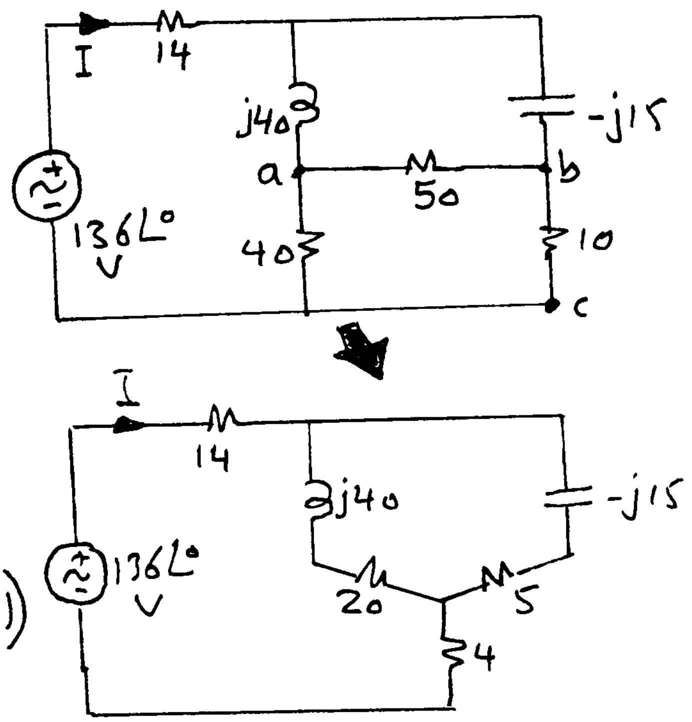
$$= \frac{45+j77.9+86.6+j50}{160+j50} = \frac{131.6+j127.9}{160+j50} \text{ Amp.}$$

$$= \frac{183.5\angle 44.1}{167.6\angle 17.35} = 1.094\angle 26.75 \text{ Amp.}$$

But $V = I_T\,(50-j60+j70) = I_T\,(50+j10)$.

$$= 1.094\angle 26.75 \times 50.99\angle 11.3 = \underline{55.8\angle 38.1 \text{ volt.}}$$

Q11 Use $\Delta \rightarrow Y$ transformations to find (I) ?



Solution:

$$R_a = \frac{50 \times 40}{10+50+40} = 20\,\Omega.$$

$$R_b = \frac{50 \times 10}{10+50+40} = 5\,\Omega.$$

$$R_c = \frac{40 \times 10}{10+50+40} = 4\,\Omega.$$

$$I_T = \frac{V_T}{Z_T}.$$

$$Z_T = 14+4+\left((20+j40)\,//\,(5-j15)\right)$$

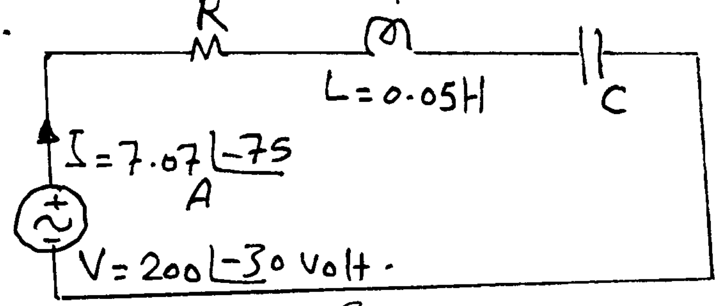
$$= 18 + \frac{(20+j40)(5-j15)}{20+j40+5-j15}$$

$$= 18 + \frac{700-j100}{25+j25} = 18 + \frac{707\angle -8.13}{35.35\angle 45} = 18 + 20\angle -53.13\ \Omega$$

$$= 18+12-j16 = (30-j16) = 34\angle -28\ \Omega.$$

$$\therefore I = \frac{136\angle 0}{34\angle -28} = 4\angle 28 \text{ Amp.}$$

Q12 For the circuit, find the value of R & C.?
If $W = 1000$ rad/sec.



Solution

$$Z_T = \frac{V_T}{I_T}$$

$$= \frac{200\angle -30}{7.07\angle -75} = 28.28\angle 45 = (\underset{R}{20} + j\underset{X_L}{20})\ \Omega$$

$$\therefore R = 20\,\Omega$$

$$X_L = 2\pi f L = WL = 1000 \times 0.05 = 50\,\Omega$$

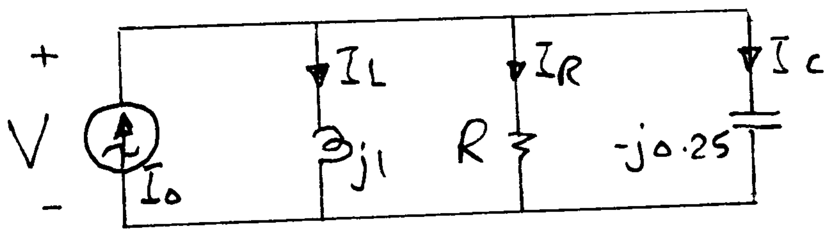
$$\therefore X_c = 50 - 20 = 30\,\Omega.$$

$$\text{But } X_c = \frac{1}{WC} = 30 = \frac{1}{1000\,C}.$$

$$\therefore C = \frac{1}{1000 \times 30} = 33.33\,\mu F$$

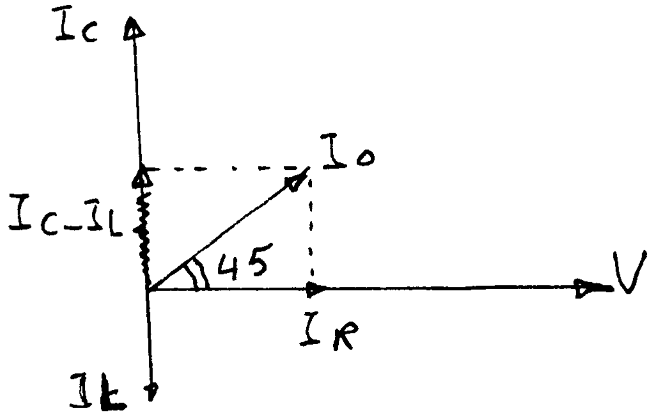
Q13 Use the phasor diagram of the circuit, find the value of (R), If the (I_R) lag (I_o) by 45°,

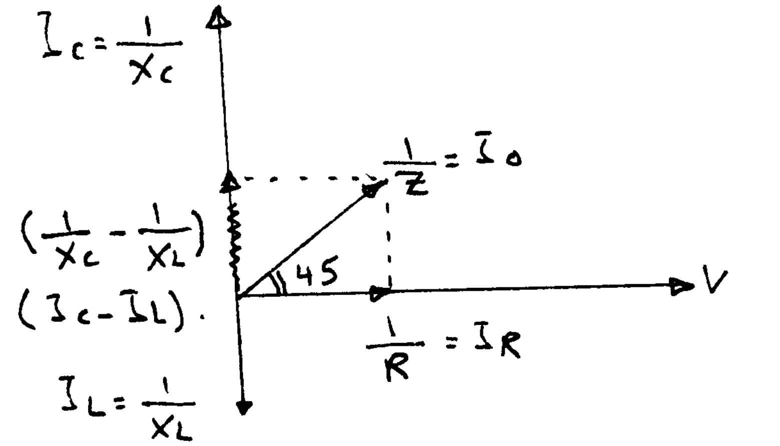
take: $V = V_m\angle 0$ volt.



Solution:

((See Condition 5)).



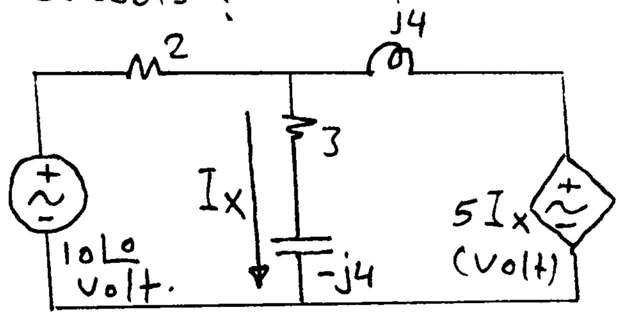


$\tan\theta = \tan 45 = 1$

$$\therefore 1 = \frac{\frac{1}{X_c} - \frac{1}{X_L}}{\frac{1}{R}}$$

$$\therefore 1 = \frac{\frac{1}{0.25} - \frac{1}{1}}{\frac{1}{R}} = \frac{4-1}{\frac{1}{R}} = \frac{3}{\frac{1}{R}} \longrightarrow R = \frac{1}{3}\,\Omega$$

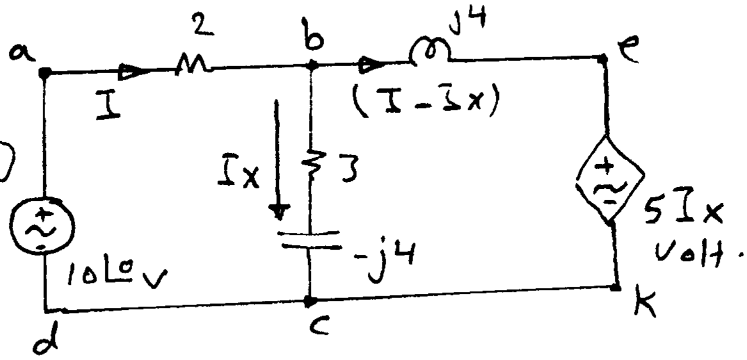
Q124 Find the (I_x) by all methods?



Solution:

* By Kirchhoff's

for circuit (abc & d)



$10\angle 0 = 2I + (3 - j4) I_x$ ..... ①

for circuit (be K & c)

$(3 - j4) I_x = j4(I - I_x) + 5 I_x$ ------ ②

From eq(2) $\rightarrow$ $3I_x - j4 I_x = j4 I - j4 I_x + 5 I_x$.

$\therefore 3 I_x - \cancel{j4 I_x} + \cancel{j4 I_x} - 5 I_x = j4 I$.

$-2 I_x = j4 I \longrightarrow \therefore I = \frac{-2 I_x}{j4} = -\frac{I_x}{j2}$

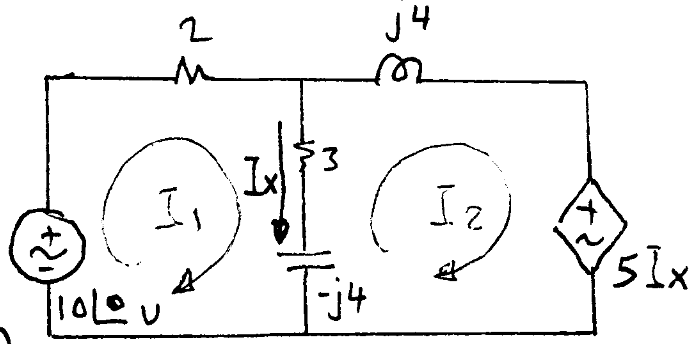
put (I) in eq(1):

$10 = 2\left(-\frac{I_x}{j2}\right) + (3 - j4) I_x$.

$10 = j I_x + 3 I_x - j4 I_x = I_x (3 - j3)$.

$\therefore I_x = \dfrac{10}{3-j3} = \dfrac{10}{4.24\angle -45} = \underline{2.35\angle 45 \text{ Amp}}.$

* **By loopmethod**



$\underline{I_x = I_1 - I_2}$

loop (1):

$10\angle 0 = (5-j4)I_1 - (3-j4)I_2$ ---------- ①

loop (2):

$-5I_x = 3I_2 - (3-j4)I_1$ ---------- ②.

$\therefore -5(I_1 - I_2) = 3I_2 - (3-j4)I_1$

$-5I_1 + 5I_2 - 3I_2 + 3I_1 - j4I_1 = 0.$

$\therefore -2I_1 - j4I_1 = -2I_2 \longrightarrow -\cancel{2}(1-j2)I_1 = -\cancel{2}I_2$

$\therefore \underline{I_2 = (1+j2)I_1} \longrightarrow$ But $I_x = I_1 - I_2$

$\underline{I_x = I_1 - (1+j2)I_1 = -j2I_1}$

put (I_2) in eq (1):

$\therefore 10\angle 0 = (5-j4)I_1 - (3-j4)(1+j2)I_1$

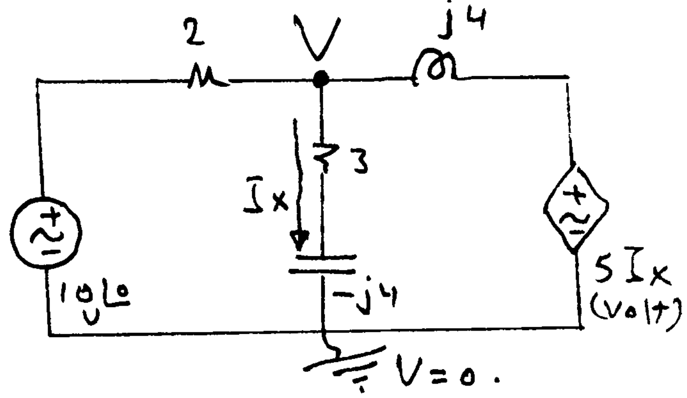
$\therefore \underline{I_1 = \dfrac{10}{-6-j6}}$. Amp.

But $I_x = -j2(I_1)$

$= -j2\left(\dfrac{10}{-6-j6}\right) = \dfrac{-j20}{-6-j6} = \dfrac{-j20(-6+j6)}{72} = \dfrac{10+j10}{6}$

$= \underline{2.35\angle 45 \text{ Amp}}.$ (Same result).

* By Nodal Voltage method!



Node (V):

$$\left(\frac{1}{2}+\frac{1}{j4}+\frac{1}{(3-j4)}\right)V-\frac{10\angle 0}{2}-\frac{5I_x}{j4}=0$$

$$\boxed{I_x=\frac{V}{3-j4}}$$

$$\left(j2(3-j4)+(3-j4)+j4\right)V-10\left(j2(3-j4)\right)-5\underset{\sim}{I_x}(3-j4)=0.$$

$$(11+j6)V-5V=80+j60.$$

$$\therefore\ V=\frac{80+j60}{6+j6}$$

$$\therefore I_x=\frac{V}{3-j4}=\frac{80+j60}{6+j6}\Big/(3-j4)=\frac{80+j60}{42-j6}$$

$$=\frac{100\angle 36.86}{42.42\angle -8.13}=\underset{\sim\sim\sim}{2.35\angle 45\ Amp}.\quad (\text{same result}).$$

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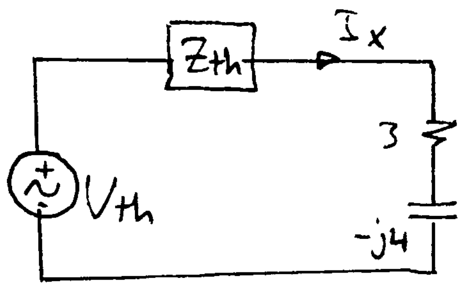
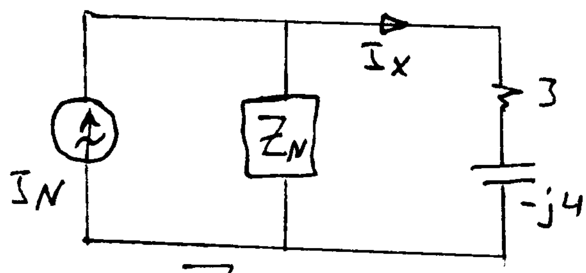
* By Thevenin's or Norton's

By Thevenin's



By Norton's

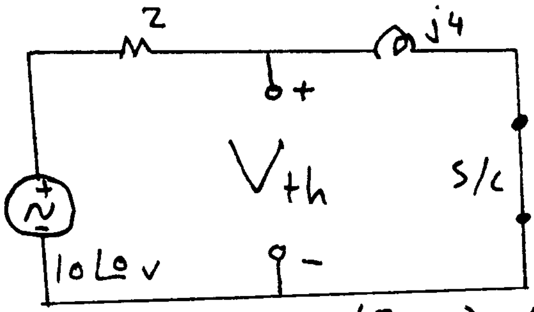


$$I_x=\frac{V_{th}}{Z_{th}+(3-j4)}.\qquad I_x=I_N\,\frac{Z_N}{Z_N+(3-j4)}$$

But $Z_{th}=Z_N$

$$\boxed{Z_{th}=Z_N=\frac{V_{th}}{I_N}}$$

* $V_{th} \longrightarrow$

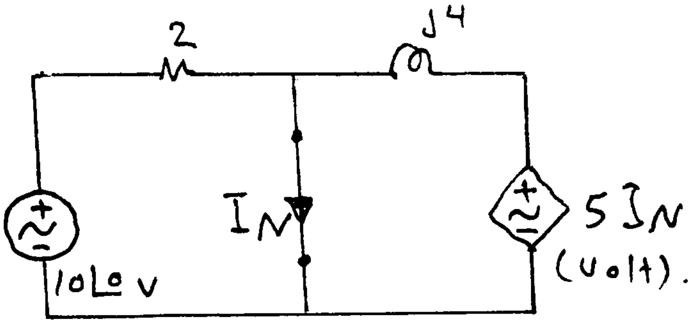


$$V_{th} = V_{j4\Omega}$$

$$= j4\left(\frac{10}{2+j4}\right) = \underbrace{\frac{j20}{(1+j2)}}\ \text{volt}.$$

Sincehere $(I_x = 0) \therefore 5I_x = 0$ (S/c)

* $I_N \longrightarrow$



$$\therefore I_N = \frac{10}{2} + \frac{5I_N}{j4}$$

$$j4I_N = j20 + 5I_N.$$

$$\therefore \underbrace{I_N = \frac{-j20}{5-j4}\ \text{Amp}}.$$

$$\therefore Z_{th} = Z_N = \frac{\frac{j20}{1+j2}}{\frac{-j20}{5-j4}} = -\frac{5-j4}{1+j2}$$

$$= \frac{3+j14}{5} = \underbrace{(0.6+j2.8)\,\Omega}.$$

$\therefore$ By thevenin:

$$I_x = \frac{V_{th}}{Z_{th}+(3-j4)} = \frac{\frac{j20}{(1+j2)}}{0.6+j2.8+(3-j4)} = \frac{\frac{j20}{1+j2}}{3.6-j1.2}$$

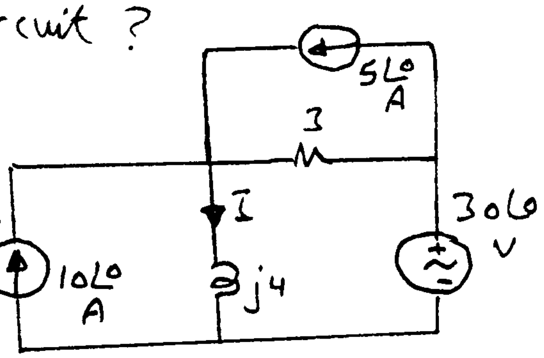
$$= \frac{j20}{6+j6} = \underbrace{2.35\angle 45\ \text{Amp}}.$$

$\therefore$ By Norton's:

$$I_x = I_N\,\frac{Z_N}{Z_N+(3-j4)} = \frac{-j20}{5-j4} \times \frac{(0.6+j2.8)}{(0.6+j2.8)+(3-j4)}$$

$$= \underbrace{2.35\angle 45\ \text{Amp}}.$$

Q15 Find the current (I) for the circuit ?



* <u>By Superposition</u> :

* I_1 By $10\angle 0$ Amp → $5\angle 0 A$ (o/c) & $30\angle 0 V$ (s/c).

$$I_1 = 10\angle 0 \,\frac{3}{3+4} = \frac{30}{3+j4} \text{ Amp}.$$

* I_2 By $5\angle 0$ Amp → $10\angle 0 A$ (o/c) & $30\angle 0$ volt (s/c).

$$I_2 = 5\angle 0 \,\frac{3}{3+j4} = \frac{15}{3+j4} \text{ Amp}.$$

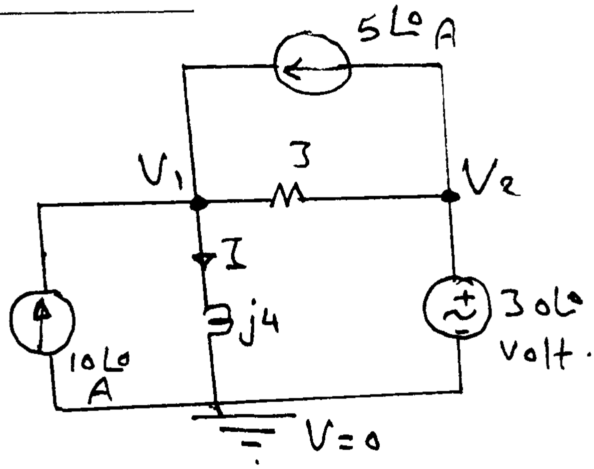
* I_3 By $30\angle 0$ Volt → $10\angle 0$ & $5\angle 0$ Amp (o/c).

$$I_3 = \frac{30\angle 0}{3+j4} \qquad \therefore I = I_1 + I_2 + I_3 = \frac{30}{3+j4} + \frac{15}{3+j4} + \frac{30}{3+j4}.$$

$$\therefore I = \frac{75}{3+j4} = \frac{75}{5\angle 53.13} = \underline{15\angle -53.13 \text{ Amp}}.$$

---

* <u>By Nodal voltage method</u>.



$V_2 = 30\angle 0$ Volt.

Node (V_1):

$$\left(\frac{1}{3} + \frac{1}{j4}\right)V_1 - \frac{30}{3} = 10 + 5$$

$$(j4+3)V_1 - j120 = 15(j12).$$

<u>j12 = م.م.ب</u>

$$\therefore (3+j4)V_1 = j300.$$

$$\therefore \underline{V_1 = \frac{j300}{3+j4}} \longrightarrow \text{But } I = \frac{V_1}{j4}$$

$$\therefore I = \frac{\frac{j300}{3+j4}}{j4} = \frac{75}{3+j4} = \underline{15\angle -53.13 \text{ Amp}}.$$

Same result.

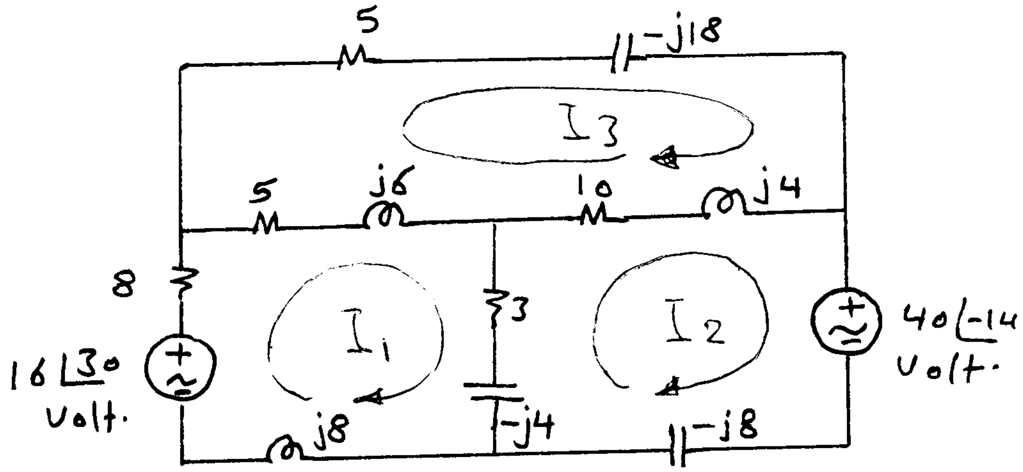
Q15 Draw the circuit corresponding to the loop current method?

$$loop(1): 16\angle 30 = (16+j10)I_1 - (3-j4)I_2 - (5+j6)I_3 \quad \text{-----} (1)$$

$$loop(2): -40\angle -14 = (13-j8)I_2 - (3-j4)I_1 - (10+j4)I_3 \quad \text{----} (2)$$

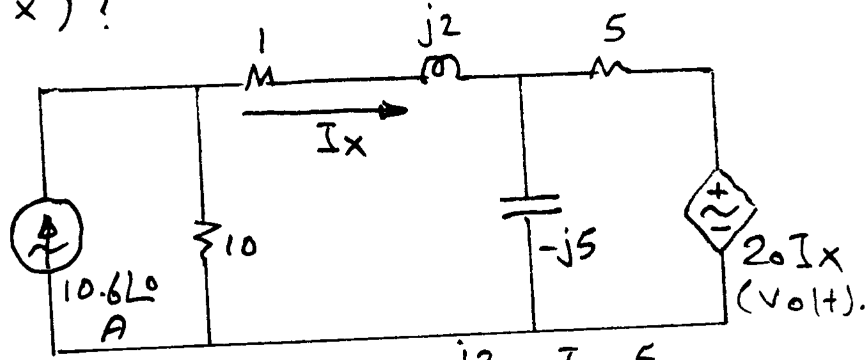
$$loop(3): 0 = (20-j8)I_3 - (5+j6)I_1 - (10+j4)I_2 \quad \text{-----} (3)$$

Solution:

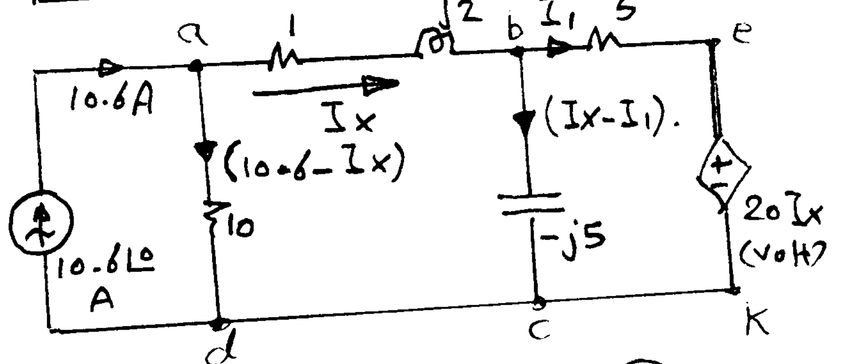


Q17 Find the value of (I_x)?

Solution:



* By Kirchhoff:



* For circuit (a,b,c & d):

$$10(10.6 - I_x) = (1+j2)I_x + (I_x - I_1)\times -j5 \quad \text{--------} (1).$$

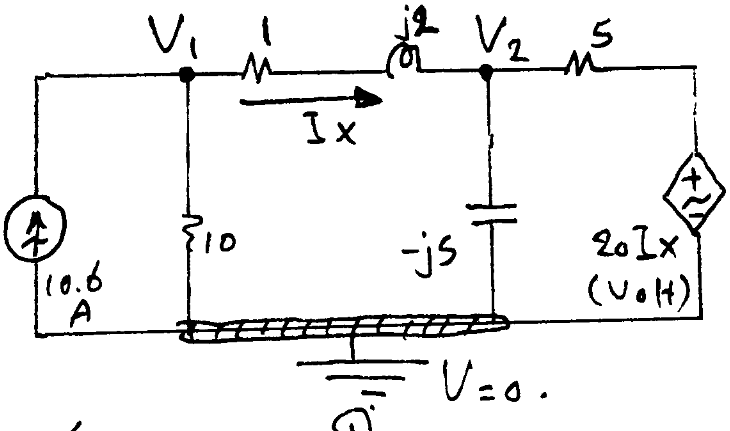
* For circuit (b,e,K & c):

$$-j5(I_x - I_1) = 5I_1 + 20I_x \quad \text{---------} (2).$$

solving the equations

$$I_x = (3.76 + j1.68)\ Amp.$$

* By Nodal voltage method



$$I_x = \frac{V_1 - V_2}{1 + j2}.$$

Node (V_1):

$$\left(\frac{1}{10} + \frac{1}{1+j2}\right) V_1 - \frac{V_2}{1+j2} = 10.6 \quad \text{------} \; (1).$$

Node (V_2):

$$\left(\frac{1}{1+j2} + \frac{1}{5} + \frac{1}{-j5}\right) V_2 - \frac{V_1}{1+j2} - \frac{20 I_x}{5} = 0 \quad \text{------} \; (2)$$

Solving the equation $I_x = (3.76 + j1.68)$ Amp.

Same result.

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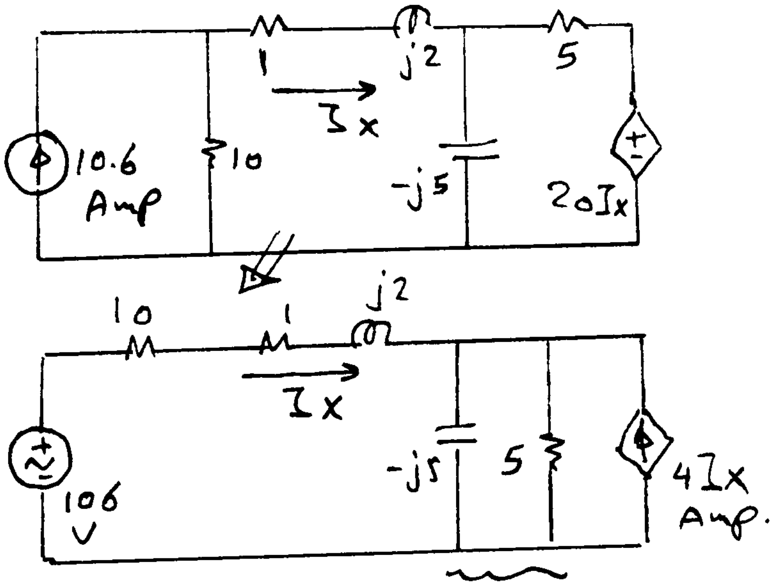
* By ohm's Law



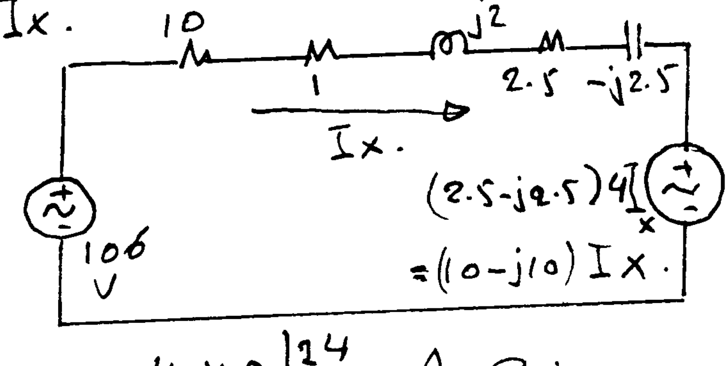
Use the (V ⇌ I) transformation as shown:

∴ By ohm's Law.

$$I_x = \frac{106 - (10 - j10) I_x}{13.5 - j0.5}$$

$$5 \,//\, -j5 = \frac{-j25}{5-j5} = 2.5 - j2.5$$

$$13.5 I_x - j0.5 I_x = 106 + j10 I_x - 10 I_x.$$



$$(23.5 - j10.5) I_x = 106$$

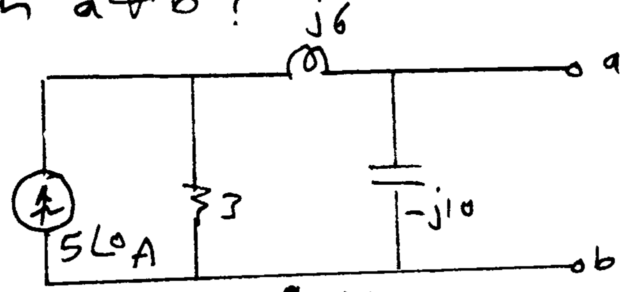
$$\therefore I_x = \frac{106}{23.5 - j10.5}.$$

$$= \frac{106}{23.5 - j10.5} = \frac{106}{25.73 \angle -24} = 4.119 \angle 24 \text{ Amp.}$$

$$= (3.76 + j1.68) \text{ Amp.}$$

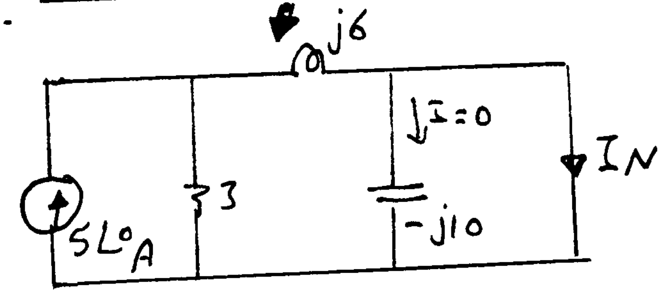
Same result.

Q18 | Find (I_N) between a & b ?



Solution:

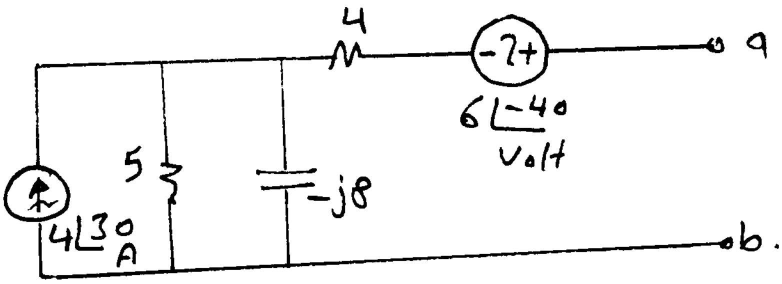
$$I_N = 5\angle 0 \; \frac{3}{3+j6} \quad (\text{C.D.R}).$$



$$= \frac{15}{3+j6} = 2.23 \angle -63.43 \; A$$

Q19 | Find (Z_{th}, V_{th} & I_N) between a & b ?

Solution:



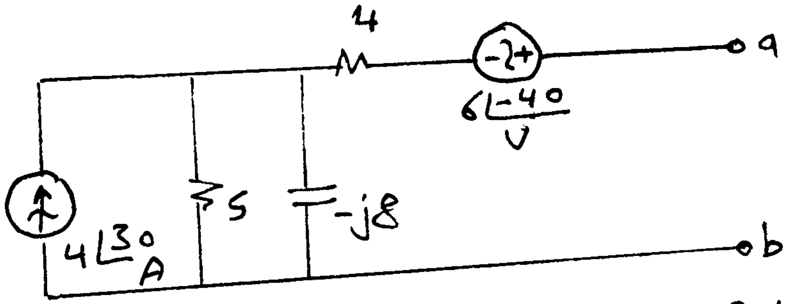
$$*\; Z_{th} = 4 + (5 // -j8).$$

$$= 4 + \frac{-j40}{5-j8}$$

$$= 4 + \frac{40\angle -90}{9.433\angle -58} = 4 + 4.24\angle -32 \;\Omega.$$

$$= 4 + 3.59 - j2.24 = 7.59 - j2.24 = 7.92 \angle -16.5 \;\Omega$$

$*\; V_{th} \rightarrow$

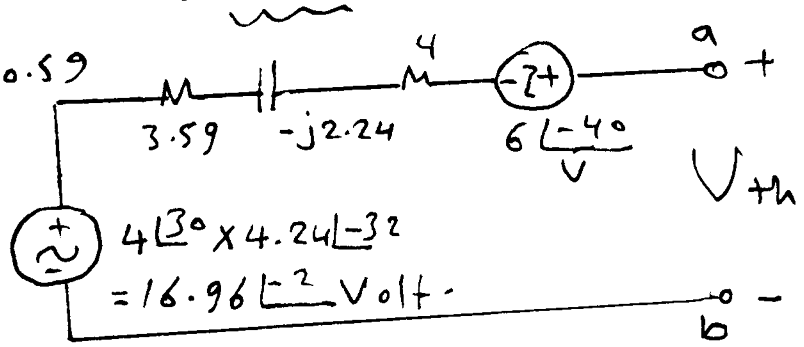


Change the current source to voltage source as shown.

$5 // -j8 = 4.24\angle -32 \;\Omega = 3.59 - j2.24$

use I → V transformation

$$\therefore V_{th} = 6\angle -40 + 16.96\angle -2$$

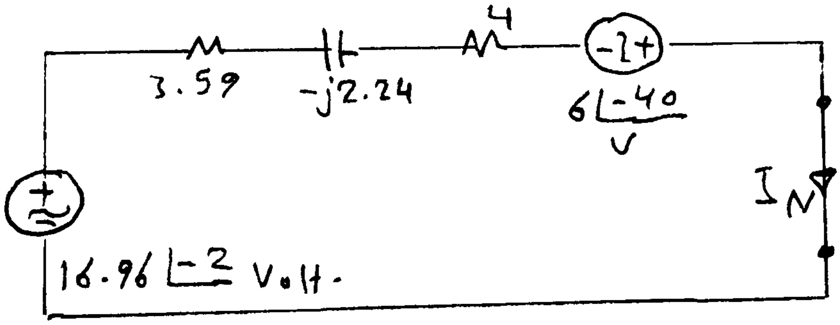


$$= 4.59 - j3.85 + 16.94 - j0.59$$

$$= 21.53 - j4.44$$

$$= 22 \angle -11.65 \text{ Volt.}$$

* $I_N \rightarrow$



$$I_N = \frac{16.96\angle{-2} + 6\angle{-40}}{3.59 - j2.24 + 4}$$

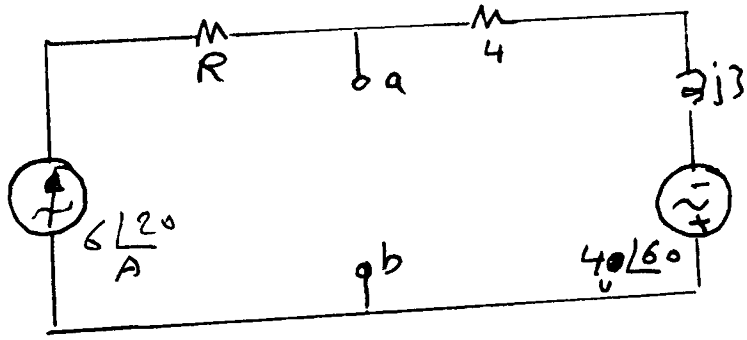
$= 2.78\angle 4.85$ Amp.

OR $I_N = \dfrac{V_{th}}{Z_{th}} = \dfrac{22\angle -11.65}{7.92\angle -16.5} = 2.78\angle 4.85$ Amp

same result.

Q20 For the circuit find (Z_{th}, V_{th} & I_N)? between a&b.

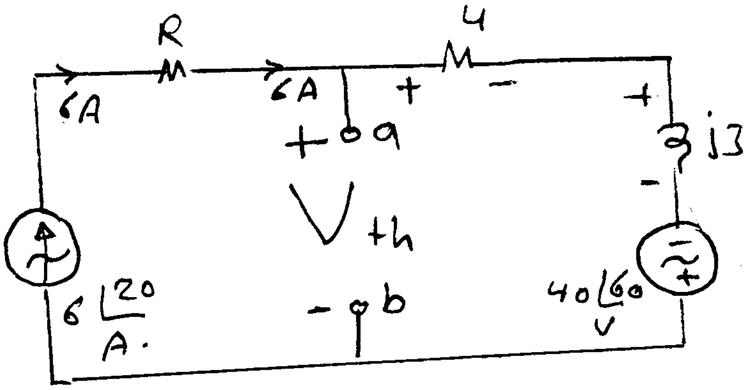
solution:



* $Z_{th} = Z_{ab} = (4+j3)\,\Omega$.

because $6\angle 20$ (o/c).

$40\angle 60$ (s/c).



* $V_{th} \rightarrow$

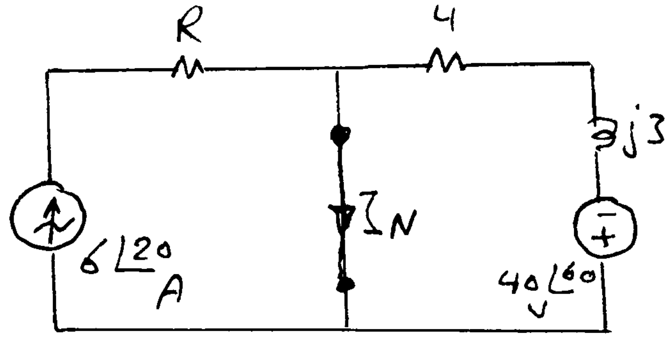
$V_{th} + 40\angle 60 = V_{(4+j3)}$

$\therefore V_{th} = 6\angle 20 \times (4+j3) - 40\angle 60$

$= 30\angle 56.86 - 40\angle 60$

$= 16.4 + j25.1 - (20 + j34.64) = -3.6 - j9.54 = -(3.6 + j9.54)$

$= -10.2\angle 69.3$ Volt.



* $I_N \rightarrow$

By $6\angle 20 \rightarrow 40\angle 60$ (s/c).

$I_{N1} = 6\angle 20$ Amp.

By $40\angle 60 \rightarrow 6\angle 20$ (o/c).

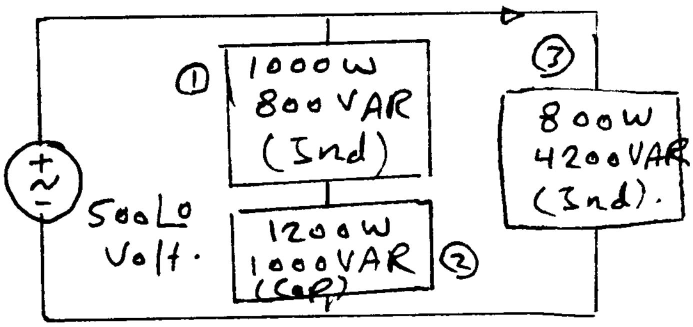
$I_{N2} = \dfrac{40\angle 60}{4+j3} = \dfrac{40\angle 60}{5\angle 36.86} = 8\angle 23.14$ Amp

$\therefore I_N = I_{N_1} - I_{N_2} = 6\angle 20 - 8\angle 23.14 = -2.04\angle 32.5$ Amp.

OR $I_N = \dfrac{V_{th}}{Z_{th}} = \dfrac{-10.2\angle 69.3}{4+j3} = -2.04\angle 32.5$ Amp.

Q2 For the circuit shown, find P_T, Q_T, S_T, i_T & P.f. then find the in box (3)?

solution:



$P_T = P_1 + P_2 + P_3$

$= 1000 + 1200 + 800 = 3000W$

$= 3\,KWatt$.

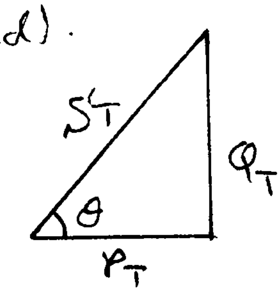
$Q_T = Q_1 - Q_2 + Q_3$

$= 800 - 1000 + 4200 = 4000VAR = 4\,KVAR$ (Ind).

$S_T = P_T + jQ_T = \sqrt{P_T^2 + Q_T^2}$

$= \sqrt{3000^2 + 4000^2} = 5000VA$

$= 5KVA$.



$S_T = V_T . I_T \longrightarrow I_T = \dfrac{5000}{500} = 10$ Amp.

$\cos\theta = \dfrac{P_T}{S_T} = \dfrac{3000}{5000} = 0.6$ (Ind).

In Box (3) $P = 800W$, $Q = 4200VAR$ $\therefore S = \sqrt{P^2 + Q^2}$

$= 4275.5VA$.

$\therefore$ the current in Box (3)

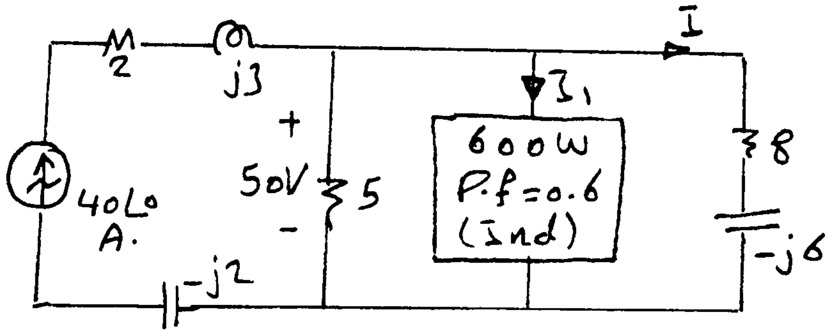
$I = \dfrac{S_3}{V} = \dfrac{4275.5}{500} = 8.55$ Amp.

$P = I^2 R \longrightarrow 800 = (8.55)^2 \times R \longrightarrow R = \dfrac{800}{(8.55)^2} = 10.94\,\Omega$

$Q = I^2 . X_L \longrightarrow 4200 = (8.55)^2 . X_L \longrightarrow X_L = \dfrac{4200}{(8.55)^2} = 57.45\,\Omega$

Q22 For the circuit, find P_T, Q_T, S'_T and element in Box?

Solution:



$P_T = P_1 + P_2 + P_3 + P_4$

$P_1 = (40)^2 \times 2 = 3200W$

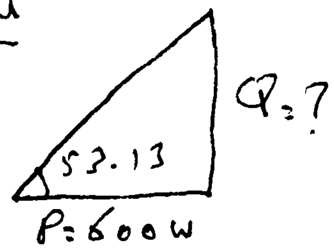
$P_2 = \frac{V^2}{5} = \frac{(50)^2}{5} = 500W.$

$P_3 = 600W$

$P_4 = I^2 \times 8 \longrightarrow I = \frac{50}{8-j6} = \frac{50}{10\angle -36.86} = 5\angle 36.86$ Amp.

$\therefore P_4 = (5)^2 \times 8 = 200W$

$\therefore P_T = 3200 + 500 + 600 + 200 = \underline{4500 \text{ Watt}}.$



In Box $P.f = 0.6 \longrightarrow \theta = 53.13$

$\tan\theta = \frac{Q}{P} = 1.333 = \frac{Q}{600}$

$\therefore Q = 800 VAR$ (Ind).

$\therefore Q_T = Q_1 + Q_2 - Q_3$

$= (40)^2 \times 1 + 800 - (5)^2 \times 6$

$= 1600 + 800 - 150 = \underline{2250 VAR}$ (Ind).

$S'_T = \sqrt{P_T^2 + Q_T^2} = \underline{5031.1 VA}$

In Box $\longrightarrow P = 600W$, $Q = 800VAR$

$\therefore S' = \sqrt{P^2 + Q^2} = \sqrt{600^2 + 800^2} = 1000 VA.$

$\therefore$ the current (I_1) in Box $\longrightarrow S' = V.I_1$

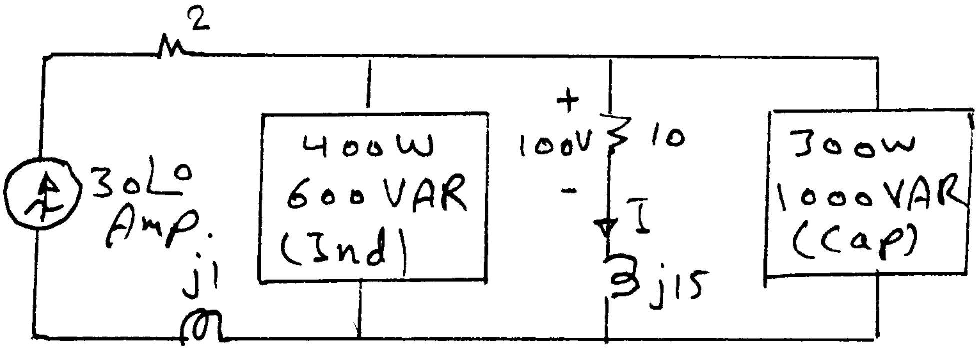
$\therefore I_1 = \frac{1000}{50} = 20$ Amp.

$P = I^2 . R = 600 = (20)^2 \times R \longrightarrow R = \frac{600}{(20)^2} = \underline{1.5 \Omega}.$

$Q = I^2 . X_L = 800 = (20)^2 \times X_L \longrightarrow X_L = \frac{800}{(20)^2} = \underline{2 \Omega}$

Q23 Find P_T, Q_T, S_T, V_T & $P.f$?

Solution:



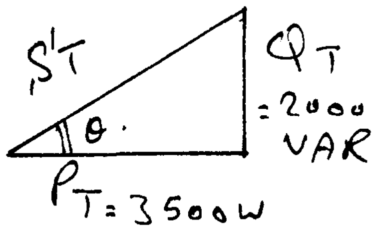
I in 10Ω

$$= \frac{100}{10} = 10 \text{ Amp}.$$

$$* \; P_T = P_1 + P_2 + P_3 + P_4$$

$$= (30)^2 \times 2 + 400 + (10)^2 \times 10 + 300$$

$$= 3500 \text{ Watt} = 3.5 \text{ KW}.$$



$$* \; Q_T = (30)^2 \times 1 + 600 + (10)^2 \times 15 - 1000$$

$$= 2000 \text{ VAR} = 2 \text{ KVAR (Ind)}.$$

$$* \; S_T = \sqrt{P_T^2 + Q_T^2}$$

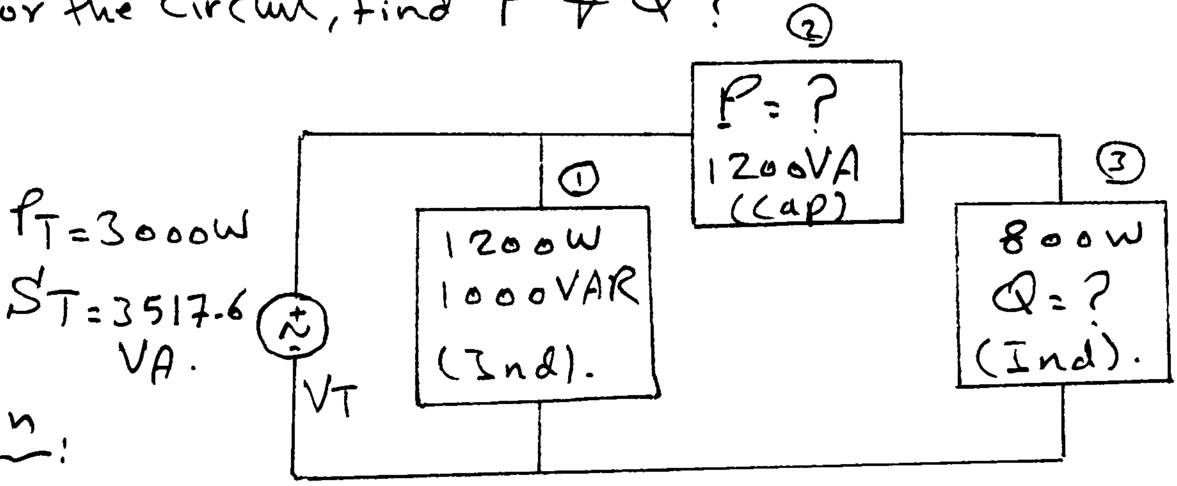
$$= \sqrt{(3500)^2 + (2000)^2} = 4031.1 \text{ VA}.$$

$$S_T = V_T . I_T$$

$$* \; \therefore V_T = \frac{S'_T}{I_T} = \frac{4031.1}{30} = 134.3 \text{ Volt}.$$

$$* \; P.f = \cos\theta = \frac{P_T}{S'_T} = \frac{3500}{4031.1} = 0.868 \text{ (Ind) or lagging.}$$

Q24 For the circuit, find P & Q ?



Solution:

$$P_T = P_1 + P_2 + P_3$$

$$\therefore 3000 = 1200 + P + 800$$

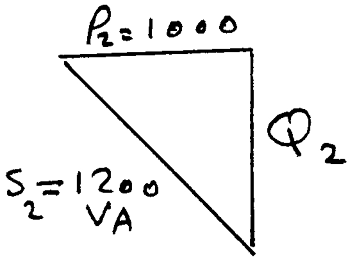
$$\therefore P = \underline{1000 \text{ Watt}}.$$

$$S_T = \sqrt{P_T^2 + Q_T^2}.$$

$$\therefore 3517.6 = \sqrt{(3000)^2 + Q_T^2}.$$

$$\therefore Q_T = 1836.7 \text{ VAR}.$$

$$Q_T = 1836.7 = 1000 + Q - Q_2.$$



Box (2)

In Box (2)

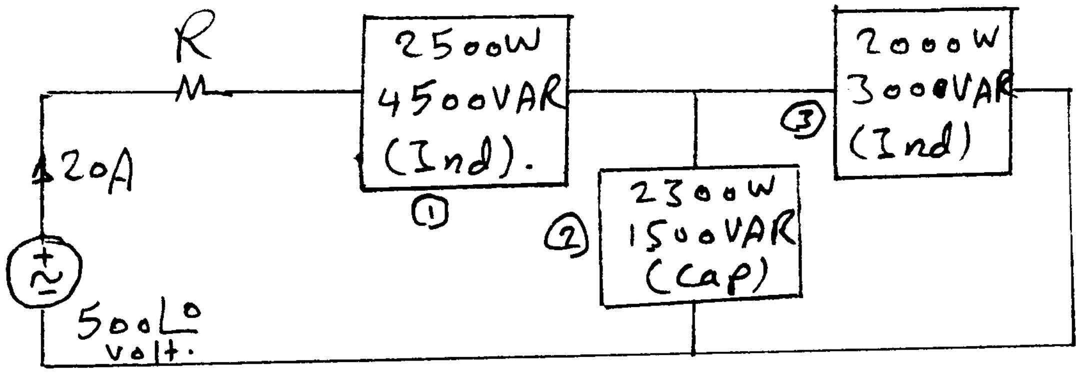
$$1200 = \sqrt{(1000)^2 + Q_2^2}$$

$$Q_2 = 663.3 \text{ VAR (Cap)}.$$

$$\therefore Q_T = 1836.7 = 1000 - 663.3 + Q.$$

$$\therefore Q = \underline{2500 \text{ VAR (Ind)}}.$$

Q25 For the circuit, find P_T, Q_T, S_T & R ?



Solution

$S_T = V_T \times I_T$

$= 500 \times 20 = 10000\,VA = 10\,KVA$.

$Q_T = 4500 + 3000 - 1500$

$= 6000\,VAR = 6\,KVAR$ (Ind).

But $S_T = \sqrt{P_T^2 + Q_T^2}$.

$\therefore 10000 = \sqrt{P_T^2 + (6000)^2}$.

$\therefore P_T = 8000\,\text{Watt} = 8\,KW$

But $P_T = P_R + P_1 + P_2 + P_3$.

$8000 = P_R + 2500 + 2300 + 2000$

$8000 = P_R + 6800$.

$\therefore P_R = 1200\,\text{Watt}$.

and $P_R = (I)^2 \cdot R$.

$1200 = (20)^2 \times R$.

$\therefore R = 3\,\Omega$