جامعة المنصور

كلية المنصور

قسم
هندسة اتصالات
المرحلة الاولى

Electrical Fundamentals

Lectuers

2017 – 2018

الاسس الكهربائية

First semester

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Circuit - Elements

We may classify circuit elements into two categories:-

*Passive elements: If it contains no source of e.m.f, and if the total energy delivered to it from the rest of the circuit is always non-negative (Resistors, Capacitors, and inductors).

*Active elements: If it contains a source of e.m.f, for example Generator, batteries, and electronic devices which required power supplies.

Voltage and Current Sources

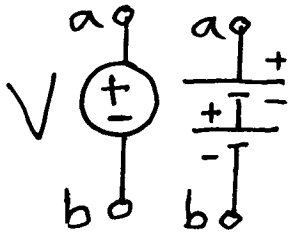
Independent and dependent Voltage and current sources.

① Independent sources:

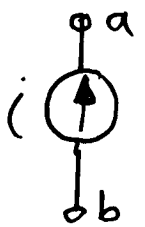
A) The independent Voltage source: such as a battery or generator that maintain a voltage between its terminals. (a) is at a higher potential than (b).

The voltage (V) may be time Varying, and may be constant. The independent voltage source is completely independent of the current through the element.

B) The independent current source: is a two terminal element through which a specified current flows. The current is completely independent of the voltage across the element. (i) is specified current. The direction of current indicated by the arrows.



Voltage source

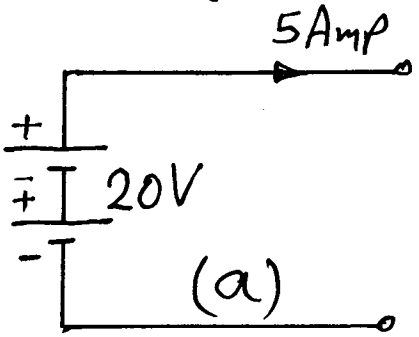


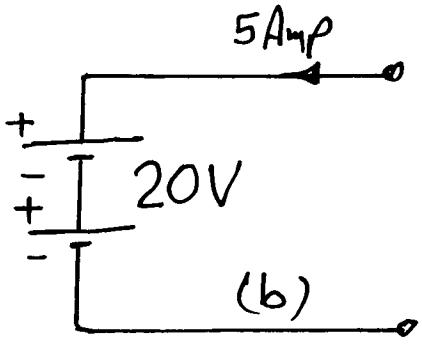
current source.

* Both independent voltage & current sources are usually meant to deliver power to the external circuit and not to absorb it.

i.e $\boxed{P = V.I}$

(V) is the voltage across the source, (i) is the current of the source directed out of the +Ve terminal, then the source is delivering power Other wise it is absorbing power, for example:-





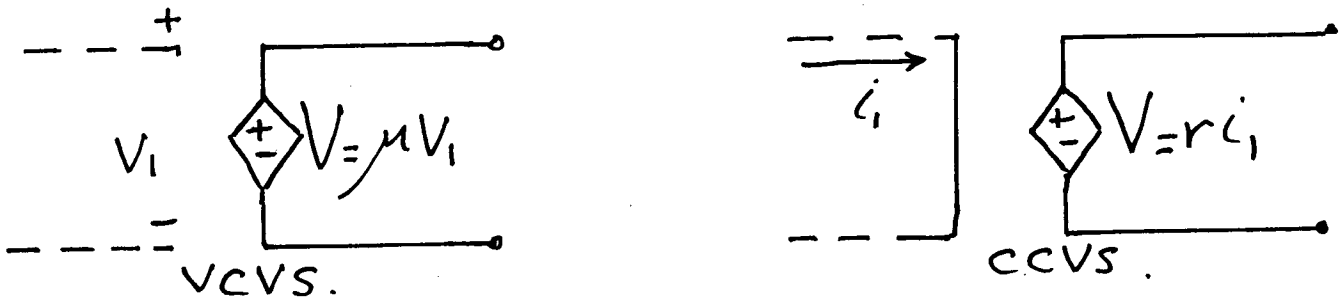
Fig(a), the battery is delivering (100W) to the external circuit, Fig(b) the battery is absorbing (100W) as would be the case when it is being charged.

② Dependent Sources:

Dependent sources whose voltage or current is controlled by another voltage or current some where in the circuit.

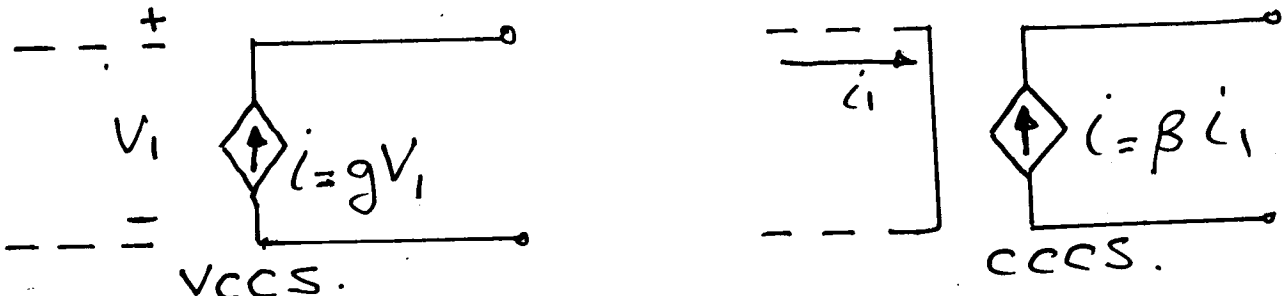
A) **Dependent or Controlled Voltage Source**, is one whose terminal voltage depends on or is controlled by a voltage or current.

VCVS: Voltage-controlled voltage source.
CCVS: current-controlled voltage source. is controlled by a current.



B) **Dependent or Controlled Current Source.**

VCCS: is controlled by a voltage.
CCCS: is controlled by a current.



The quantities (μ & β) are dimensionless constants commonly referred to as the voltage and current gain.

The constants (r & g) have units of ohms and mhos.

Dependent sources are essential components in «Amplifier» Circuit.

Resistance (R)

The unit of resistance is the ohm (Ω), the value of resistance can be determined by the factors:-

1- Material 2- Cross-Sectional area.
3- Length and 4- Temperature.

$$\therefore R \propto \frac{l}{A}$$

Where:

l: The length of conductor in (m).
A: Cross-Sectional area in (m^2).

$$\therefore \boxed{R = \rho \frac{l}{A}}$$

ρ: is the resistivity in ($\Omega.m$).

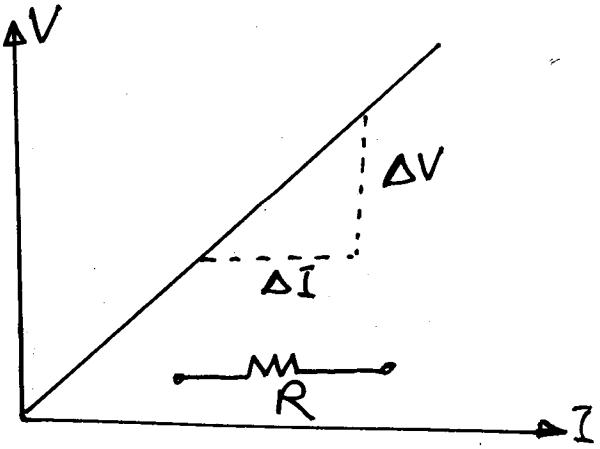
« Ohm's Law »

Ohm's Law defined as « The voltage across a resistor is directly proportional to the current flowing through the resistor ».

$$\therefore V \propto I \quad \text{or} \quad \boxed{V = I.R} \quad R \geq 0 \text{ in } \Omega.$$

The function $V = f(I)$, the Voltage-current characteristic is a straight line passing through the origin.
The slope will be the resistance (R).

$$\text{Slope} = R = \frac{\Delta V}{\Delta I} \text{ .... ohm.}$$



Electric power (P)

Electric power is defined as the time rate at which work is being done or energy is being expended. Since power is the rate of doing work and a rate is any physical quantity divided by time, we may define the average power (P) as:

$$P = \frac{W}{t} = \frac{\text{total Work done (J)}}{\text{total time taken (S)}} = \frac{J}{S}$$

$1\ J/S = 1\ \text{Watt} = 1\ N.m/S$

But $V = \frac{W}{Q}$ and $I = \frac{Q}{t}$.

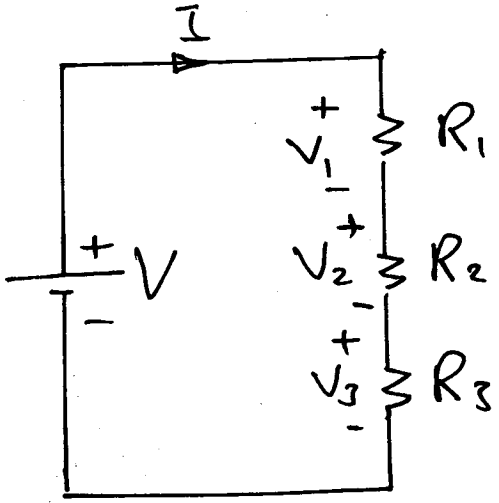
$$\therefore P = \frac{W}{t} = \frac{V.Q}{t} = V.I \quad \text{(Watt)}.$$

But $V = I.R$ (ohm's Law).

$$\therefore \boxed{P = V.I = I^2.R = \frac{V^2}{R}} \quad \text{Watt}.$$

((Resistance Connections))

① Series Connection: If the resistance connected as shown, they are said to be in Series.



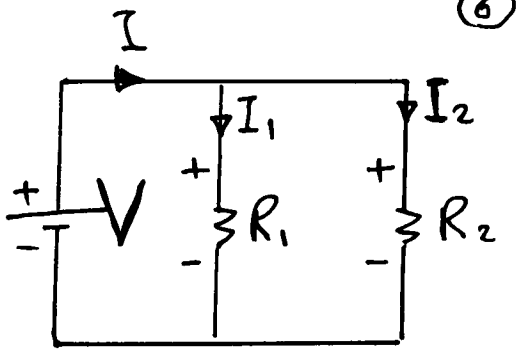
$$V = V_1 + V_2 + V_3.$$
$$= I.R_1 + I.R_2 + I.R_3$$
$$= I(R_1 + R_2 + R_3).$$

But $V = I.R_{eq}$ (ohm's Law).

$$\therefore \boxed{R_{eq} = R_1 + R_2 + R_3}$$

(2) Parallel connection:

resistances connected as shown, they are said to be in parallel



$$I = I_1 + I_2.$$

$$\therefore I = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

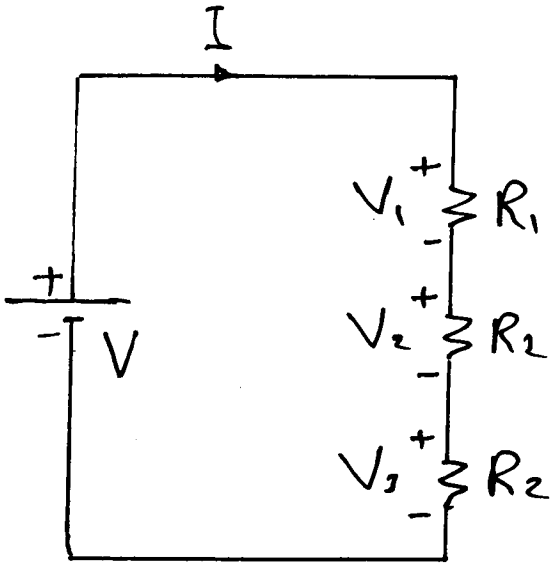
But $I = \frac{V}{R_{eq}}$ (ohm's Law).

$$\therefore \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}} \quad \underline{OR} \quad \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}.$$

** Voltage Divider Rule

we have $I = \frac{V}{R_{eq}}$.

$$= \frac{V}{R_1 + R_2 + R_3}.$$



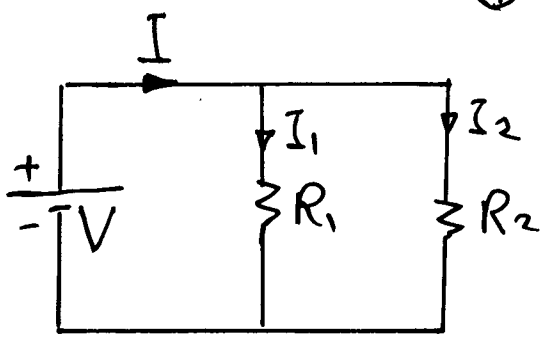
But $V_1 = I_1 R_1 = I . R_1$

$$\therefore V_1 = V \frac{R_1}{R_1 + R_2 + R_3}.$$

and $V_2 = V \frac{R_2}{R_1 + R_2 + R_3}.$

In General $V_x = V \frac{R_x}{R_1 + R_2 + R_3 + \cdots + R_x}.$

** Current Divider Rule



We have $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$.

and $V = I.R_{eq}$

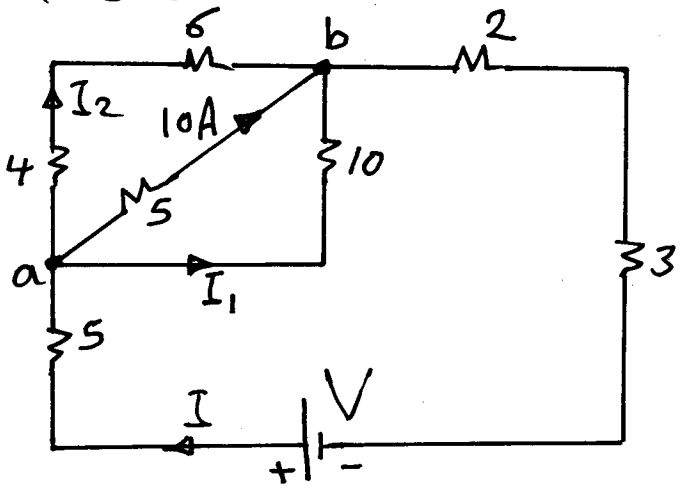
$= I \frac{R_1 R_2}{R_1 + R_2}$.

$\therefore I_1 = I \frac{R_2}{R_1 + R_2}$ & $I_2 = I \frac{R_1}{R_1 + R_2}$.

Home work: prove that $I_1 = I \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$.

For R_1, R_2, R_3 connected in parallel.

Ex: Find (V) & (I) for the circuit.



$V_{ab} = 10 \times 5 = 50$ Volt.

$\therefore I_1 = 50/10 = 5$ Amp.

$I_2 = 50/4+6 = 5$ Amp.

$\therefore I = I_1 + I_2 + I_3$

$= 10 + 5 + 5 = \underline{20 \text{ Amp}}$.

$R_{eq} = 5 + (10 // 10 // 5) + 2 + 3$

$= 12.5 \Omega$.

$\therefore V = I.R_{eq} = 20 \times 12.5 = \underline{250}$ Volt

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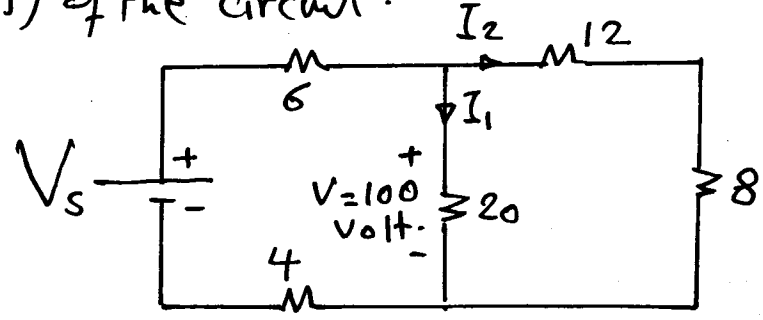
Ex: Find the value of (V_s) of the circuit.



$I_1 = \frac{100}{20} = 5$ Amp.

$I_2 = \frac{100}{12+8} = 5$ Amp.

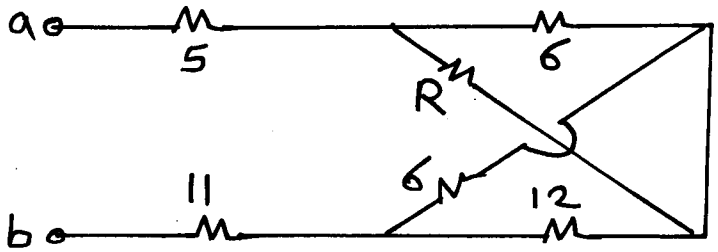
$I_{Total} = 5 + 5 = 10$ Amp.

$R_{eq} = 6 + 4 + (20 \| 20) = 20\,\Omega$.

$\therefore V_s = I \cdot R_{eq} = \underline{200\ \text{Volt}}$.

Ex: Find the value of (R), if the equivalent resistance between a & b is ($R_{ab} = 22\,\Omega$).

ans: $R = 3\,\Omega$.



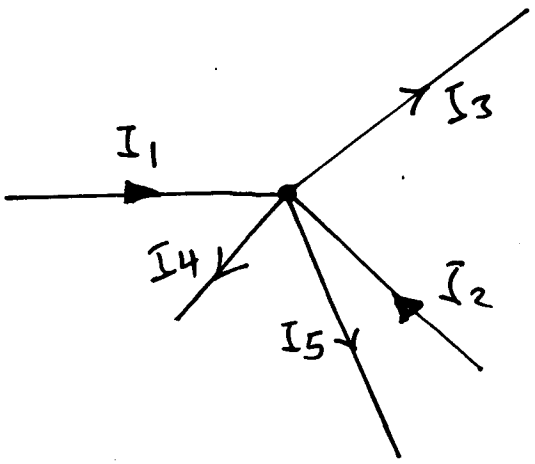
(Kirchhoff's Law)

① Kirchhoff's current Law (K.C.L):

It states as ((The algebraic Sum of the incoming currents and Outgoing currents at anode (Point) is Zero)).

$$\sum I_{\text{in Coming}} = \sum I_{\text{out going}}.$$

$$I_1 + I_2 = I_3 + I_4 + I_5.$$

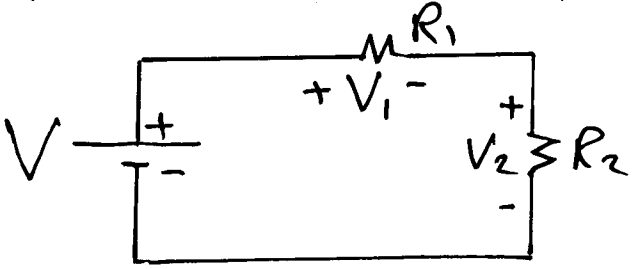


② Kirchhoff's Voltage Law (K.V.L). ⑨

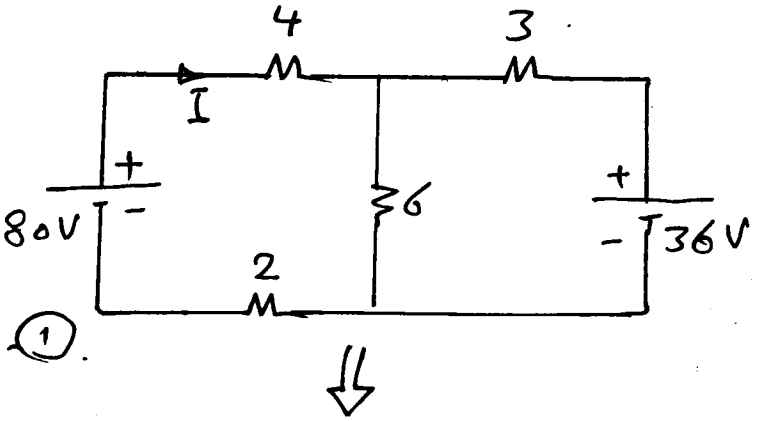
Kirchhoff's Voltage Law states as ((The algebraic Sum of the potential rises and drops around a closed loop (or circuit) is Zero)).

$$\sum V = 0.$$

OR $V = V_1 + V_2.$



Ex: Find the current (I) for the circuit shown.



For the circuit (abcd).

$80 = (4+2)I + 6I_1$

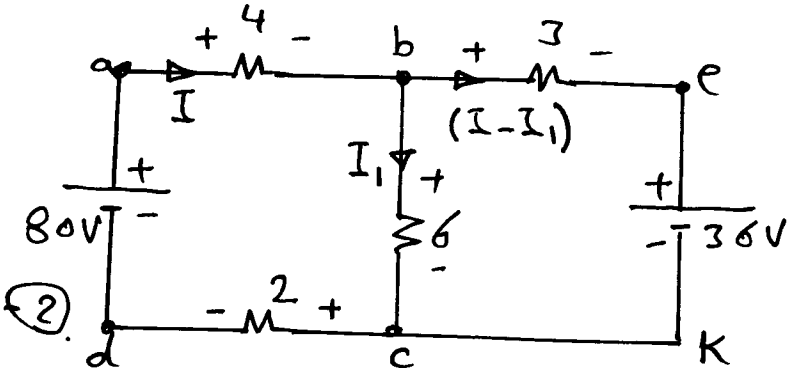
$80 = 6I + 6I_1$ ------ (1).

For the circuit (bekc).

$6I_1 = 36 + 3(I - I_1)$.

$6I_1 = 36 + 3I - 3I_1$

$9I_1 = 36 + 3I$ ------ (2).



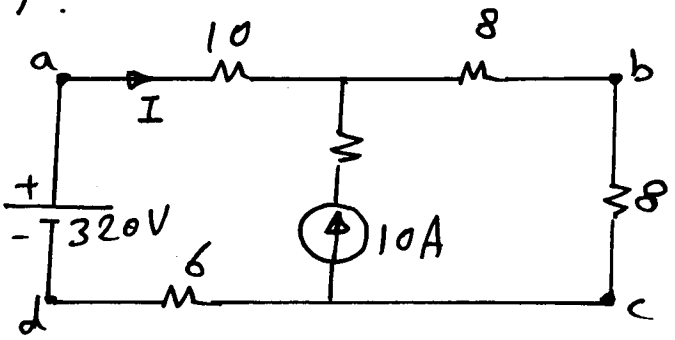
From eq(2) $I_1 = \frac{12+I}{3}$.

put I_1 in eq.(1).

$80 = 6I + 6\left(\frac{12+I}{3}\right)$.

$\therefore \underline{I = 7 \text{ Amp}}$.

Ex: Find (I) and power consumed in (8Ω), and Power supply by (320V)?



by K.V.L the circuit(abcd)

$320 = (10+6)I + (8+8)(I+10)$.

$320 = 16I + 16I + 160$

$160 = 32I$.

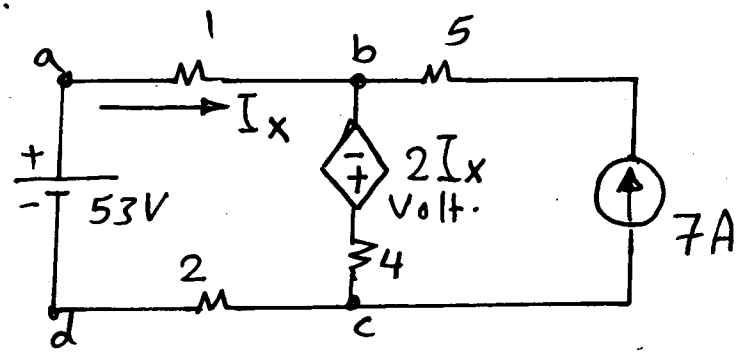
$\therefore I = 5$ Amp

$\therefore I$ in $8\Omega = 10 + 5 = 15$ Amp.

$\therefore P_{in}(8\Omega) = (15)^2 \times 8 = 1800$ watt.

Power Supply by $320V = V.I = 320 \times 5 = 1600$ Watt.

Ex: Find the value of (I_x)?



For the circuit of (abcd):

$53 + 2I_x = 3I_x + 4(I_x + 7)$.

$53 + 2I_x = 3I_x + 4I_x + 28$.

$25 = 5I_x$.

$\therefore I_x = 5$ Amp

Home work: Find the power supply by (7 Amp).

ans: $P_{in\ 7amp} = 511$ Watt.

$P_{1+2} = 5^2 \times 3 = 75$ Watt.

$P_{4\Omega} = (12)^2 \times 4 = 576$ Watt.

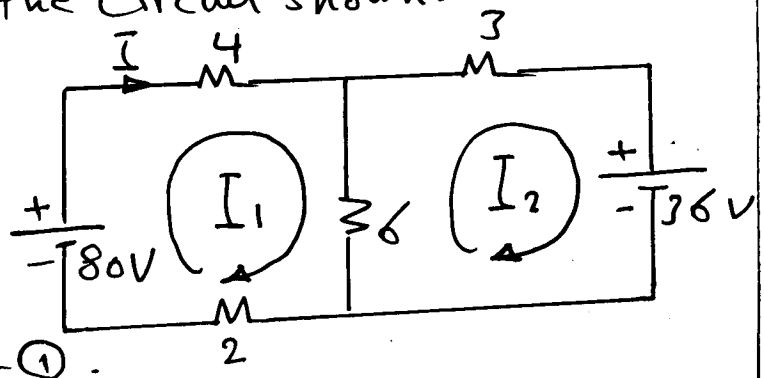
$P_{5\Omega} = 7^2 \times 5 = 245$ Watt.

$P_{53V} = V.I = 53 \times 5 = 265$ Watt.

$P_{in\ 2I_x} = 10 \times 12 = 120$ Watt.

(Mesh current (Loop Current) method)

Ex: Find the current (I) for the circuit shown.



* $I = I_1$

loop(1):

$80 = (4+6+2)I_1 - 6I_2$

$80 = 12I_1 - 6I_2$ ---------- (1).

loop(2):

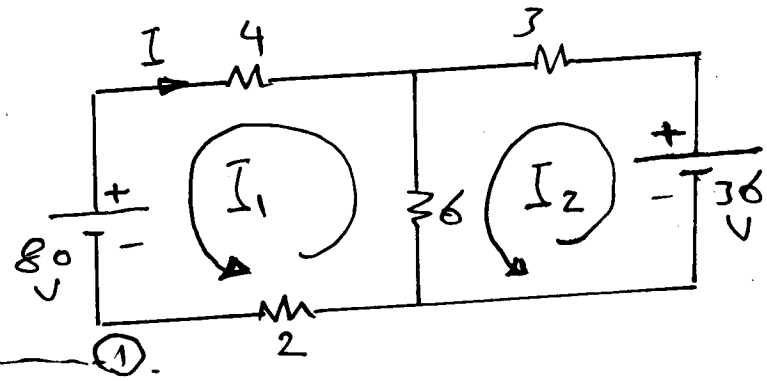
$-36 = (3+6)I_2 - 6I_1$

$-36 = 9I_2 - 6I_1$ ---------- (2).

Solving eq(1) & eq(2)

We get $I = I_1 = 7$ Amp.

OR $I_1 = -I$



loop(1):

$-80 = 12I_1 - 6I_2$

$-40 = 6I_1 - 3I_2$ ---------- (1).

loop(2):

$36 = 9I_2 - 6I_1$

$12 = 3I_2 - 2I_1$ ---------- (2)

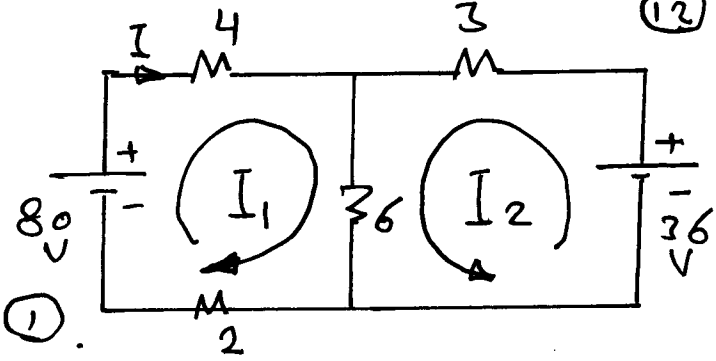
Solving eq(1) & eq(2).

We get $I_1 = -I = -7$ Amp.

$I = 7$ Amp.

(12)

*OR $I = I_1$



loop(1):

$80 = (4+6+2)I_1 + 6I_2$

$80 = 12I_1 + 6I_2$ ——— (1).

loop(2):

$36 = (3+6)I_2 + 6I_1$

$36 = 9I_2 + 6I_1$ ——— (2).

Solving eq(1) & eq(2) we get

$I = 7$ Amp.

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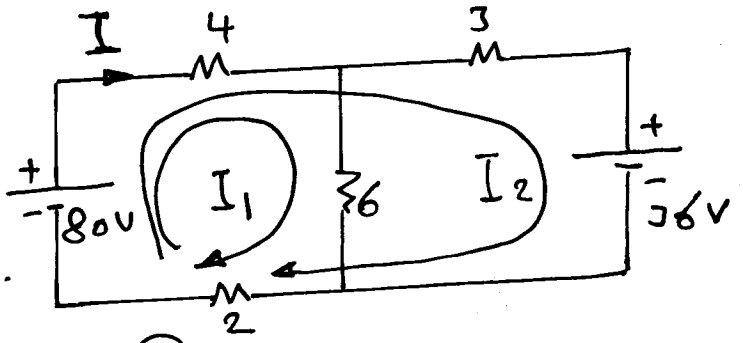
*OR $I = I_1 + I_2$.



loop(1):

$80 = (4+6+2)I_1 + (4+2)I_2$.

$80 = 12I_1 + 6I_2$ ——— (1).

loop(2):

$80 - 36 = (4+3+2)I_2 + (4+2)I_1$.

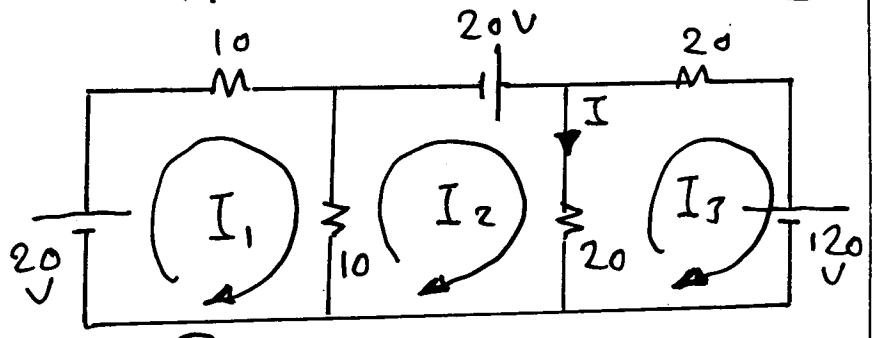
$44 = 9I_2 + 6I_1$ ——— (2).

Solving eq(1) & eq(2) we get:

$I_1 = \frac{76}{12}$ Amp , $I_2 = \frac{8}{12}$ Amp.

$\therefore I = 7$ Amp.

Ex. For the circuit shown find (I)?



$I = I_2 - I_3$

loop (1):

$20 = (10+10)I_1 - 10I_2$.

$20 = 20I_1 - 10I_2$ ---------- (1).

loop (2):

$20 = (20+10)I_2 - 10I_1 - 20I_3$

$20 = 30I_2 - 10I_1 - 20I_3$ ---------- (2).

loop (3):

$-120 = 40I_3 - 20I_2$ ---------- (3).

From eq(1) $I_1 = \frac{2 + I_2}{2}$

put the value of I_1 in eq(2).

$2 = 3I_2 - \left(\frac{2+I_2}{2}\right) - 2I_3$

$\therefore 6 = 5I_2 - 4I_3$ ---------- (4).

Solving eq(3) & eq(4), we get:

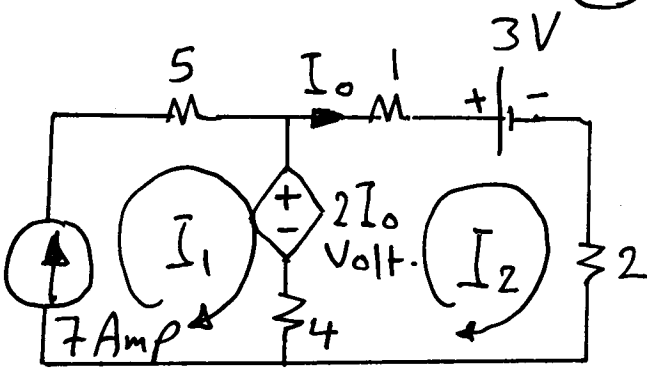
$I_3 = -4$ Amp.

$I_2 = -2$ Amp.

$\therefore I = I_2 - I_3 = \underline{2 \text{ Amp}}$.

Home work: Solve this example by Kirchhoff's Law.

Ex: Find (I_o) for the circuit?



$I_1 = 7$ Amp.

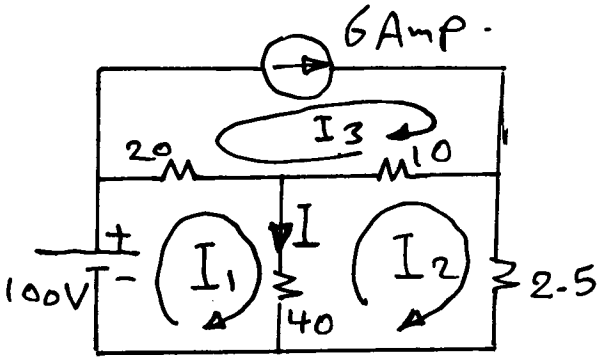
loop(2): $I_2 = I_o$

$2I_o - 3 = (1+2+4)I_o - 4I_1$

$25 = 7I_o - 2I_o$

$\therefore I_o = 25/5 = 5$ Amp

Ex: Find (I) for the circuit?



$I_3 = 6$ Amp.

loop(1):

$100 = 60I_1 - 40I_2 - 20I_3$ ------ (1)

loop(2):

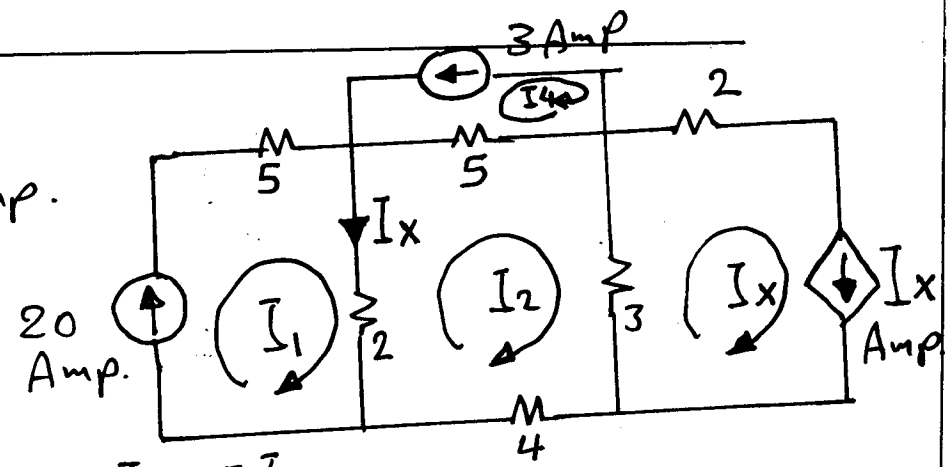
$0 = (52.5)I_2 - 40I_1 - 10I_3$ ------- (2).

and $I = I_1 - I_2$.

Solving eq(1) & eq(2). we get

$I = 1$ Amp

Ex: Find (I_x)?



$I_1 = 20A$, $I_4 = -3$ Amp.

$I_3 = I_x$.

loop(2):

$0 = (5+3+4+2)I_2 - 2I_1 - 3I_3 - 5I_4$.

$0 = 14I_2 - 2\times 20 - 3(20 - I_2) - 5(-3)$.

$0 = 17I_2 - 85$

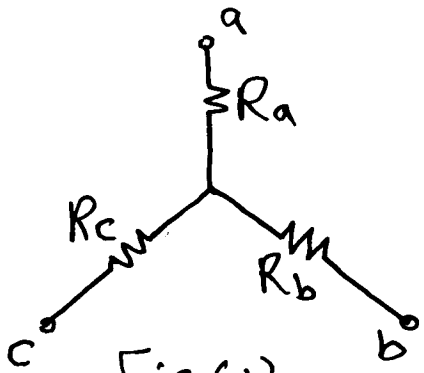
$\therefore I_2 = 5$ Amp.

$\therefore I_x = I_1 - I_2 = 20 - 5 = 15$ Amp.

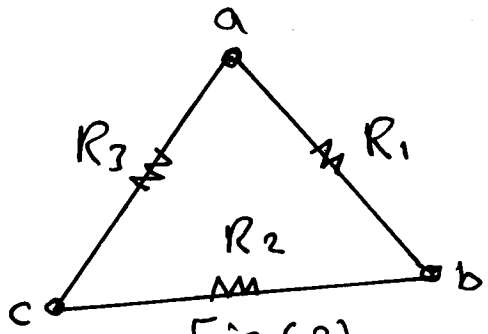
Star ⇌ Delta Transformation.

Y ⇌ Δ

If the three resistors are connected as shown in Fig(1) they are said to be connected as (Star or Y), and if they are connected as shown in fig(2) they are in (Delta or Δ).

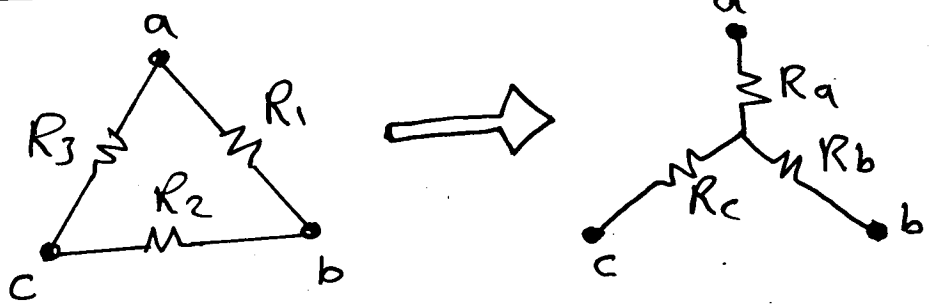


Fig(1)



Fig(2).

a) Delta ⟶ Star Transformation
Δ ⟶ Y Transformation



For Delta Connection:

$$R_{ab} = R_1 \,//\, (R_2 + R_3).$$

$$= \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3}.$$

$$= \frac{R_1 R_2 + R_1 R_3}{R_1 + R_2 + R_3}.$$

$$\therefore R_{bc} = \frac{R_1 R_2 + R_2 R_3}{R_1 + R_2 + R_3}.$$

and $R_{ca} = \frac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$.

For Star Connection:

$R_{ab} = R_a + R_b$.

$R_{bc} = R_b + R_c$.

$R_{ca} = R_c + R_a$.

Since the points a, b & c for both connection are the same, then.

$\therefore R_a + R_b = \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3}$ ---------- (1).

$R_b + R_c = \frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$ ---------- (2).

$R_c + R_a = \frac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$ ---------- (3).

** Subtract eq(2) from eq(1) we get:-

$R_a - R_c = \frac{R_1R_3 - R_2R_3}{R_1 + R_2 + R_3}$ ---------- (4).

** Add eq(3) with eq(4) we get:

$2R_a = \frac{2R_1R_3}{R_1 + R_2 + R_3}$

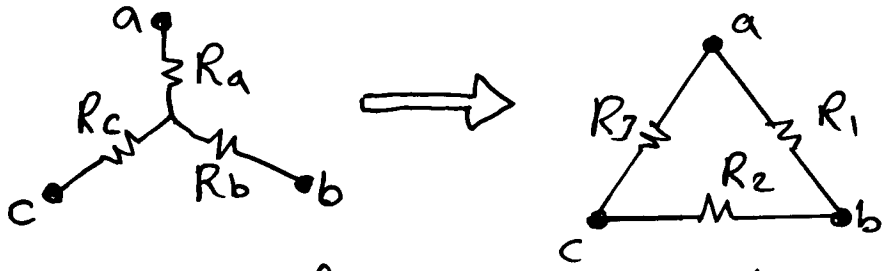
$\therefore \boxed{R_a = \frac{R_1R_3}{R_1 + R_2 + R_3}}$.

$\boxed{R_b = \frac{R_1R_2}{R_1 + R_2 + R_3}}$.

and $\boxed{R_c = \frac{R_2R_3}{R_1 + R_2 + R_3}}$

b) Star ⟶ Delta Transformation

Y ⟶ Δ



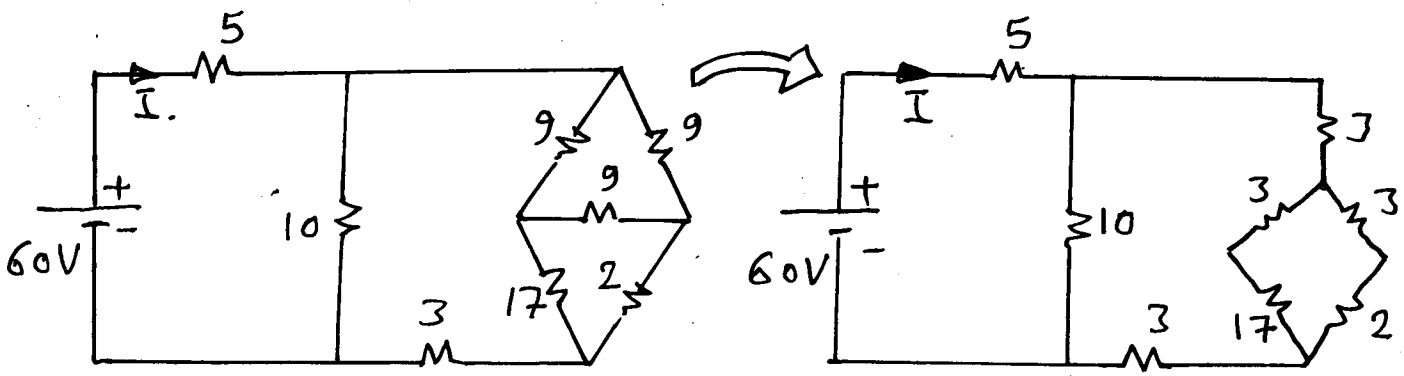
The expressions for the three Delta-connected resistors as function of the three Y-connected resistors are;

$$R_1 = \frac{R_a R_b + R_a R_c + R_b R_c}{R_c}$$

$$R_2 = \frac{R_a R_b + R_a R_c + R_b R_c}{R_a}$$

$$R_3 = \frac{R_a R_b + R_a R_c + R_b R_c}{R_b}$$

Ex: Find the current (I) for the circuit shown?



$$R_{eq} = \left(\left\{ \left[(3+17) /\!/ (3+2) \right] + 3 + 3 \right\} /\!/ 10 \right) + 5 .$$

$$= 10\,\Omega .$$

$$\therefore I = \frac{60}{10} = \underline{6\ \text{Amp}} .$$

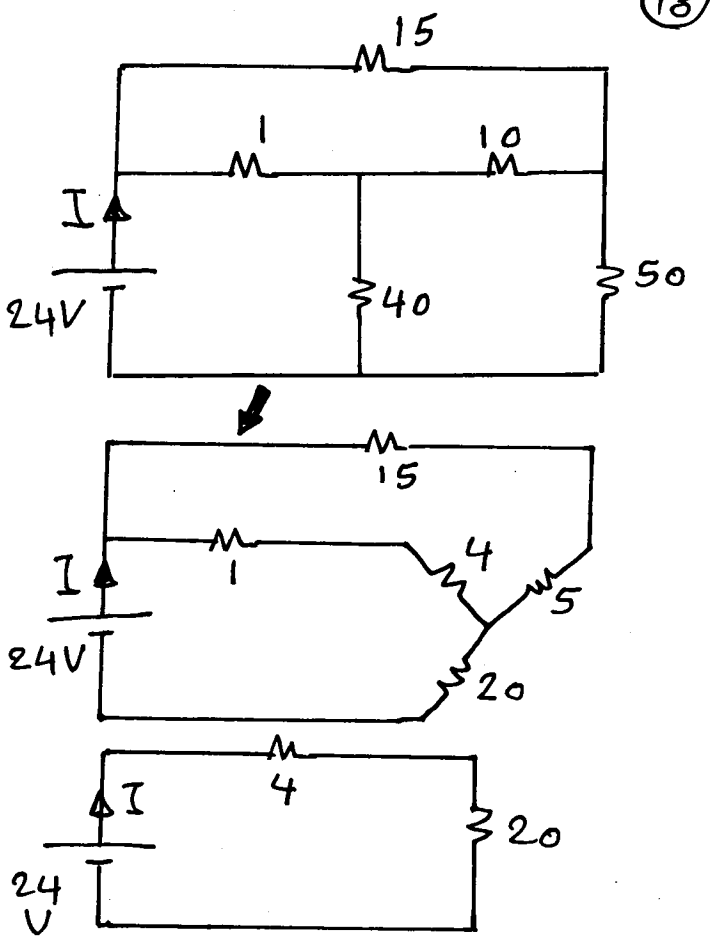
Ex: Use Δ-Y to find (I)?

change the Δ to Y resistance as shown.

$$R_{eq} = [(15+5)\,||\,(1+4)] + 20$$

$$= 4 + 20 = 24\,\Omega.$$

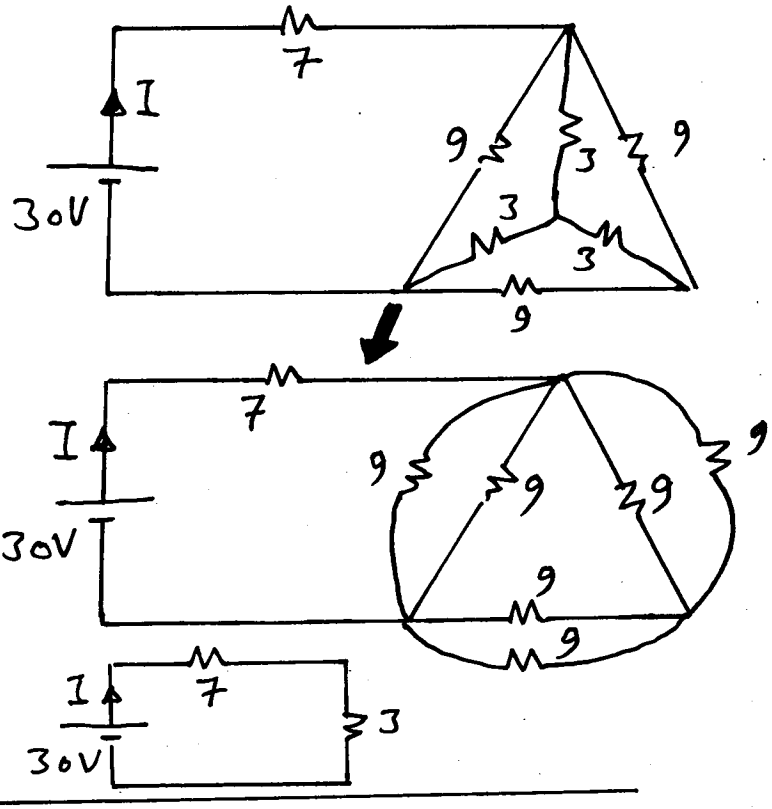
$$\therefore I = \frac{V}{R_{eq}} = \frac{24}{24} = 1\,\text{Amp}.$$



Ex: Find (I)?

change from Y to Δ.

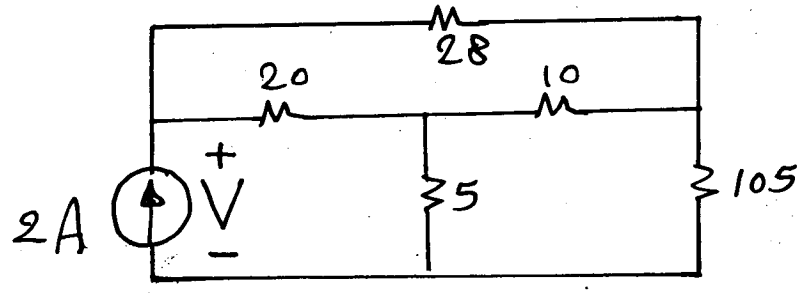
$$I = \frac{30}{10} = 3\,\text{Amp}.$$



Homework:

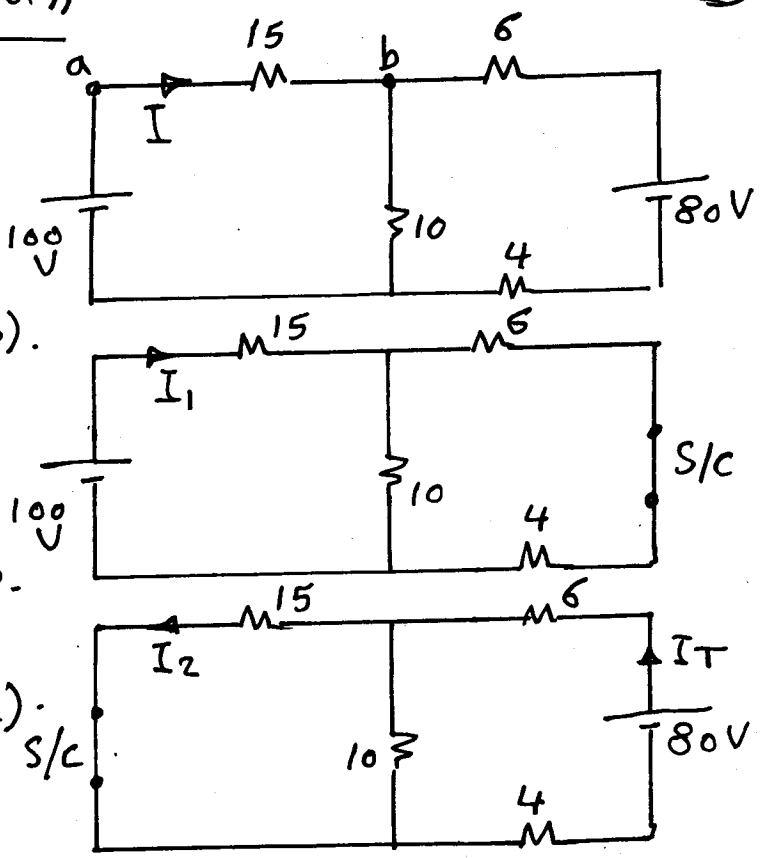
Find (V)?

ans: V = 35 Volt.



((Superposition method))

Ex: Find (I) ?



1) By effect of 100V → 80V (S/c).

$$I_1 = \frac{100}{15+(10//10)} = 5\,Amp\,(a \to b).$$

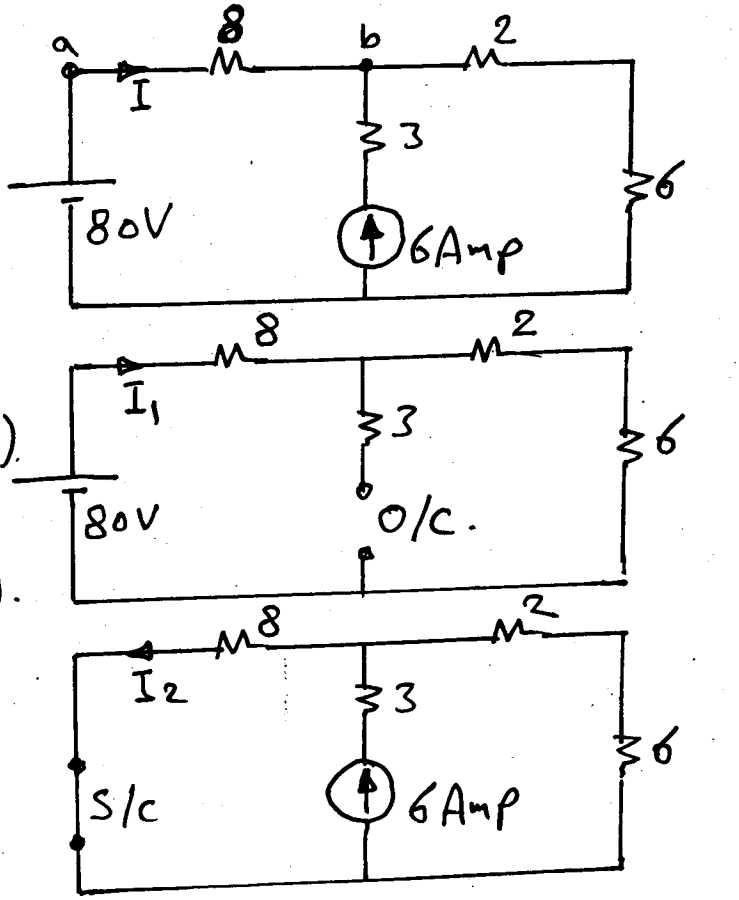
2) By effect of 80V → 100V (S/c).

$$I_T = \frac{80}{6+4+(15//10)} = \frac{80}{16} = 5\,Amp.$$

$$\therefore I_2 = 5\,\frac{10}{10+15} = 2\,Amp\,(b \to a).$$

$$\therefore I = I_1 - I_2 = 5-2 = \underline{3\,Amp}$$

Ex: Find (I) ?



1) By effect of 80V → 6A (O/c).

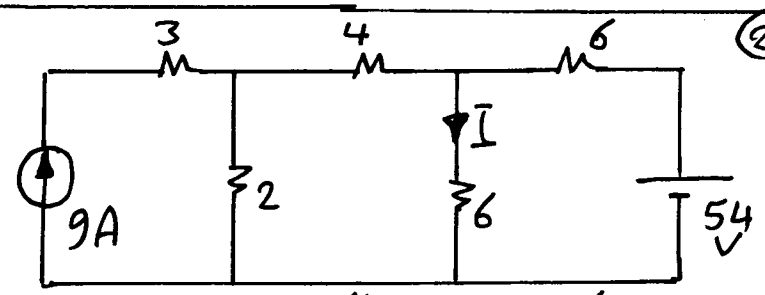
$$I_1 = \frac{80}{16} = 5\,Amp\,(a \to b).$$

2) By effect of 6Amp → 80V (S/c).

$$I_2 = 6\,\frac{8}{8+8} = 3\,Amp\,(b \to a).$$

$$\therefore I = I_1 - I_2 = 5-3 = \underline{2\,Amp}.$$

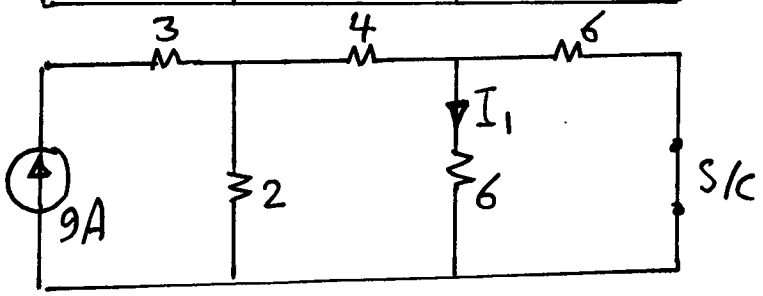
Ex: find (I)?



1) By effect of 9A → 54V (S/C).

$I_{4\Omega} = 9\,\frac{2}{2+7} = 2\,Amp.$

$\therefore I_1 = 2\,\frac{6}{6+6} = 1\,Amp.$

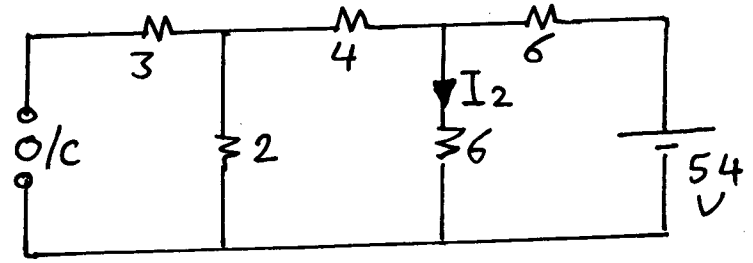


2) By effect of 54V → 9A (O/C).

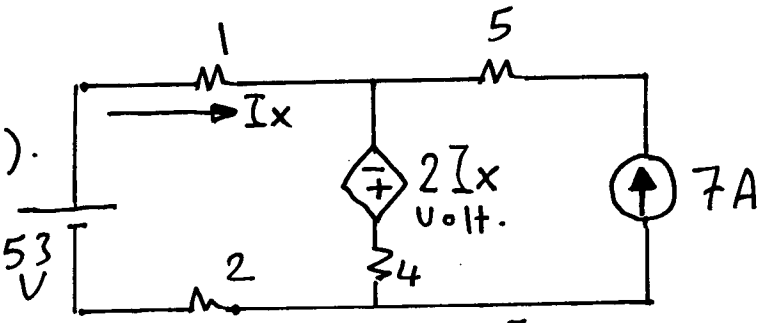
$I_T = \frac{54}{6+(6\|6)} = 6\,Amp.$

$I_2 = 6\,\frac{6}{6+6} = 3\,Amp.$

$\therefore I = I_1 + I_2 = \underline{4\,Amp}$



Ex: Find (Ix)?

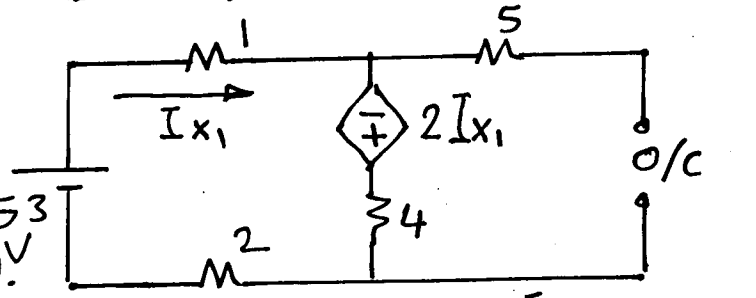


1) By effect of 53V → 7A (O/C).

$I_{x_1} = \frac{53 + 2I_{x_1}}{1+2+4}$

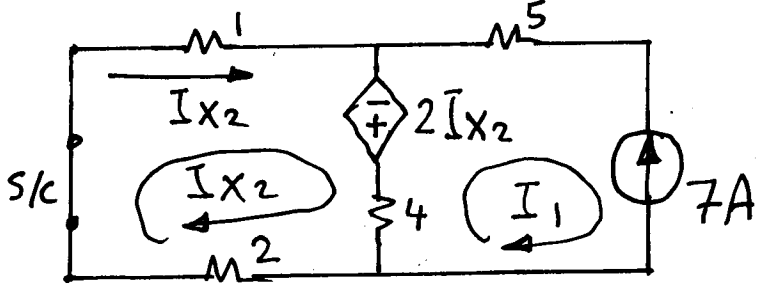
$7I_{x_1} = 53 + 2I_{x_1}$

$\therefore I_{x_1} = \frac{53}{5}\,Amp.$



2) By effect of 7Amp → 53V (S/C).

loop (I_{x_2}):



$2I_x = 7I_{x_2} - 4I_1$

$2I_{x_2} = 7I_{x_2} - 4(-7).$

$-5I_{x_2} = 28$

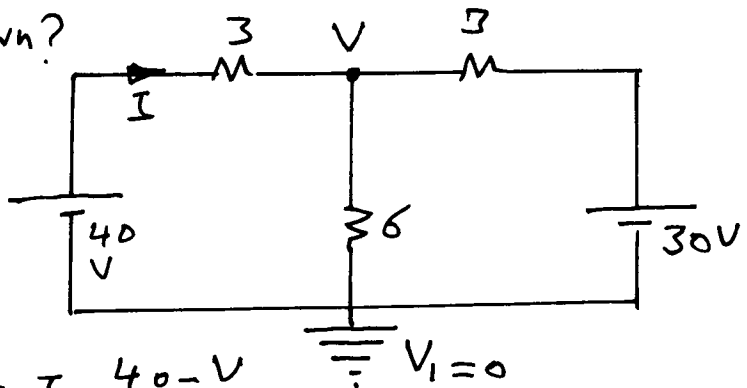
$\therefore I_{x_2} = -\frac{28}{5}\,Amp.$

$I_1 = -7\,Amp.$

$\therefore I_x = I_{x_1} + I_{x_2} = \frac{53}{5} + \left(-\frac{28}{5}\right) = \frac{25}{5} = \underline{5\,Amp.}$

(Nodal Voltage method)

Ex: Find (I) for the circuit shown?



$$\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{6}\right)V - \frac{40}{3} - \frac{30}{3} = 0$$

$$(2+2+1)V - 80 - 60 = 0$$

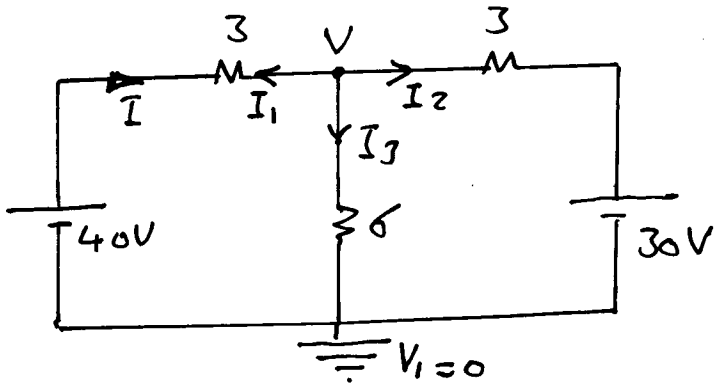
$$5V = 140$$

$$\therefore V = 28 \text{ volt}.$$

But $I = \frac{40-V}{3}$

$$\therefore I = \frac{40-28}{3} = 4 \text{ Amp}.$$

OR



By K.C.L

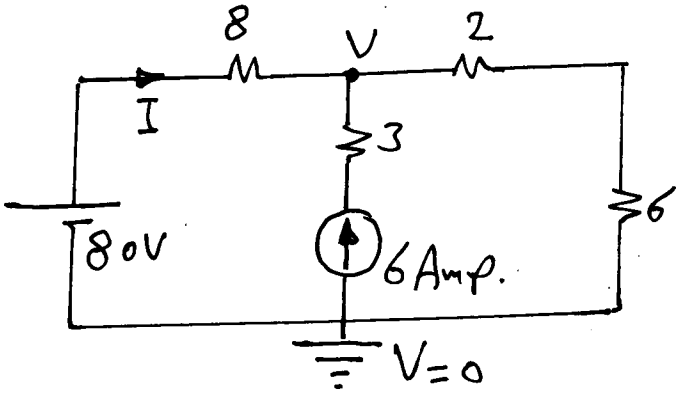
$$I_1 + I_2 + I_3 = 0$$

$$\frac{V-40}{3} + \frac{V-30}{3} + \frac{V}{6} = 0$$

$$\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{6}\right)V - \frac{40}{3} - \frac{30}{3} = 0.$$

$$\therefore V = 28 \text{ volt}.$$

Ex Find (I)?



Node (V):

$$\left(\frac{1}{8}+\frac{1}{8}\right)V - \frac{80}{8} = 6$$

$$(1+1)V - 80 = 48.$$

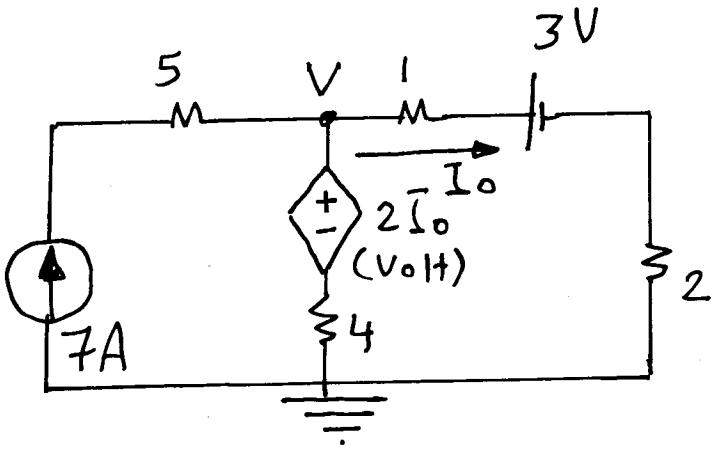
$$2V = 128.$$

$$V = 64 \text{ Volt}.$$

But $I = \frac{80-V}{8}$.

$$\therefore I = \frac{80-64}{8} = 2 \text{ Amp}.$$

Ex: Find (I_o)?



Node(V)

$$\left(\frac{1}{3}+\frac{1}{4}\right)V-\frac{2I_o}{4}-\frac{3}{3}=7.$$

$$(4+3)V-6I_o-12=84.$$

$$7V-6I_o=96$$

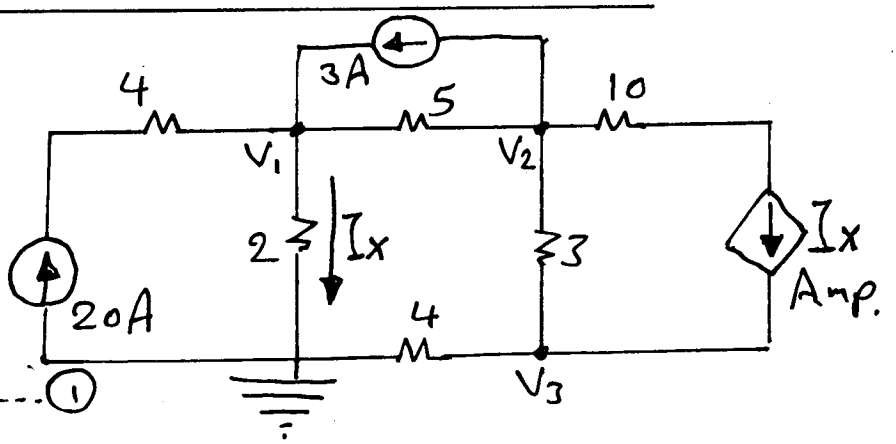
$$7V-2V+6=96$$

$$\therefore V=18 \text{ Volt}.$$

But $I_o=\dfrac{V-3}{3}$.

$$\therefore I_o=\frac{18-3}{3}=5 \text{ Amp}.$$

Ex: Find (I_x)?



Node(V_1):

$$\left(\frac{1}{2}+\frac{1}{5}\right)V_1-\frac{V_2}{5}=23$$

$$7V_1-2V_2=230 \text{ ----- } (1)$$

Node(V_2):

$$\left(\frac{1}{3}+\frac{1}{5}\right)V_2-\frac{V_1}{5}-\frac{V_3}{3}=-3-I_x$$

But $I_x=\dfrac{V_1}{2}$.

$$\therefore 16V_2+9V_1-10V_3=-90 \text{ ------ } (2)$$

Node(V_3):

$$\left(\frac{1}{3}+\frac{1}{4}\right)V_3-\frac{V_2}{3}=I_x \text{ ------ } (3)$$

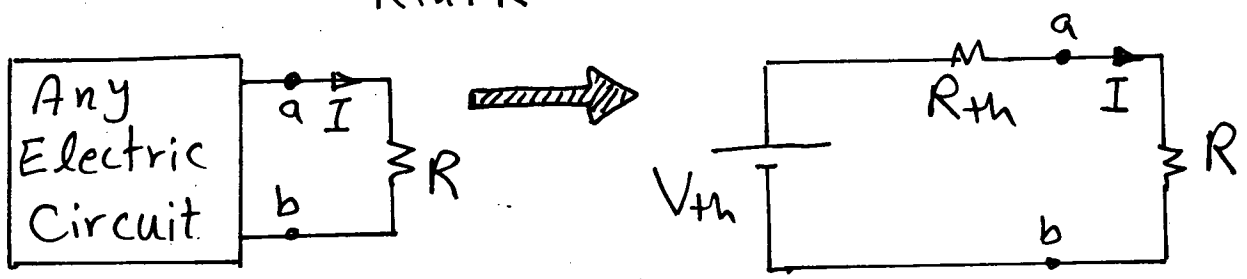
Solving: $V_1=30$ Volt.

$$\therefore I_x=\frac{30}{2}=15 \text{ Amp}.$$

((Thevenin's Theorem))

This theory states as ((The current (I) passing through a resistance (R) of any electric circuit is

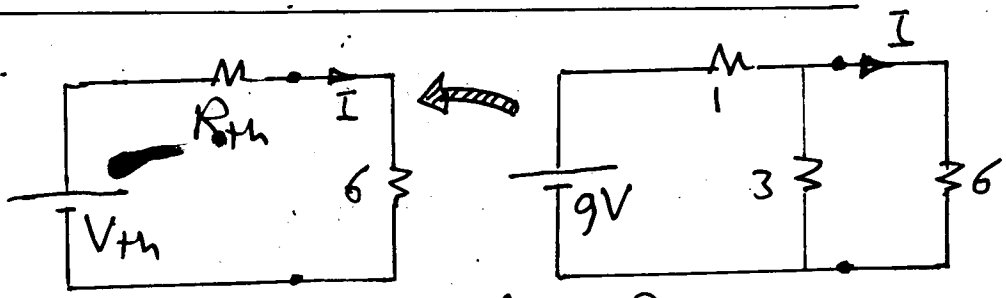
$$I = \frac{V_{th}}{R_{th} + R} \cdot))$$



Where: V_{th}: The voltage between a & b with (R) disconnected.

R_{th}: The circuit resistance between a & b with (R) also disconnected.

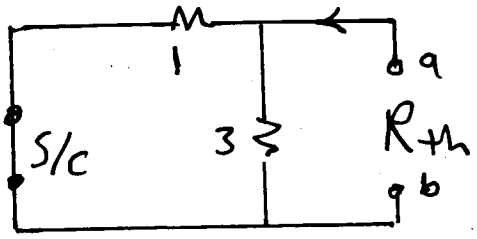
Ex: Find (I)?



$$I = \frac{V_{th}}{R_{th} + 6}$$

To find Rth?

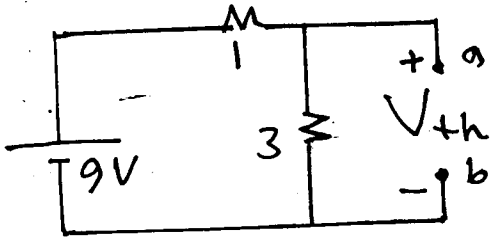
* $R_{th} = 1 \,||\, 3 = \frac{1 \times 3}{1+3} = \frac{3}{4}\ \Omega$.



To find Vth?

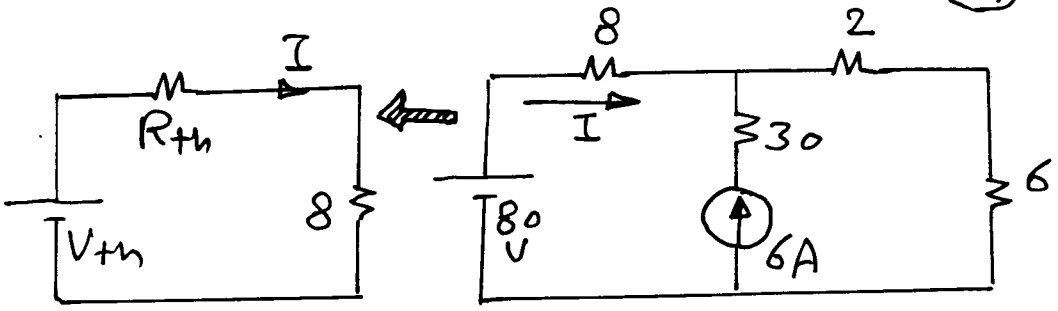
* $V_{th} = V_{3\Omega} = \frac{9}{4} \times 3 = \frac{27}{4}$ Volt.

$$\therefore I = \frac{27/4}{\frac{3}{4} + 6} = \frac{27/4}{27/4} = 1\ \text{Amp}.$$

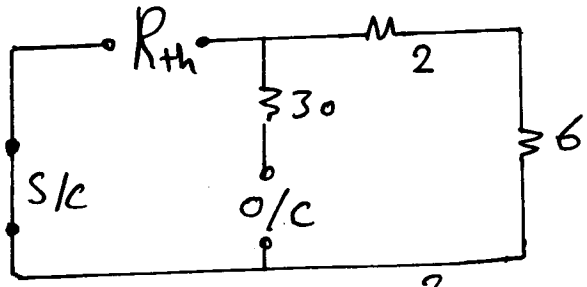


Ex: Find (I)?

$$I = \frac{V_{th}}{R_{th}+8}.$$



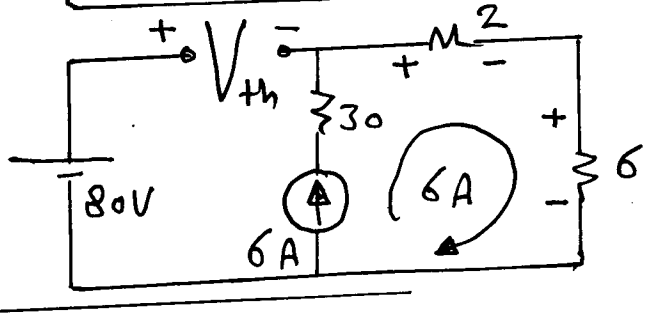
* $R_{th} = 2 + 6 = 8\ \Omega$.



* $V_{th} = 80 - V_{2\Omega} - V_{6\Omega}$.

$= 80 - 2\times6 - 6\times6$.

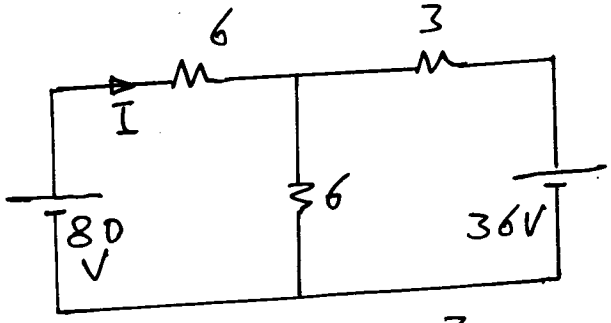
$= 32$ Volt.



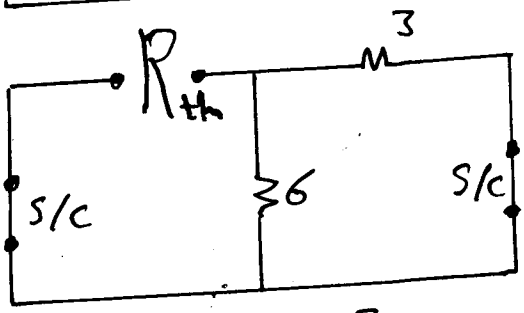
$\therefore I = \frac{32}{8+8} = \underline{2\ \text{Amp}}$.

Ex: Find (I)?

$$I = \frac{V_{th}}{R_{th}+6}.$$

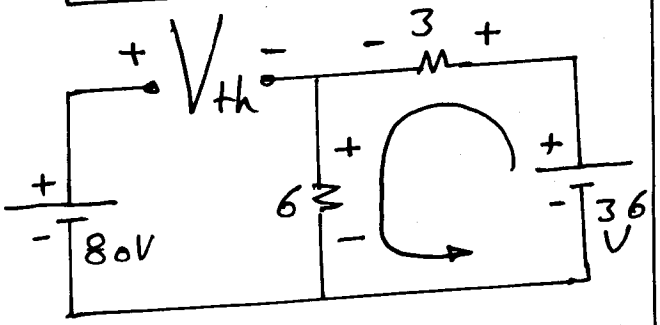


* $R_{th} = 3 // 6 = 2\ \Omega$.



* $V_{th} = 80 - V_{6\Omega}$.

$= 80 - \frac{36}{3+6}\times6 = 56$ Volt.



OR $V_{th} + 36 = 80 + V_{3\Omega}$.

$\therefore V_{th} = 80 - 36 + \frac{36}{9}\times3$

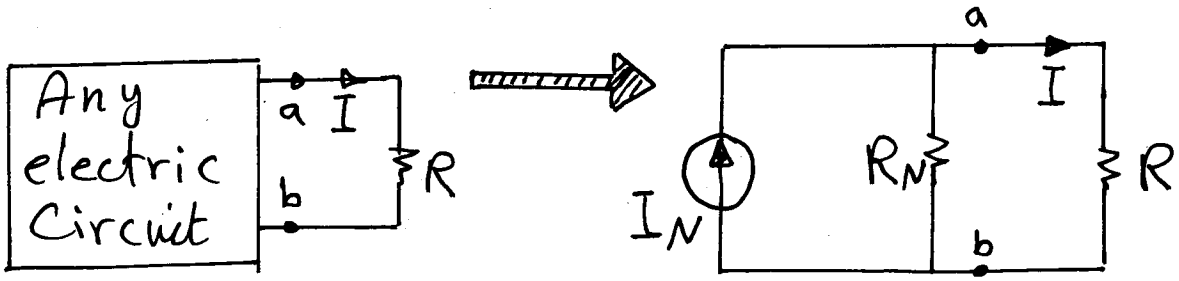
$= 56$ Volt.

$\therefore I = \frac{56}{2+6} = \underline{7\ \text{Amp}}$.

((Norton Theory))

This theory states as ((The current (I) passing through a resistance (R) of any electric circuit is

$$I = I_N \frac{R_N}{R_N + R} \text{ .)) .}$$



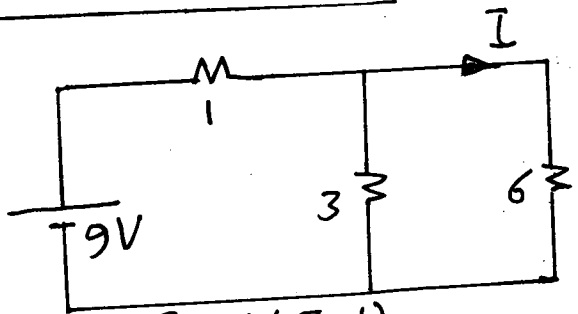
Where:

I_N : (The current passing through a & b after replacing the resistor (R) by a short-circuit)

R_N : The circuit resistance between a & b with (R) disconnected.

$$\boxed{R_N = R_{th}}$$

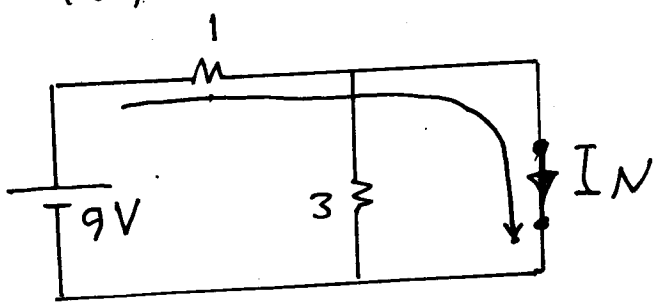
Ex:



To find (I_N) .

$$I = I_N \frac{R_N}{R_N + 6} \text{ .}$$

* $R_N = R_{th} = 1 \| 3 = 3/4\ \Omega$.

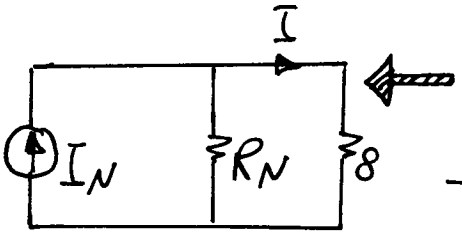


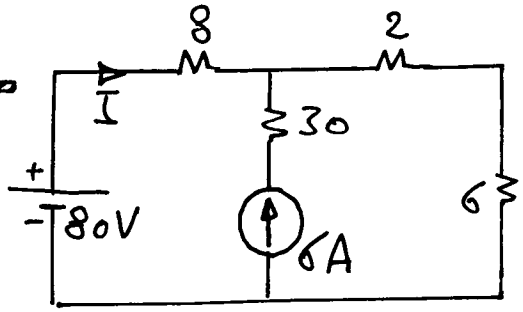
* $I_N \longrightarrow$

$$I_N = \frac{9}{1} = 9 \text{ Amp .}$$

$$\therefore I = 9 \frac{3/4}{3/4 + 6} = \frac{27/4}{27/4} = 1 \text{ Amp .}$$

Ex: Find (I)?

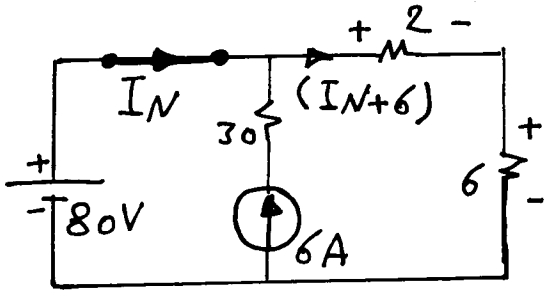




$$I = I_N \frac{R_N}{R_N + 8}.$$

* $R_N = R_{th} = 2 + 6 = 8\ \Omega$.

* $I_N \longrightarrow$



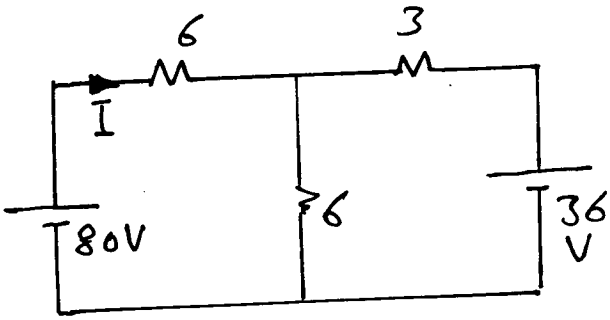
$80 = 2(I_N + 6) + 6(I_N + 6)$.

$32 = 8 I_N$

$\therefore I_N = 4$ Amp.

$\therefore I = 4 \frac{8}{8+8} = \underline{2}$ Amp.

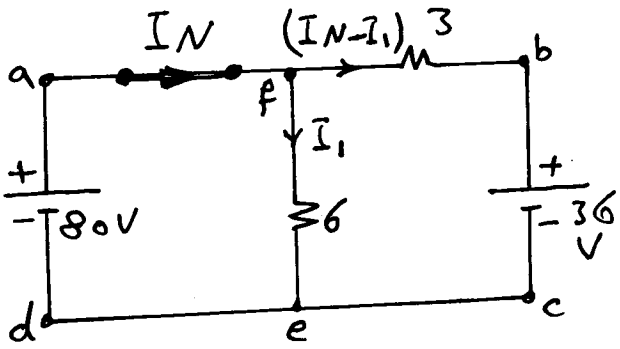
Ex: Find (I)?



$$I = I_N \frac{R_N}{R_N + 6}.$$

* $R_N = R_{th} = 3 // 6 = 2\ \Omega$.

* $I_N \longrightarrow$



for circuit abcd

$80 = 3(I_N - I_1) + 36$

$44 = 3 I_N - 3 I_1$ ---------- (1).

for circuit afed

$80 = 6 I_1$ ---------- (2)

$\therefore I_1 = 80/6$

from eq (1):

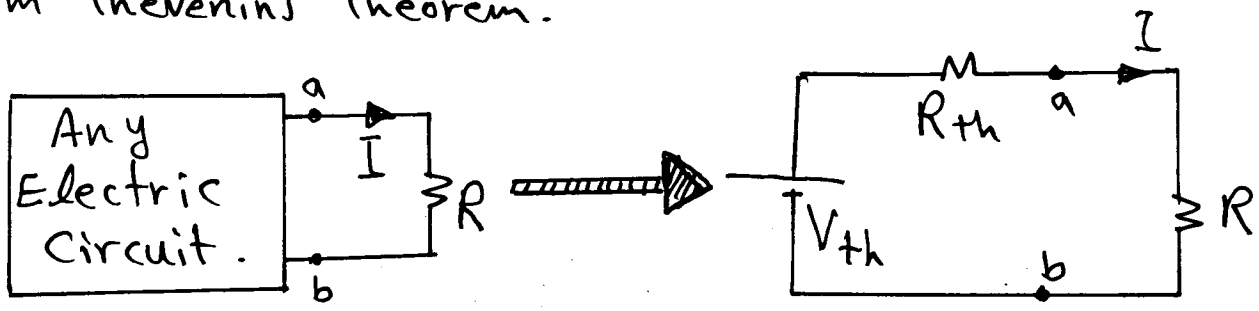
$44 = 3 I_N - 3 \left(\frac{80}{6}\right)$.

$\therefore I_N = 28$ Amp.

$\therefore I = 28 \frac{2}{2+6} = \underline{7}$ Amp.

Voltage Sources ⇌ Current Sources Transformation
V.S ⇌ C.S Transformations

* From Thevenin's Theorem.



$$I = \frac{V_{th}}{R_{th}+R} \quad \text{---------- (1)}$$

* From Norton's Theorem.

$$I = I_N \frac{R_N}{R_N+R} \quad \text{---------- (2)}$$

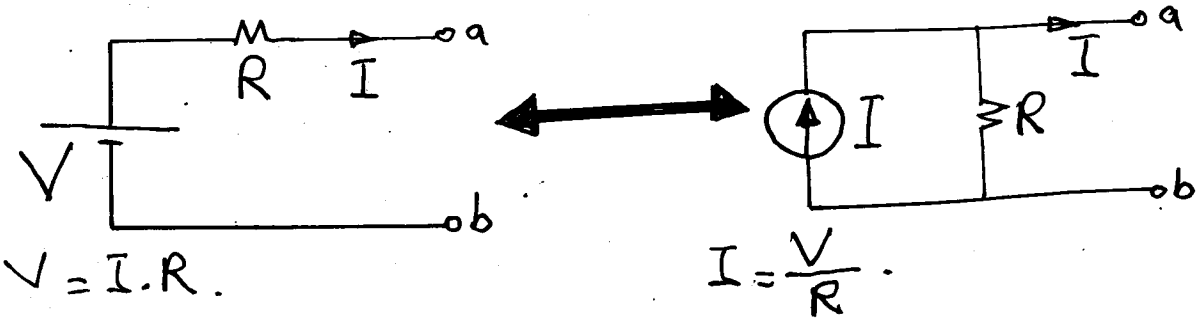
** Since $R_{th} = R_N$

∴ From eq(1) & eq(2).

$$V_{th} = I_N R_N$$

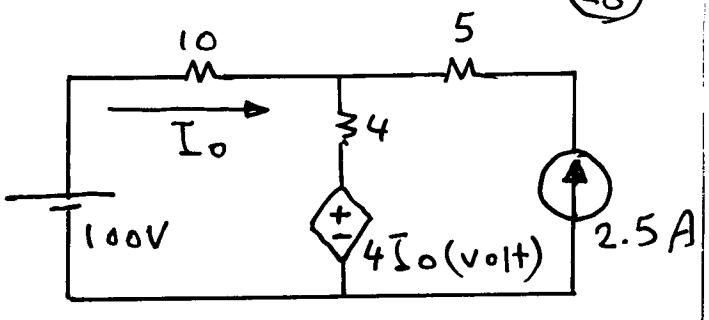
$$\therefore \boxed{R_{th} = R_N = \frac{V_{th}}{I_N}}$$

* In General:



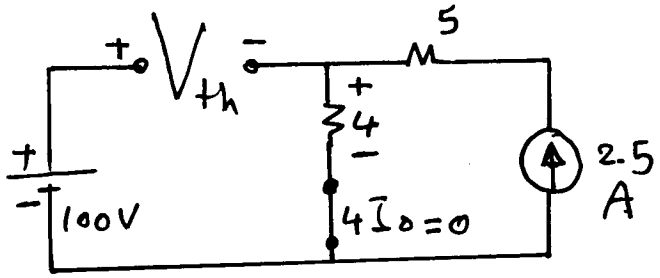
$V = I.R.$ $\qquad I = \frac{V}{R}.$

Ex: Find (I_o) By using Thevenin's Theory?



$R_{th} = R_N = \frac{V_{th}}{I_N}$.

$\therefore V_{th} = 100 - 4 \times 2.5 = 90V$.



$I_N \longrightarrow$

$I_N = I_1$

$I_2 = -2.5A$.

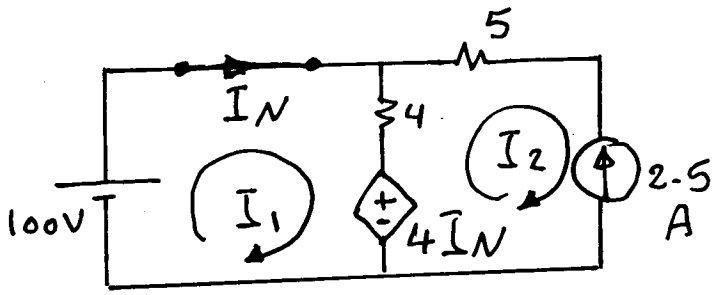
loop (1):

$100 - 4I_N = 4I_N - 4(-2.5)$.

$90 = 8I_N$.

$\therefore I_N = 90/8$.

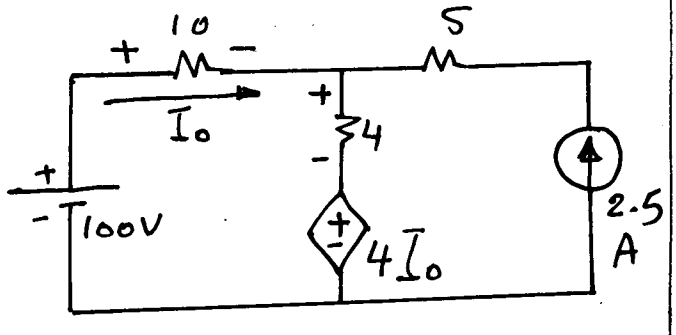
$\therefore R_{th} = R_N = 8\Omega$.



$I = \frac{V_{th}}{R_{th}+10} = \frac{90}{8+10} = 5$ Amp.

OR $I = I_N \frac{R_N}{R_N+10} = \frac{90}{8} \times \frac{8}{10+8} = 5$ Amp.

Solve the Same Ex by Kirchhoff's Law.



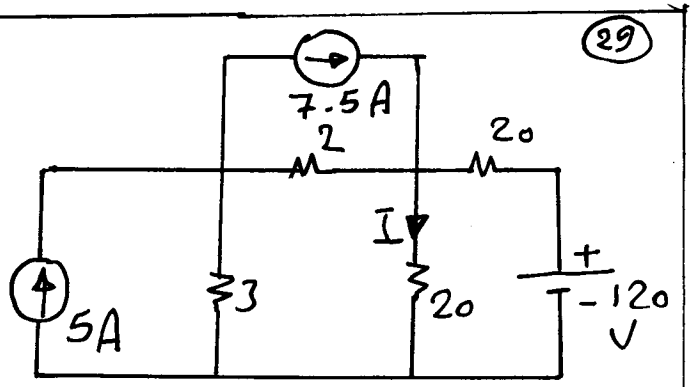
a b c d

$100 = 10I_o + 4(I_o + 2.5) + 4I_o$

$90 = 18I_o$.

$\therefore I_o = 5$ Amp.

Ex: Find the current (I)?



*By Node method

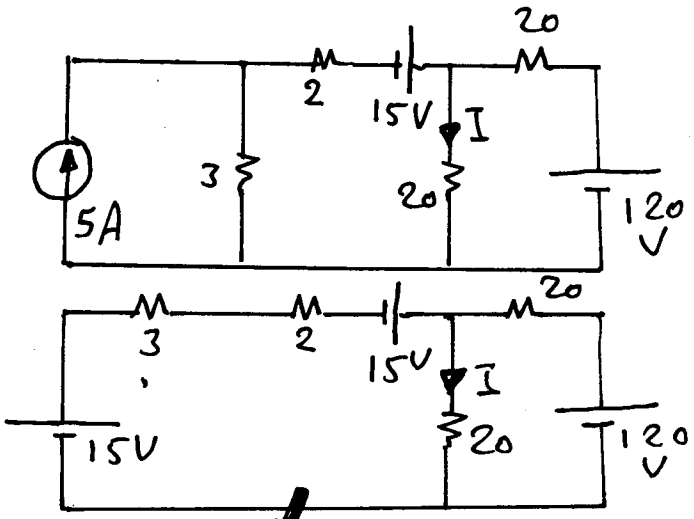
$$\left(\frac{1}{20}+\frac{1}{20}+\frac{1}{5}\right)V-\frac{30}{5}-\frac{120}{20}=0.$$

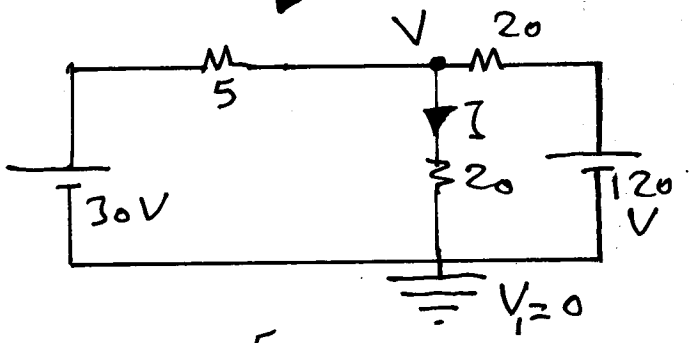
$$6V-120-120=0.$$

$$6V=240.$$

$$\therefore V=40 \text{ volt}.$$

$$\therefore I=\frac{V}{20}=\frac{40}{20}=\underline{2 \text{ Amp}}.$$





*By Thevenin's Theory:

$$R_{th}=5//20=4\,\Omega.$$

*V_{th} ⟶

$$I=\frac{120-30}{25}=\frac{90}{25} \text{ Amp}.$$

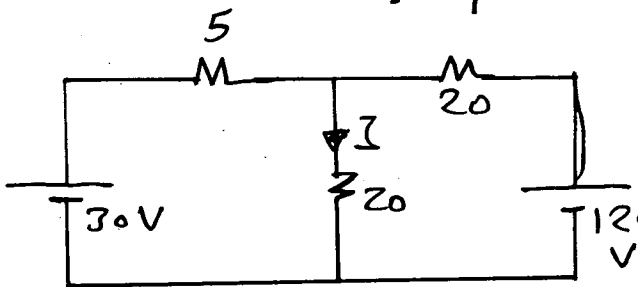
$$\therefore V_{th}=30+V_{5\Omega}.$$

$$=30+\frac{90}{25}\times 5=48 \text{ Volt}.$$

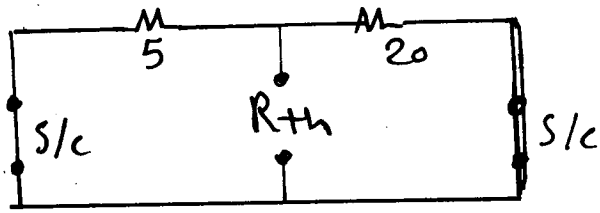
OR $V_{th}=120-V_{20\Omega}.$

$$=120-\frac{90}{25}\times 20=48 \text{ Volt}.$$

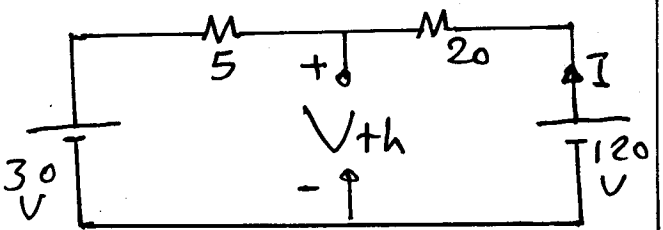
$$\therefore I=\frac{V_{th}}{R_{th}+20}=\frac{48}{4+20}=\underline{2 \text{ Amp}}$$



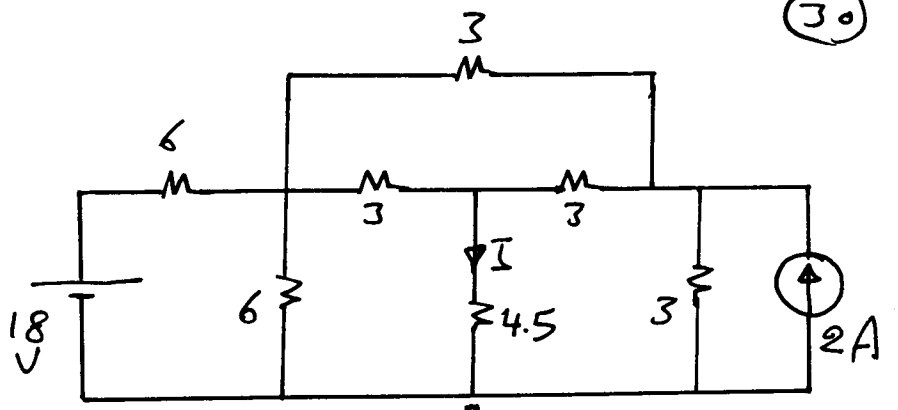
To find R_{th}



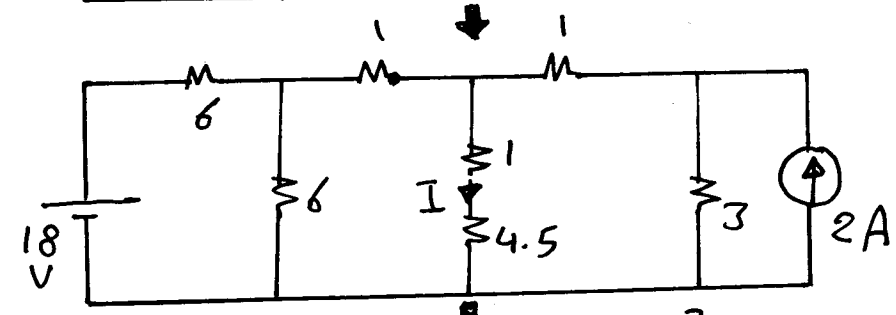
To find V_{th}

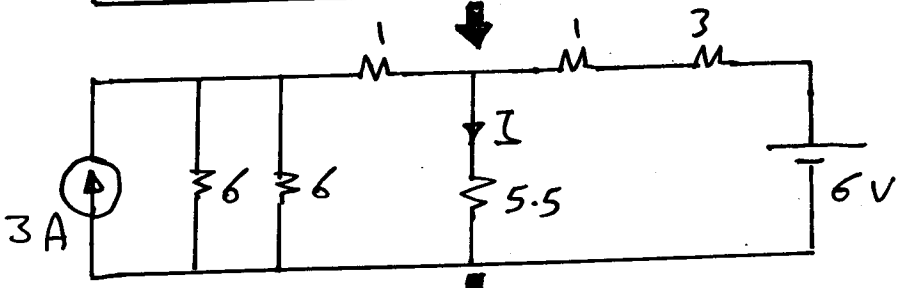


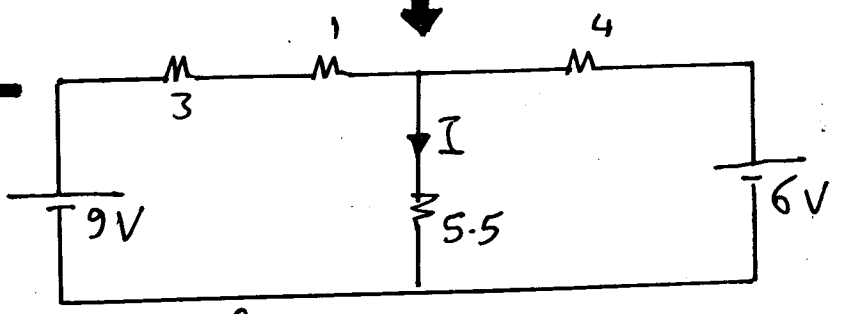
Ex: Find (I)?

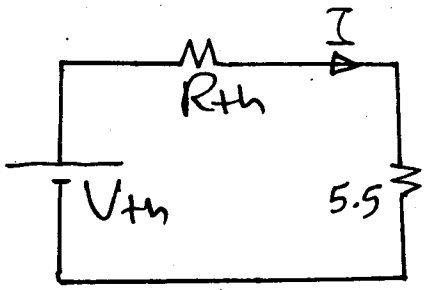


By Thevenin's



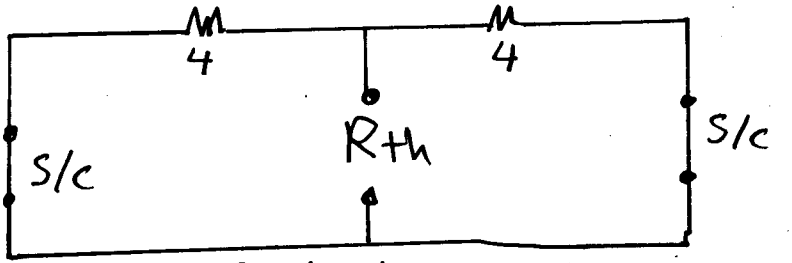






$$I = \frac{V_{th}}{R_{th}+5.5}.$$

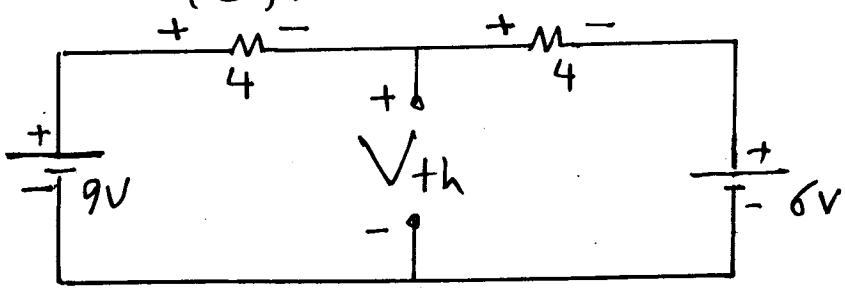
To find Rth



* $R_{th} = 4 \| 4 = 2\,\Omega$.

* $V_{th} \longrightarrow$

To find Vth

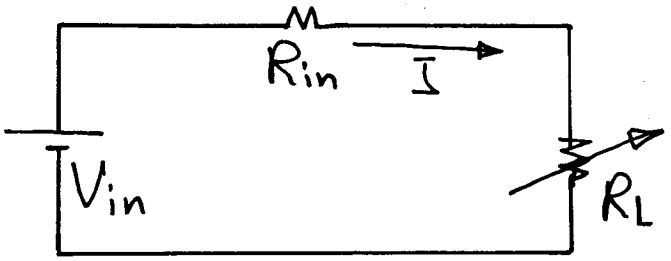


$V_{th} = 9 - V_{4\Omega}$.

$= 9 - \frac{3}{8} \times 4 = 7.5$ Volt.

$\therefore I = \frac{7.5}{5.5+2} = 1$ Amp.

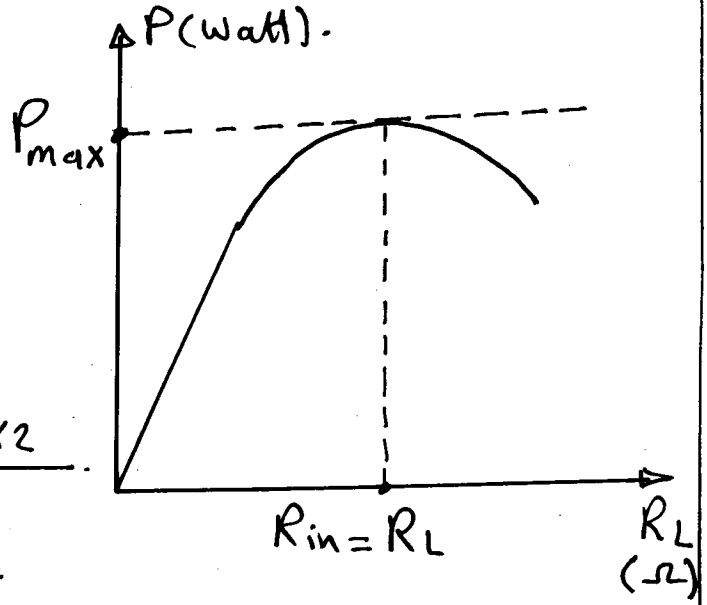
(Maximum power Transfer)



The power delivered to the load $(R_L) = I^2 . R_L$.

But $I = \frac{V_{in}}{R_{in}+R_L}$.

$$\therefore \boxed{P_L = \left[\frac{V_{in}}{(R_{in}+R_L)}\right]^2 \times R_L}$$



Since $P = \frac{V_{in}^2}{(R_{in}+R_L)^2} \times R_L$.

$$\frac{dP_{max}}{dR_L} = \frac{(R_{in}+R_L)^2 - R_L(R_{in}+R_L)\times 2}{(R_{in}+R_L)^4}.$$

But $\frac{dP_{max}}{dR_L} = 0$.

$\therefore (R_{in}+R_L)^2 - 2R_L(R_{in}+R_L) = 0$.

$(R_{in}+R_L)[(R_{in}+R_L) - 2R_L] = 0$.

$\therefore (R_{in}+R_L)(R_{in}-R_L) = 0$.

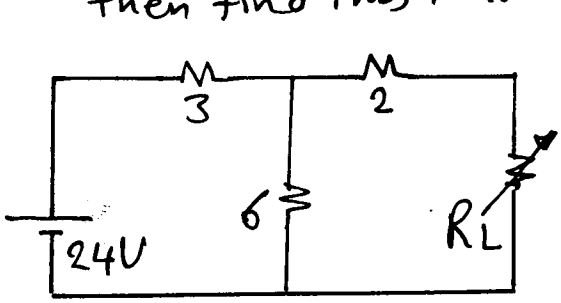
* either $R_{in} = -R_L$ (neglected).
* or $\boxed{R_{in} = R_L}$

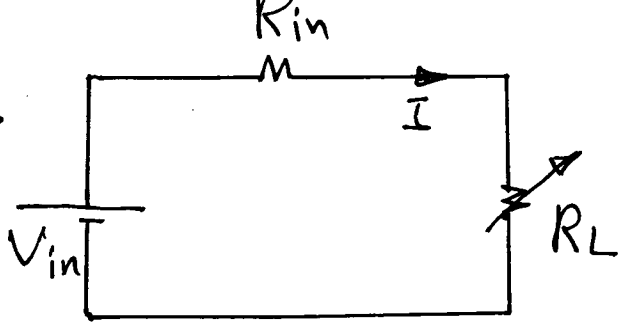
$\therefore$ Maximum power transfer takeplace only when

$$R_L = R_{in}$$

and $R_L = R_{in} = R_{th}$

Ex: Find the value of (R_L) for maximum power to (R_L), and then find this max. power.





at maximum power.

$R_L = R_{in} = R_{th}$

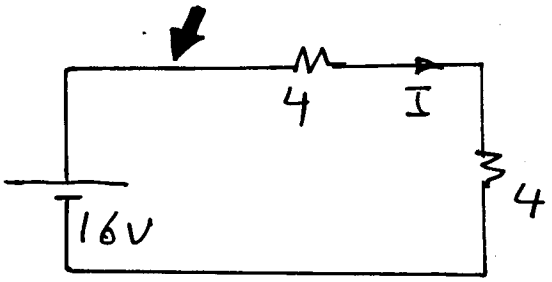
$= (3 // 6) + 2 = \underline{4\,\Omega}$.

$\therefore V_{in} = V_{th} = V_{6\Omega}$.

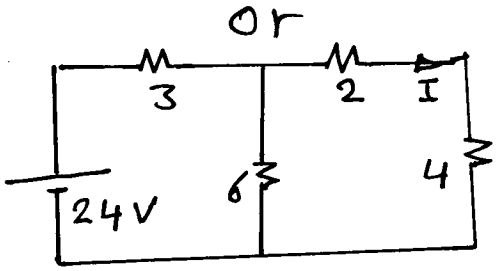
$= \frac{24}{3+6} \times 6 = 16$ Volt.

and $I = \frac{V_{th}}{R_{th}+4} = \frac{16}{4+4} = 2$ Amp.

and $P = I^2 \cdot R = (2)^2 \times 4 = \underline{16\ \text{Watt}}$



Or



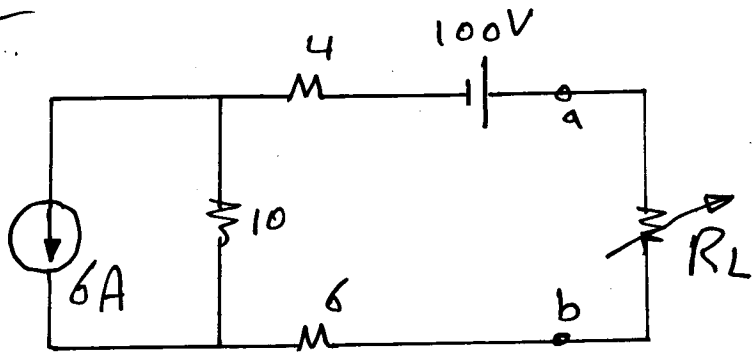
Ex: Find (R_L) for max. power

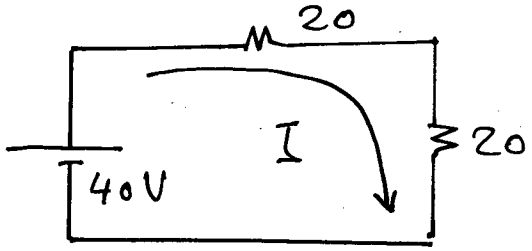
$R_L = R_{th} = 6 + 10 + 4 = \underline{20\,\Omega}$

$V_{in} = V_{th} = V_{ab} = 100 - 60$

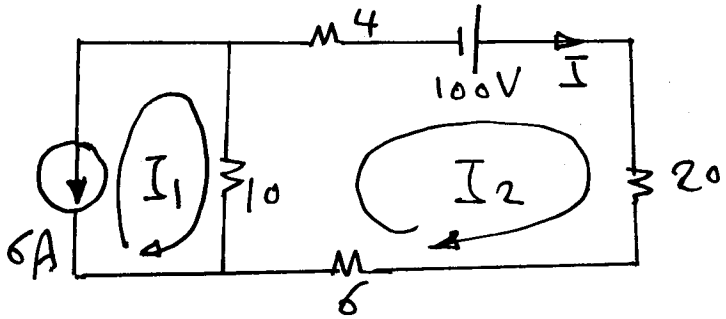
$= 40$ volt.

$\therefore I = \frac{40}{20+20} = 1$ Amp.





By loop method.



OR

loop(1) $\rightarrow I_1 = -6$ A.

loop(2)

$100 = (20+4+6+10) I_2 - 10 I_1$

$100 = 40 I_2 - 10(-6)$.

$40 = 40 I_2$.

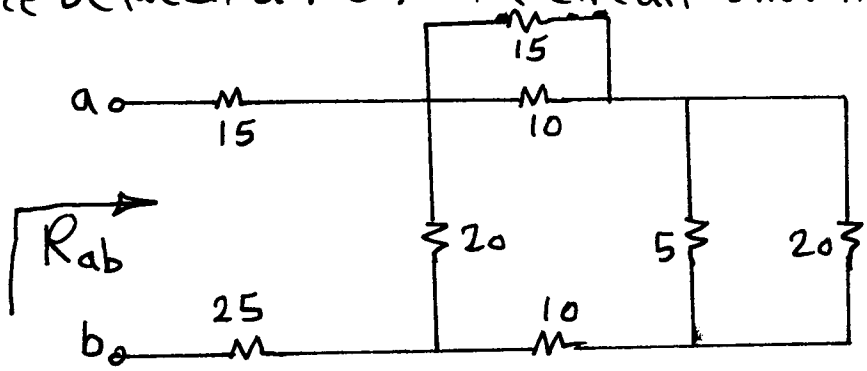
$\therefore I_2 = I = \underline{1\ \text{Amp}}$

Q1 Find the resistance between a & b for the circuit shown?

Solution:

$5 // 20 = \frac{5 \times 20}{5+20} = 4\,\Omega$.

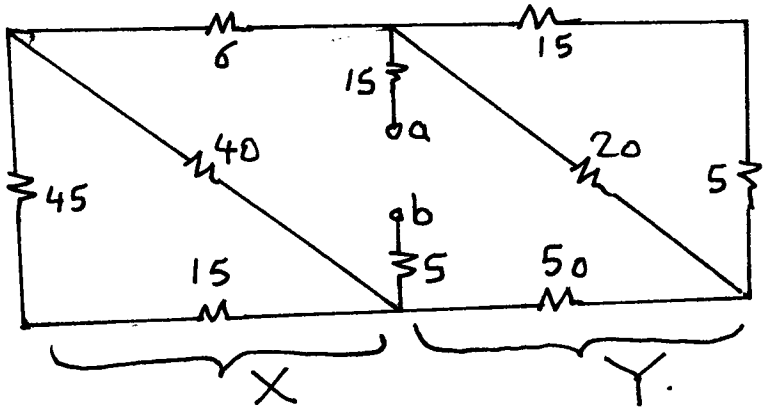
$10 // 15 = \frac{10 \times 15}{10+15} = 6\,\Omega$.



$\therefore R_{ab} = 15 + (20 // (4+6+10) + 25 = \underline{50\,\Omega}$.

Q2 Find the resistance between a & b for the Circuit shown?

Solution:



$X = [(45+15) // 40] + 6$

$= (60 // 40) + 6 = 30\,\Omega$.

$Y = [(15+5) // 20] + 50$

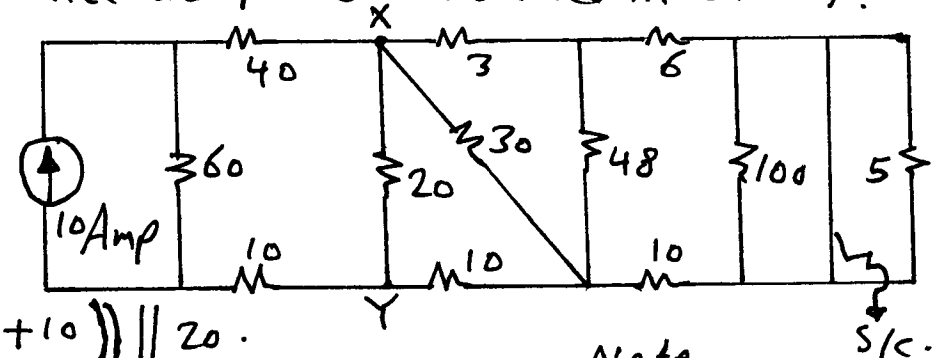
$= (20 // 20) + 50 = 60\,\Omega$.

$X // Y = 30 // 60 = \frac{30 \times 60}{30+60} = 20\,\Omega$.

$\therefore R_{ab} = 5 + 15 + 20 = \underline{40\,\Omega}$

Q3 Find the total resistance and power consumed in ($40\,\Omega$)?

Solution:



$R_{XY} = \{\{[\{(6+10) // 48\} + 3] // 30\} + 10\} // 20$.

$= \{\{[\{(16 // 48) + 3\} // 30] + 10\} // 20 = 20 // 20 = 10\,\Omega$.

Note: $100 // 0 // 5 = 0\,\Omega$.

$\therefore R_{eq} = 60 // (40 + 10 + R_{XY}) = 60 // 60 = \underline{30\,\Omega}$.

$\therefore I_{in\,40\Omega} = 10\,\frac{60}{60+60} = 5\,Amp$ (current Divider rule) (C.D.R).

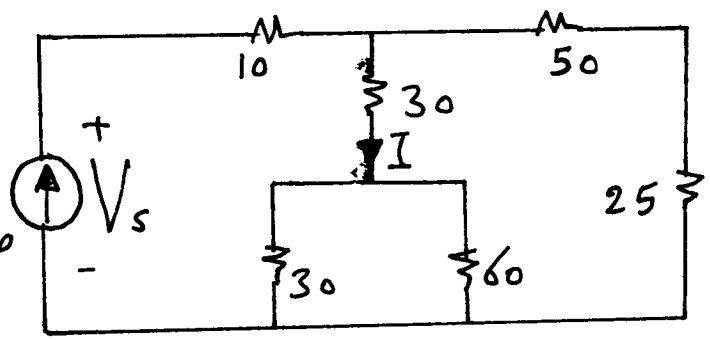
$\therefore P_{in\,40\Omega} = I^2 . R = (5)^2 \times 40 = \underline{1000\,watt} = \underline{1\,Kwatt}$.

=1=

Q4 Find the Value of (V_s) and power consumed in (60Ω)?

Solution:

$30 // 60 = 20\Omega$.



$\therefore R_{eq} = 10 + [(30+20) // (50+25)]$

$= 10 + (50 // 75) = 10 + 30 = 40\Omega$.

$\therefore V_s = I \cdot R_{eq} = 25 \times 40 = 1000$ Volt.

$I = 25 \frac{75}{75+50} = 15$ Amp (C.D.R).

$\therefore I_{60\Omega} = 15 \frac{30}{30+60} = 5$ Amp. (C.D.R).

$\therefore P_{in\,60\Omega} = I^2 \cdot R = (5)^2 \times 60 = 1500$ Watt $= 1.5$ KW.

Q5 Find the resistance between a & b for the circuit?

Solution:

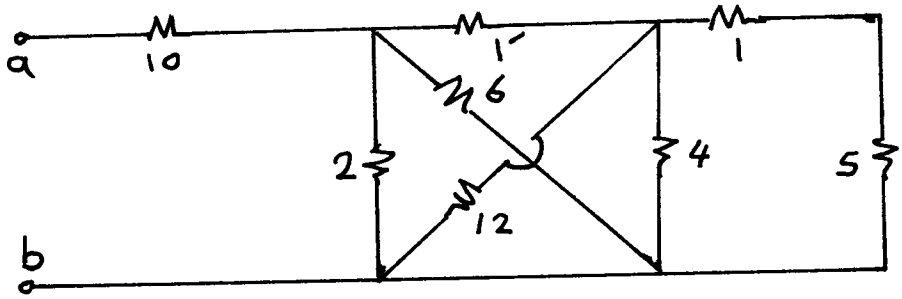
$1 + 5 = 6\Omega$.

$12 // 4 // (1+5)$

$12 // 4 // 6 = 2\Omega$.

$2 + 1' = 3\Omega$

$\therefore R_{ab} = 10 + (2 // 3 // 6) = 11\Omega$.



Q6 Find the resistance between a & b for the circuit?

Solution:

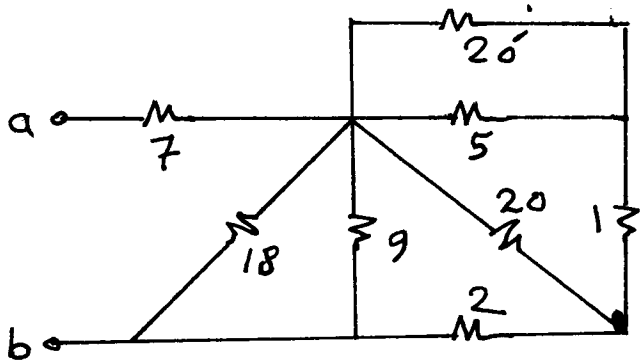
$20' // 5 = \frac{20' \times 5}{20'+5} = 4\Omega$.

$4 + 1 = 5'\Omega$.

$5' // 20 = 4\Omega$.

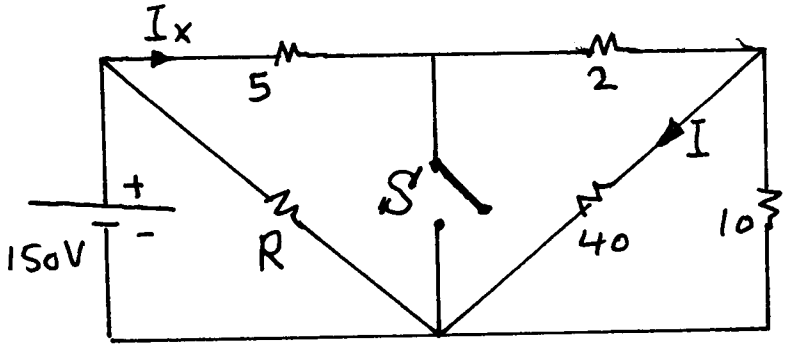
$4 + 2 = 6\Omega$.

$\therefore R_{ab} = 7 + (18 // 9 // 6) = 10\Omega$



Q7 Find the current (I) and power consumed in (5Ω) ? when

1- The switch (S) open.
2- " " (S) closed.



Solution:

1- $10 // 40 = \frac{10 \times 40}{10+40} = 8\Omega$.

$\therefore I_x = \frac{150}{5+2+8} = 10$ Amp.

$\therefore P_{in\ 5\Omega} = 10^2 \times 5 = \underline{500}$ Watt.

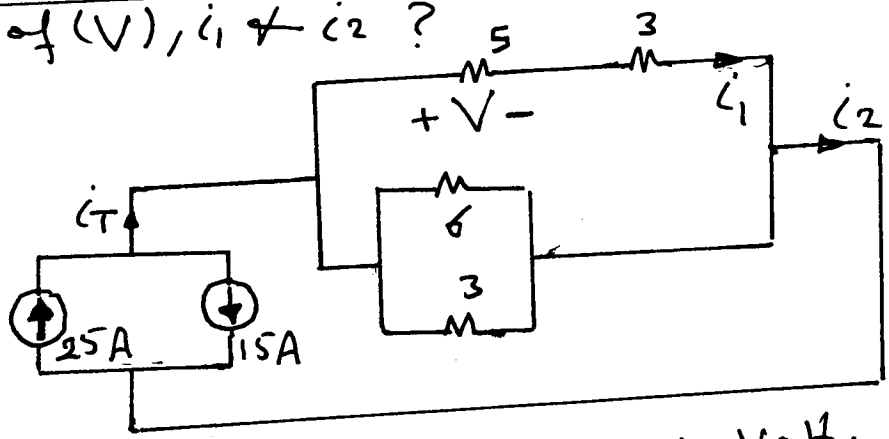
$\because I = 10\ \frac{10}{10+40} = \underline{2}$ Amp (C.D.R).

2- $I_x = \frac{150}{5} = 30$ Amp.

$\therefore P_{in\ 5\Omega} = 30^2 \times 5 = \underline{4500}$ Watt.

$\because I = 0$ ⟶ because (S) closed (S/c).

Q8 Find the value of (V), i_1 & i_2 ?



Solution:

$i_T = i_2$ (same current)

$= 25 - 15 = \underline{10}$ Amp.

$5 + 3 = 8\Omega$.

$6 // 3 = 2\Omega$.

$\therefore i_1 = 10\ \frac{2}{2+8} = \underline{2}$ Amp.

$\therefore V = I.R = 2 \times 5 = \underline{10}$ Volt.

Q9 Find the Voltage (V_{ab}) For the circuit shown?

Solution:

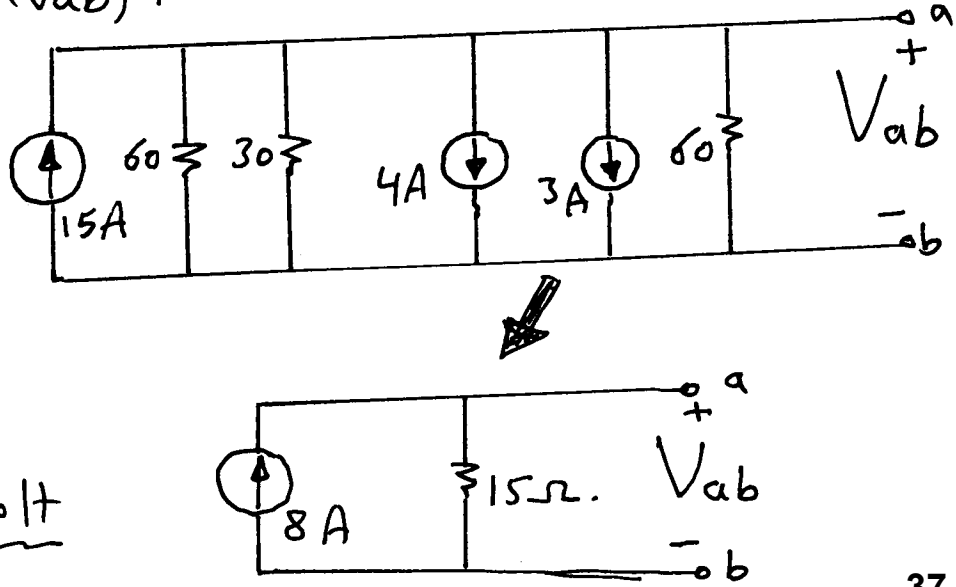
$I_T = 15 - 4 - 3 = 8$ A.

$Req = 60 // 30 // 60$

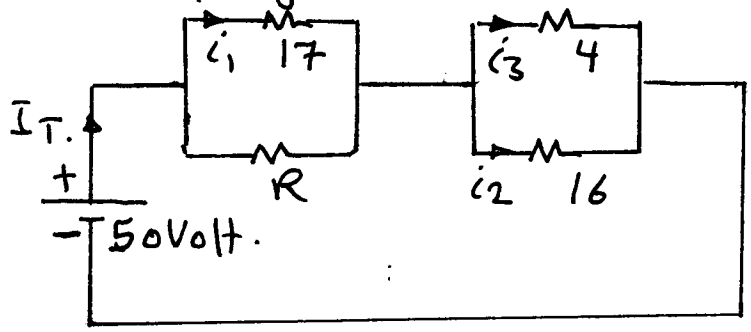
$= 15\Omega$.

$\therefore V_{ab} = I_T \times Req$

$= 8 \times 15 = \underline{120}$ Volt



Q10 Given that the total power supply is (250 W), find R, i_1, i_2 & i_3?



Solution:

Since $P_T = 250$ watt.

$\therefore P_T = V_T . I_T$

$250 = 50 \times I_T$

$\therefore I_T = \frac{250}{50} = 5$ Amp.

$\therefore i_T = i_2 + i_3 \longrightarrow i_2 = 5 \frac{4}{4+16} = 1$ Amp. (C.D.R).

$\therefore i_3 = 5 - 1 = 4$ Amp OR $i_3 = 5 \frac{16}{4+16} = 4$ Amp.

$V_{4\Omega} = V_{16\Omega} = 4 \times 4 = 16$ Volt OR $1 \times 16 = 16$ Volt.

$V_{17\Omega} = V_R = 50 - 16 = 34$ Volt.

$\therefore i_1 = \frac{34}{17} = 2$ Amp & $R = \frac{34}{3} = 11.333\ \Omega$.

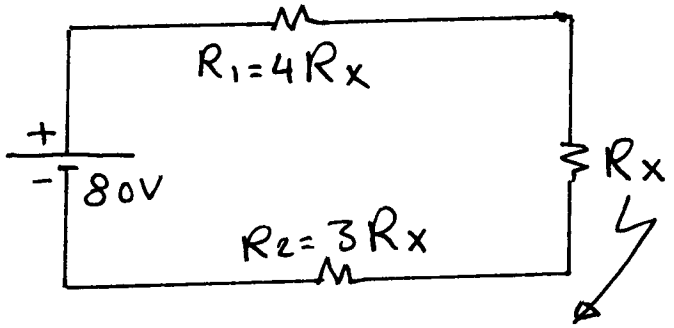
$\longrightarrow 3 = 5 - 2 = 3$ Amp.

Q11 Find the value of (R_x) and the power supply by (80V)?

Solution:

$R_T = 4R_x + R_x + 3R_x = 8R_x$.

$\therefore I_T = \frac{V_T}{R_T} = \frac{80}{8R_x} = \frac{10}{R_x}$ Amp.



$50 = (I_T)^2 \times R_x$

$50 = \left(\frac{10}{R_x}\right)^2 . R_x$.

$\therefore R_x = \frac{100}{50} = 2\ \Omega$.

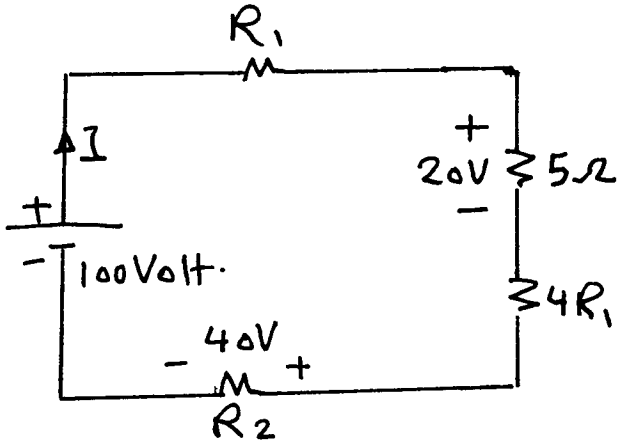
$\therefore I_T = \frac{10}{2} = 5$ Amp.

& Power supply (P_T) $= V . I = 80 \times 5 = 400$ watt.

$P_{in} R_x = 50$ watt.

Q12 Find the value of R_1 & R_2?

Solution:



$I_T = I_{5\Omega} = \frac{20}{5} = 4\,Amp.$

$\therefore 40 = 4 \times R_2.$

$\therefore R_2 = 10\,\Omega$

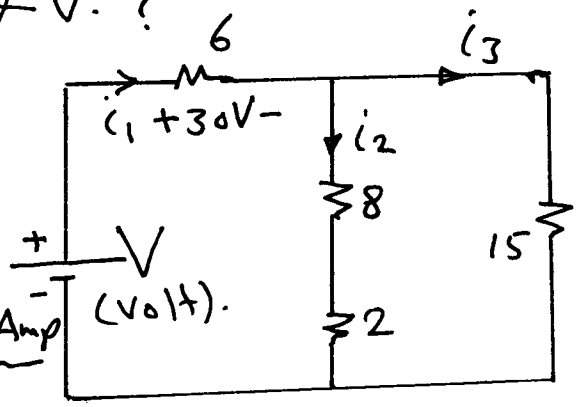
$Req = \frac{V_T}{I_T} = \frac{100}{4} = 25\,\Omega.$

$25 = R_1 + 5 + 4R_1 + R_2 = 5R_1 + 15.$

$\therefore 10 = 5R_1 \longrightarrow \therefore R_1 = 2\,\Omega.$

Q13 For the circuit find i_1, i_2, i_3 & V. ?

Solution:



$i_1 = \frac{30}{6} = 5\,Amp.$

$i_2 = 5\,\frac{15}{15+(8+2)} = 3\,Amp.$

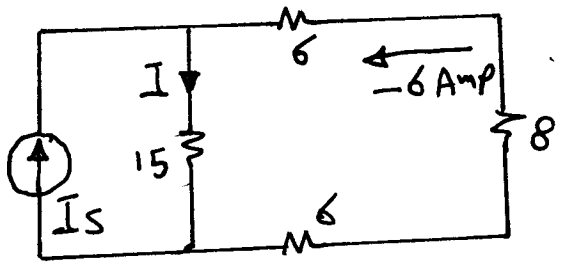
$i_3 = 5 - 3 = 2\,Amp$ OR $i_3 = 5\,\frac{10}{25} = 2\,Amp$

$Req = 15 // (8+2) + 6 = 12\,\Omega.$

$\therefore V = I \cdot Req = 5 \times 12 = 60\,volt.$

Q14 For the circuit, find (I & Is)?

Solution: $6 + 8 + 6 = 20\,\Omega.$



$\therefore V_{20\Omega} = 20 \times 6 = 120\,Volt.$

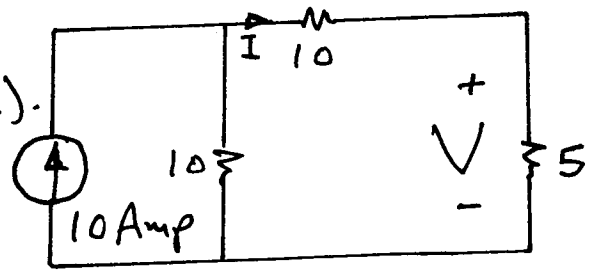
$V_{20\Omega} = V_{15\Omega} = 120\,Volt$ (parallel).

$\therefore I = \frac{120}{15} = 8\,Amp.$

& $I_S = 8 + 6 = 14\,Amp$ (Kirchhoff current Law) (K.C.L).

Q15 For the circuit, find (V) & Power supply by (10 Amp)?

Solution:



$I = 10\,\frac{10}{10+(10+5)} = 4\,Amp$ (C.D.R).

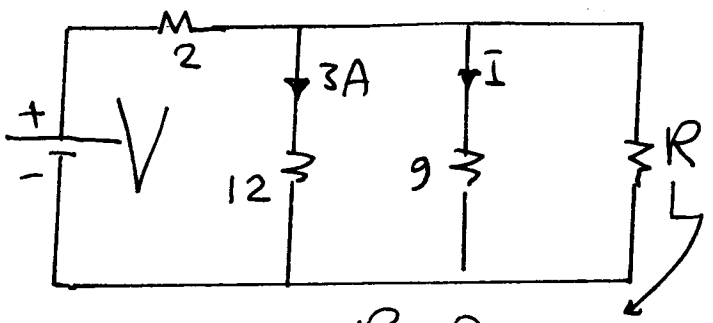
$\therefore V_{5\Omega} = 5 \times 4 = 20\,Volt.$

$Req = 10 // (10+5) = 10 // 15 = 6\,\Omega.$

$\therefore P_T = I^2 \cdot R = (10)^2 \times 6 = 600\,Watt.$

Q16 Find the Unknown quantities for the circuit?

Solution:



$V_{12} = V_{9\Omega} = V_R$ (parallel)

$= 3 \times 12 = 36$ Volt.

$\therefore I = \frac{36}{9} = 4$ Amp.

$P_{in\,R} = 72$ watt.

$P = 72 = \frac{V^2}{R} = \frac{(36)^2}{R} \longrightarrow \therefore R = \frac{(36)^2}{72} = 18\Omega$

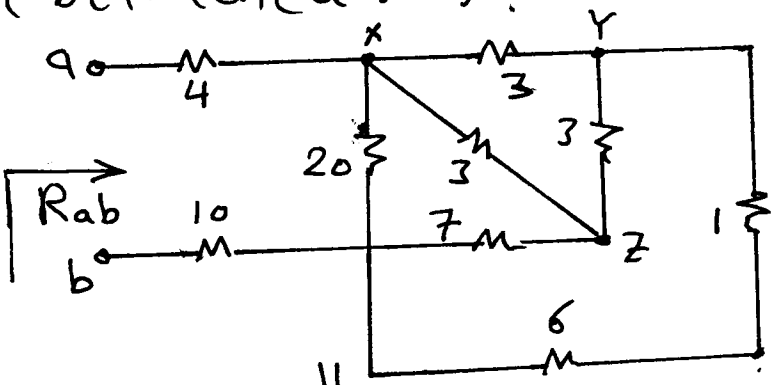
$\therefore I_{18\Omega} = \frac{36}{18} = 2$ Amp $\longrightarrow \therefore I_T = 3+4+2 = 9$ Amp.

$R_{eq} = 2 + (12 \| 9 \| 18) = 2 + 4 = 6\Omega$.

$\therefore V = I_T \cdot R_{eq} = 9 \times 6 = 54$ Volt.

Q17 Find the resistance between (a & b)?

$\Delta_{xyz} \Rightarrow Y$



$R_x = \frac{3 \times 3}{3+3+3} = 1\Omega$.

$1+1+6 = 8\Omega$.

$1+7 = 8\Omega$.

$8 \| 8 = 4\Omega$.

$4+1 = 5\Omega$.

$5 \| 20 = \frac{5 \times 20}{5+20} = 4\Omega$.

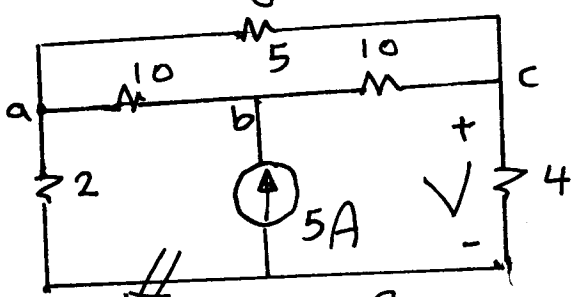
$\therefore R_{eq} = 4 + 4 + 10 = 18\Omega$.

Q18 Use $\Delta \Rightarrow Y$ transformation, find the voltage (V)?

$R_a = \frac{5 \times 10}{5+10+10} = 2\Omega$

$R_b = \frac{10 \times 10}{5+10+10} = 4\Omega$.

$R_c = \frac{5 \times 10}{5+10+10} = 2\Omega$.

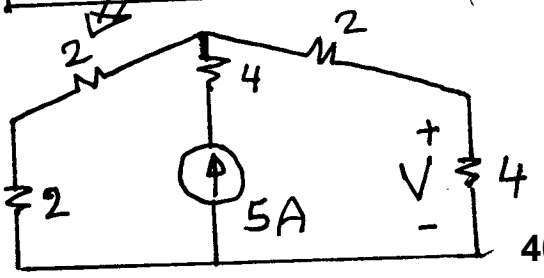


$2+4 = 6\Omega$

$2+2 = 4\Omega$.

$\therefore I_{4\Omega} = 5\,\frac{4}{4+6} = 2$ Amp.

$\therefore V = 2 \times 4 = 8$ Volt.



Q19 If $R_{eq} = 50\Omega$, find the value of (R)?

Solution:

$12 // 12 // 12 = 4\Omega$.

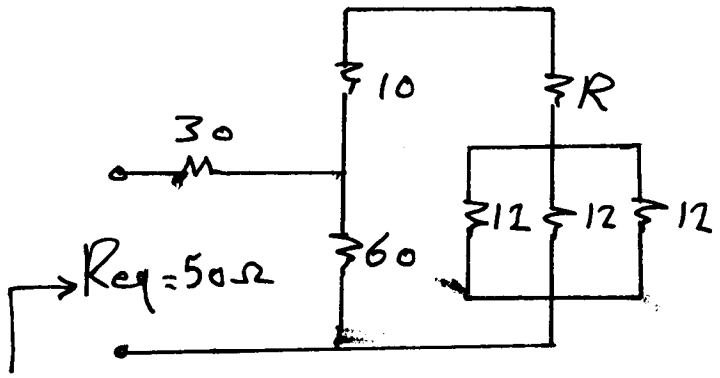
$10 + R + 4 = 14 + R$.

$R_{eq} = 50 = 30 + (60 // 14+R)$.

$20 = 60 // (14+R)$

$20 = \frac{60(14+R)}{60+(14+R)}$.

$1480 + 20R = 840 + 60R \longrightarrow \therefore 40R = 640 \longrightarrow \therefore R = 16\Omega$



Q20 Find Rab ?

Solution:

$12 // 60 = \frac{12 \times 60}{12+60} = 10\Omega$.

$10 + 20 = 30\Omega$.

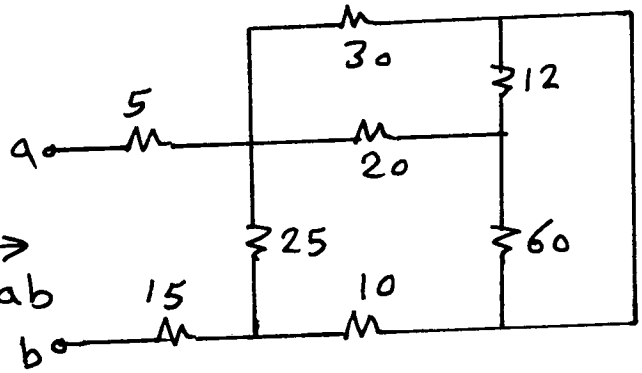
$30 // 30 = 15\Omega$.

$15 + 10 = 25$

$\therefore R_{ab} = 5 + 15 + 25 // 25 = 32.5\Omega$.

OR $R_{ab} = 5 + 15 + \left[25 // \left(\{(12 // 60) + 20\} // 30\right) + 10\right]$.

$= 32.5\Omega$.



Q21 Find the value of (R)?

Solution:

$V_{10\Omega} = 10 \times 5 = 50$ Volt.

$\therefore V_{(4+5)} = 140 - 50 = 90V$.

$\therefore I_{(4+5)} = \frac{90}{9} = 10$ Amp.

$(a \rightarrow b)$

(K.C.L) at point a.

$\therefore I_{2\Omega} = 10 - 5 = 5$ Amp.

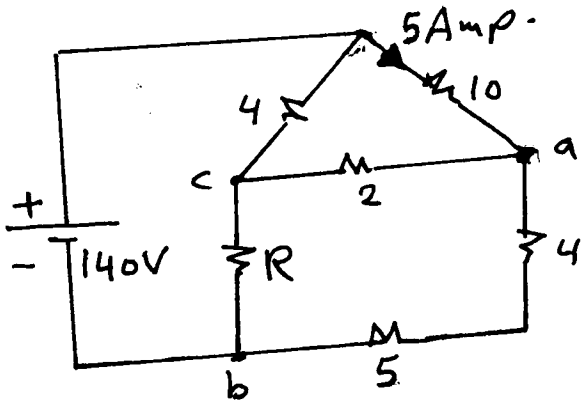
$\therefore V_{2\Omega} = 10$ Volt. $\longrightarrow \therefore I_{4\Omega} = \frac{40}{4} = 10$ Amp.

$\therefore V_{4\Omega} = 50 - 10 = 40$ Volt.

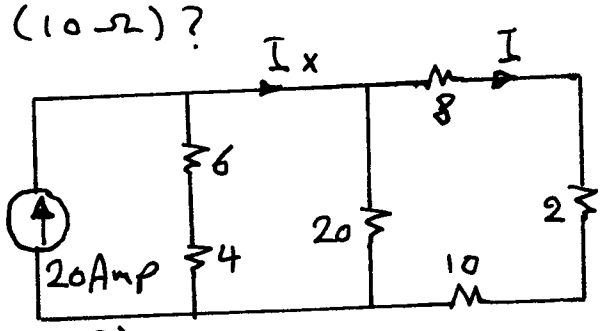
$\therefore$ K.C.L at b $\longrightarrow I_R = 10 - 5 = 5$ Amp.

$\therefore$ K.V.L at (cab) $\longrightarrow V_R = V_{2\Omega} + V_{(4-5)} = 10 + 90 = 100V$

$\therefore R = \frac{V_R}{I_R} = \frac{100}{5} = 20\Omega$.



Q22 Find the power consumed in (10Ω)?



Solution:

$6+4=10\Omega$.

$8+2+10=20\Omega$.

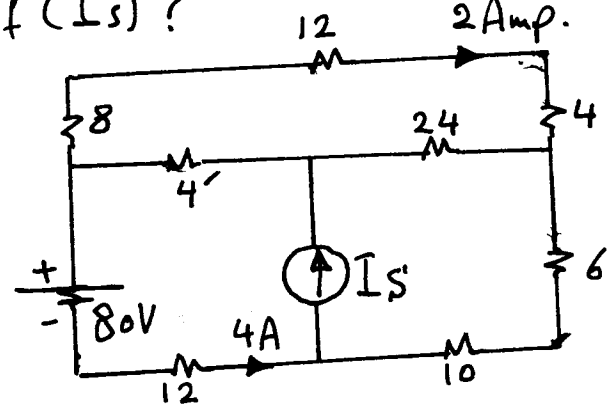
$$I_x = 20\frac{10}{10+(20\|20)} = 20\frac{10}{10+10}$$

$= 10$ Amp. (C.D.R).

$$I = 10\frac{20}{20+20} = 5 \text{ Amp}.$$

$\therefore P_{in\,10\Omega} = (5)^2 \times 10 = 250$ watt.

Q23 For the circuit find the value of (Is)?



Solution:

$8+12+4=24\Omega$.

$\therefore V_{24\Omega} = 2\times 24 = 48$ Volt.

$V_{12\Omega} = 12\times 4 = 48$ Volt.

$6+10=16\Omega$.

$\therefore V_{16\Omega} = 80$ Volt.

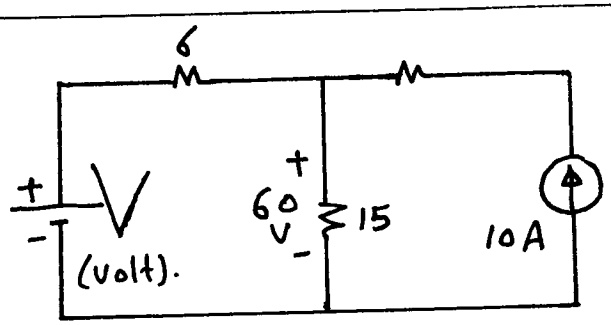
$\therefore I_{16\Omega} = \frac{80}{16} = 5$ Amp.

$\therefore I_{24\Omega} = 5-2 = 3$ Amp. $\longrightarrow$ $\therefore V_{24\Omega} = 3\times 24 = 72$ Volt.

& $V_{4\Omega} = 72-48 = 24$ Volt $\longrightarrow$ $\therefore I_{in\,4\Omega} = \frac{24}{4} = 6$ Amp.

$\therefore I_S = 3+6 = 9$ Amp (Kirchhoff current Law) (K.C.L).

Q24 Find the value of (V)?



Solution:

$I_{15\Omega} = \frac{60}{15} = 4$ Amp.

$\therefore I_{6\Omega} = 10-4 = 6$ Amp.

& $V_{6\Omega} = 6\times 6 = 36$ Volt.

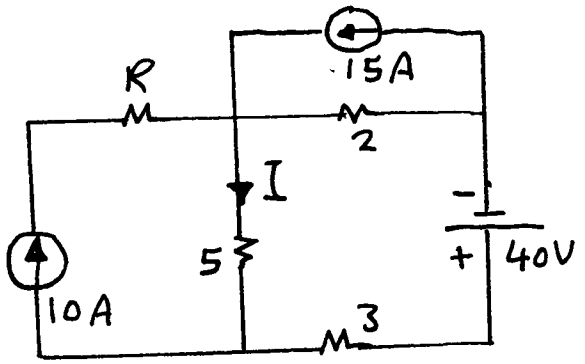
By Kirchhoff Voltage Law (K.V.L).

$V+36=60$

$\therefore V = 24$ Volt.

Q25 Find the current (I)?

Solution:



① By Kirchhoff's Law:

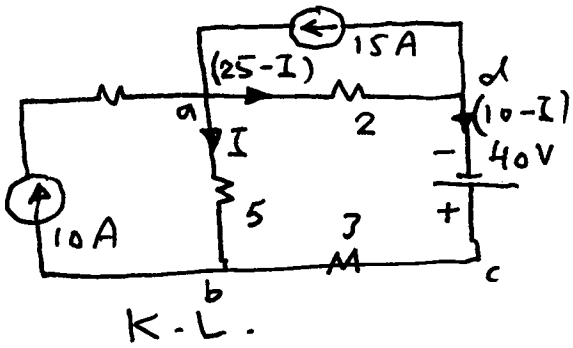
for the circuit of a, b, c & d:

$40+5I = 2(25-I)+3(10-I)$.

$40+5I = 50-2I+30-3I$.

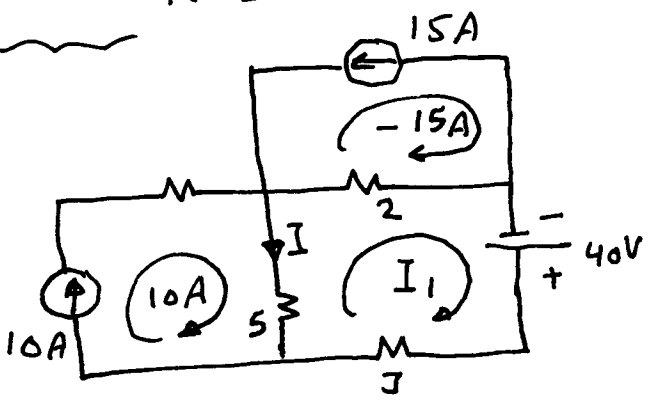
$10I = 40$

$\therefore I = 4$ Amp.



K.L.

② By loop method:



$I = 10 - I_1$

loop (1):

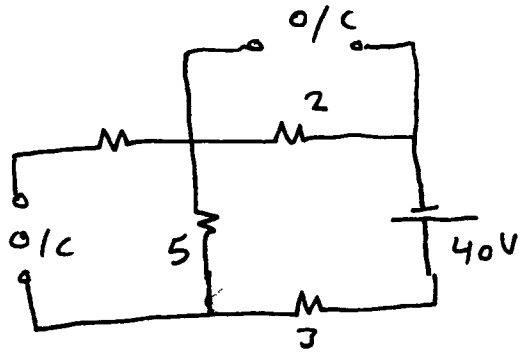
$40 = (2+3+5)I_1 - 5\times10 - 2(-15)$.

$40 = 10I_1 - 50 + 30$

$60 = 10I_1 \longrightarrow I_1 = 6$ Amp.

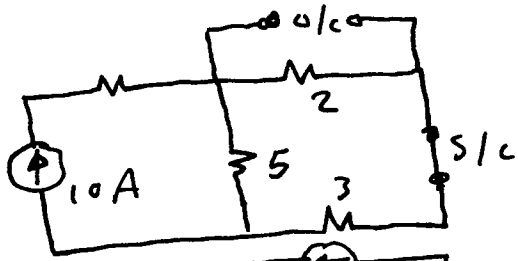
$\therefore I = 10 - 6 = 4$ Amp.

③ By superposition:



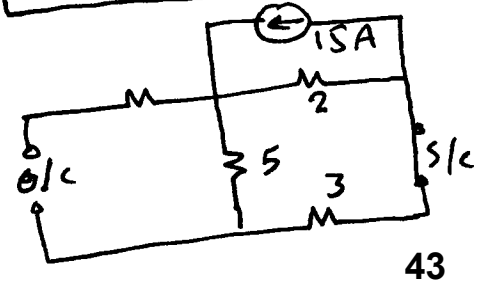
* I by 40 Volt $\longrightarrow$ 10A & 15A (o/c).

$= \frac{40}{2+5+3} = \frac{40}{10} = 4$ Amp ↑



* I by 10 Amp $\longrightarrow$ 15A (o/c) & 40V (s/c).

$= 10\,\frac{(2+3)}{5+(2+3)} = 5$ Amp ↓ (C.D.R).

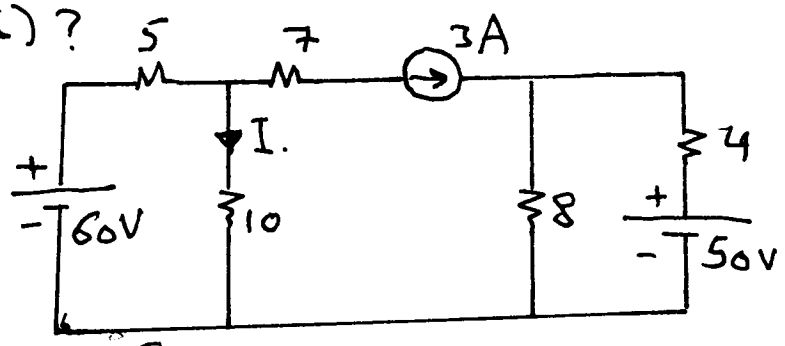


* I by 15 Amp $\longrightarrow$ 10 Amp (o/c) & 40V (s/c).

$= 15\,\frac{2}{2+(3+5)} = 3$ Amp ↓ (C.D.R).

$\therefore I = 5+3-4 = 4$ Amp.

Q26 Find the current (I)?



Solution:

① By K.L:

take (abcd)

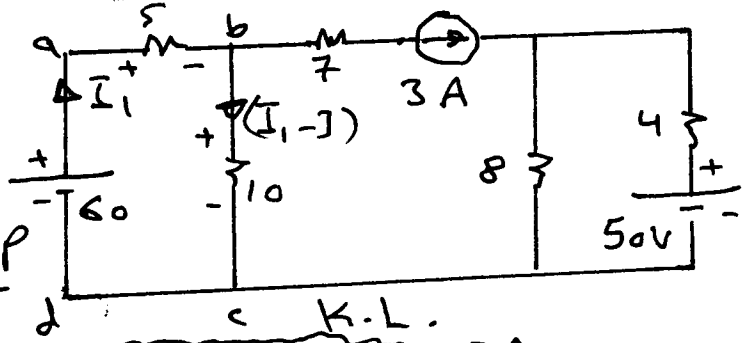
$60 = 5I_1 + 10(I_1 - 3)$.

$60 = 15I_1 - 30$

$\therefore 90 = 15I_1$

$I_1 = 6$ Amp.

$\therefore I = (I_1 - 3) = 6 - 3 = 3$ Amp



K.L.

② By loop method:

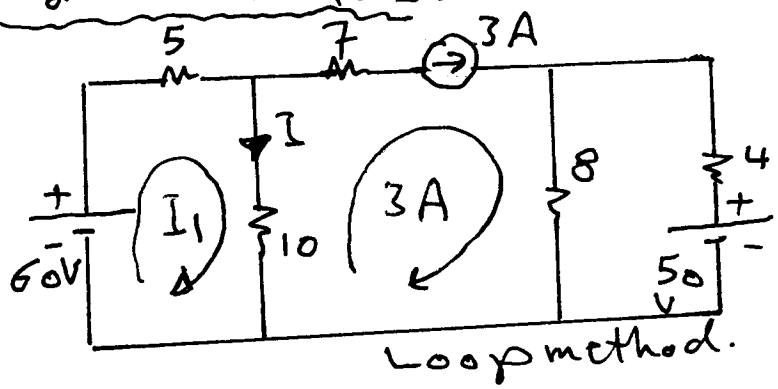
$I = I_1 - 3$

loop (1):

$60 = (5+10)I_1 - 10 \times 3$

$90 = 15I_1$

$\therefore I_1 = 6$ Amp $\longrightarrow$ $\therefore I = 6 - 3 = 3$ Amp

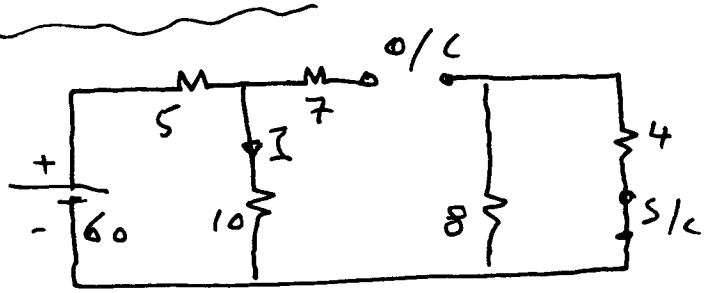


Loop method.

③ By superposition.

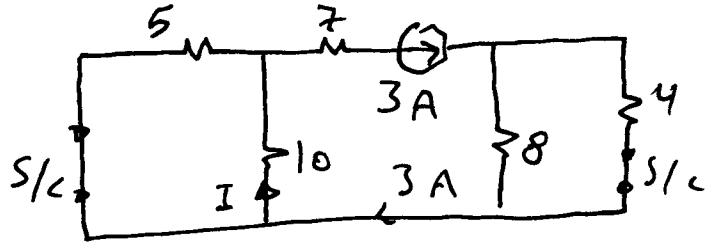
* By 60V $\longrightarrow$ 3A (o/c), 50 (s/c).

$I = \frac{60}{5+10} = 4$ Amp ↓



* By 3A $\longrightarrow$ 60V + 50V (s/c)

$I = 3\frac{5}{5+10} = 1$ Amp ↑

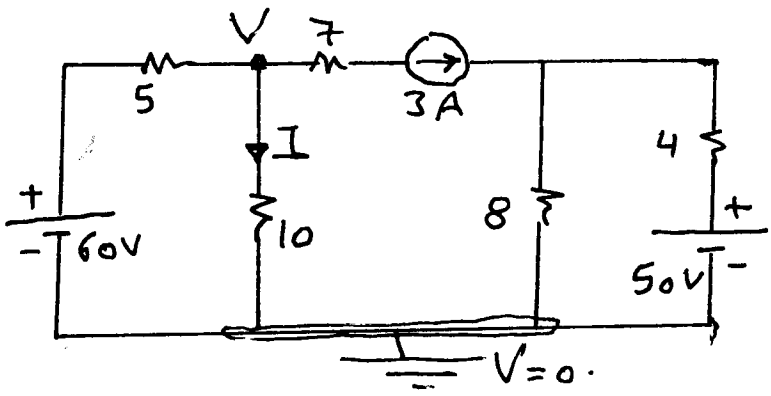


* By 50V $\longrightarrow$ 60V (s/c), 3A (o/c).

$I = 0$ (open circuit).

$\therefore I = 4 - 1 = 3$ Amp.

④ By Nodal Voltage



$I = \frac{V}{10}$.

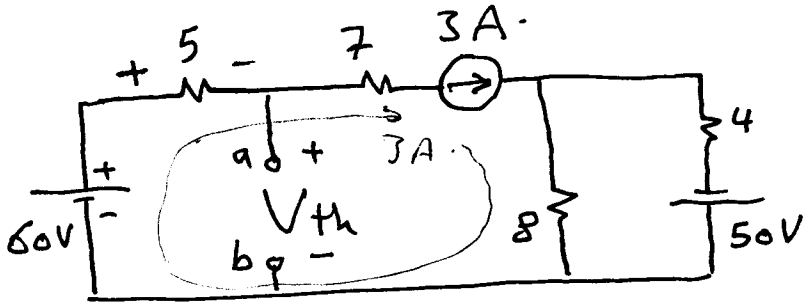
Node (V):

$(\frac{1}{5}+\frac{1}{10})V - \frac{60}{5} = -3$

$(2+1)V - 120 = -30$.

$3V = 90 \longrightarrow V = 30 \text{ Volt} \longrightarrow \therefore I = \frac{30}{10} = 3 \text{ Amp}$.

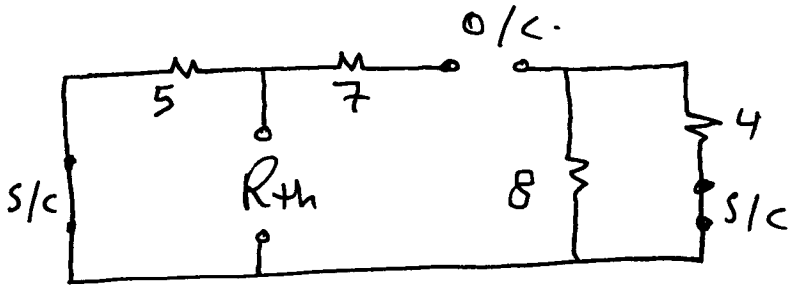
⑤ By Thevenins Theorem



$I = \frac{V_{th}}{R_{th}+10}$.

* $V_{th} \longrightarrow$

$V_{th} = 60 - 5\times3$ } K.V.L.
$= 45 \text{ Volt}$ }



* $R_{th} \longrightarrow$

$\therefore R_{th} = 5\,\Omega$. (only).

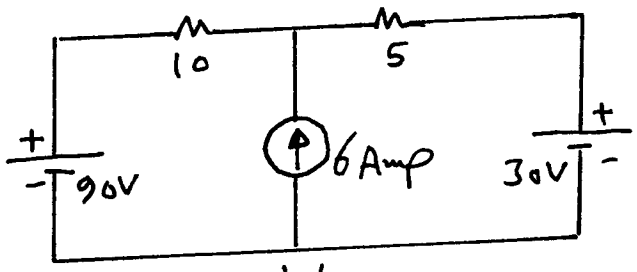
$\therefore I = \frac{45}{5+10} = 3 \text{ Amp}$.

Q27 Find the power delivered by (6 Amp)?

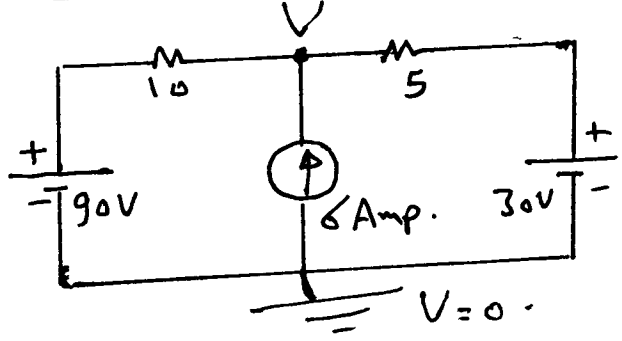
Solution:

$P = V \times I = V \times 6 = 6V$. (watt).

$\therefore$ find (V) across the 6Amp.



① By Nodal Voltage



$(\frac{1}{5}+\frac{1}{10})V - \frac{90}{10} - \frac{30}{5} = 6$.

$(2+1)V - 90 - 60 = 60$

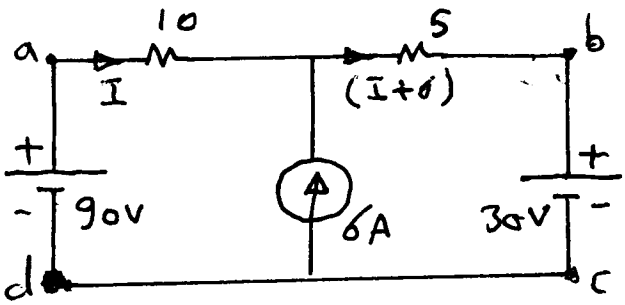
$3V = 210$

$\therefore V = 70 \text{ Volt}$.

$\therefore P = 70 \times 6 = 420 \text{ Watt}$.

② By K-L

By the circuit (a,b,c & d).



$90 = 10I + 5(I+6) + 30$

$30 = 15I$.

$\therefore I = 2$ Amp.

$\therefore$ The voltage across (6Amp) $= 90 - 10 \times I =$ 70 Volt

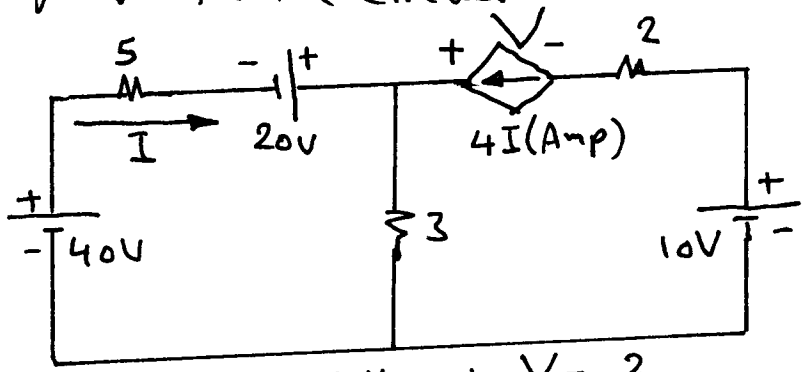
OR $V_{6Amp} = 30 + 5(3+6) = 30 + 40 =$ 70 volt.

$\therefore P = 6 \times 70 =$ 420 watt.

Q28 Find the value of I & V for the circuit.

① By K-L:

take abcd:

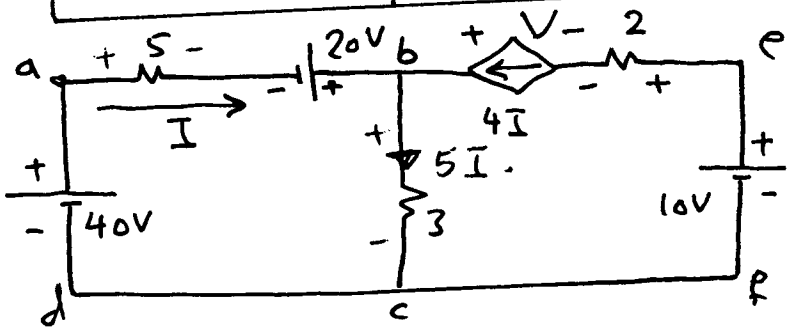


$40 + 20 = 5I + 3(5I)$.

$60 = 20I$.

$\therefore I = 3$ Amp.

to find (V) take aefd



$10 + 15 + V = 40 + 20 + 2 \times 12$.

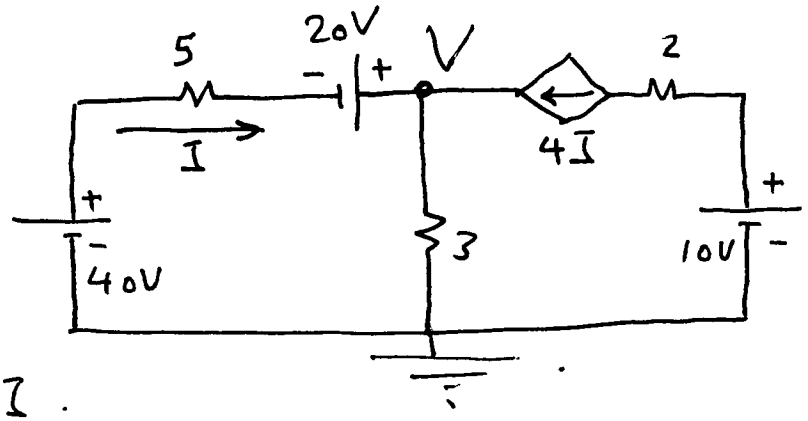
$25 + V = 84$.

$\therefore V =$ 59 Volt.

---

② By Node method

to find (I).



$I = \frac{40 + 20 - V}{5} = \frac{60 - V}{5}$.

Node (V)

$\left(\frac{1}{5} + \frac{1}{3}\right)V - \frac{60}{5} = 4I$.

$(3+5)V - 180 = 60I$

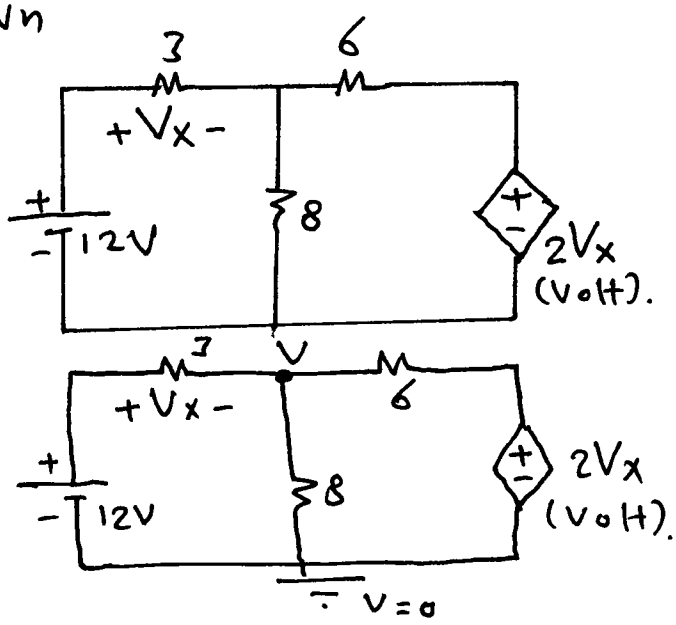
$8V - 180 = 60\left(\frac{60 - V}{5}\right)$

$8V - 180 = 12(60 - V)$

$\therefore 20V = 900 \longrightarrow \therefore V = 45$ Volt.

$\therefore I = \frac{60 - 45}{5} =$ 3 Amp

Q29 Find (V_x) for the circuit shown



Solution:

① By Node Voltage

$V_x = 12 - V$.

Node (V):

$$\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{8}\right)V - \frac{12}{3} - \frac{2V_x}{6} = 0$$

$$(8+4+3)V - 96 - 8V_x = 0.$$

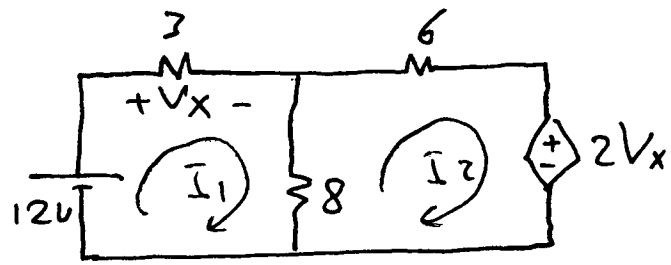
$$15V - 8V_x = 96.$$

$$\therefore 15V - 8(12 - V) = 96.$$

$$23V = 192 \longrightarrow \therefore V = \frac{192}{23} = 8.347$$

and $V_x = 12 - 8.347 = 3.653$ Volt.

② By loop method.



loop (1):

$$12 = (3+8)I_1 - 8I_2 \quad \text{---} (1).$$

loop (2):

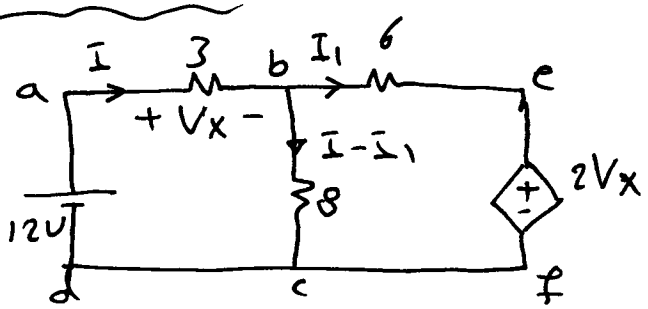
$$-2V_x = (6+8)I_2 - 8I_1 \quad \text{---} (2).$$

But $V_x = 3I_1$

$$\therefore -2(3I_1) = 14I_2 - 8I_1$$

Solving eq (1) & (2) $\longrightarrow I_1 = 1.217$ Amp.

③ By K.L.



take abcd

$$12 = 3I + 8(I - I_1) \quad \text{---} (1).$$

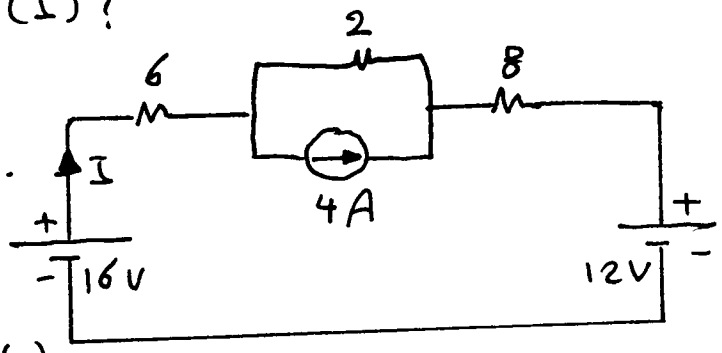
take aefd

$$12 = 3I + 6I_1 + 2V_x \quad \text{---} (2)$$

But $V_x = 3I$.

Solving eq (1) & eq (2) $\longrightarrow I = 1.217$ Amp.

Q30 Use superposition to find (I)?



Solution:

* take (16V) → 4A (o/c), 12V (S/c).

$\therefore I = \frac{16}{6+2+8} = \frac{16}{16}$ Amp. ↑

* take (12V) → 16V (S/c), 4A (o/c).

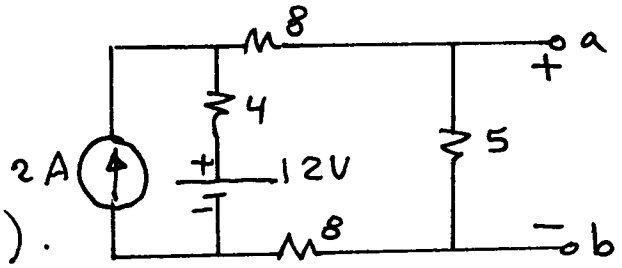
$\therefore I = \frac{12}{6+2+8} = \frac{12}{16}$ A ↓

* take (4A) → 12V & 16V (S/c).

$\therefore I = 4\ \frac{2}{2+6+8} = \frac{8}{16}$ A ↑ (C.D.R).

$\therefore I = \frac{16}{16} + \frac{8}{16} - \frac{12}{16} = \frac{12}{16} = 0.75$ Amp.

Q31 Find V_{th} & R_{th} of the circuit between a & b?



Solution:

$V_{th} = V_{ab} = V_{5\Omega}$.

use superposition to find (V_{th}).

* take (12V) → 2A (o/c).

$V_{th_1} = \frac{12}{4+8+5+8} \times 5 = \frac{12}{5}$ Volt.

* take (2A) → 12V (S/c).

$V_{th2} = 2\ \frac{4}{4+8+5+8} \times 5 = \frac{8}{5}$ Volt → use current divider rule.

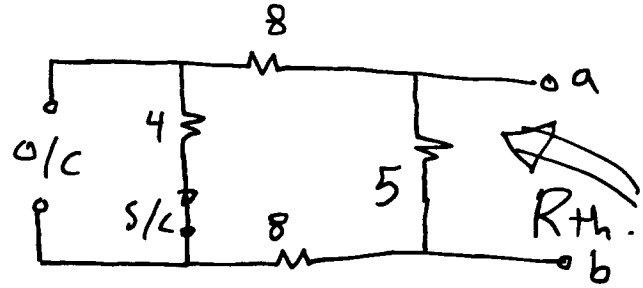
$\therefore V_{th} = V_{th_1} + V_{th2} = \frac{12}{5} + \frac{8}{5} = \frac{20}{5} = 4$ Volt

R_{th} →

$R_{th} = 5 // (8+4+8)$

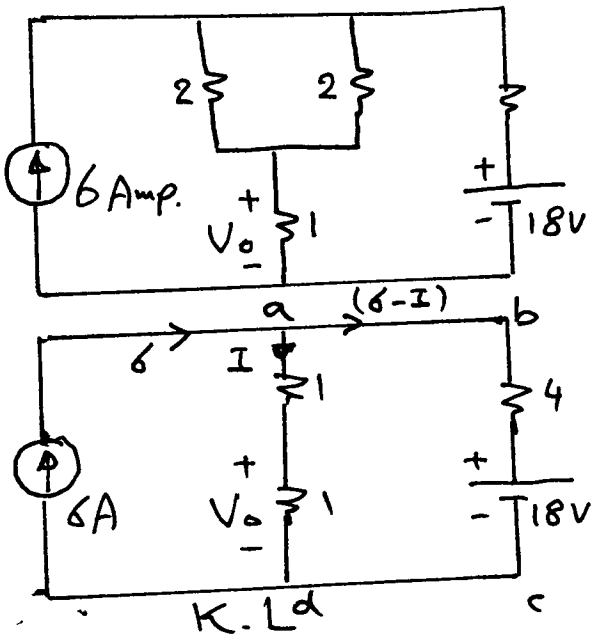
$= 5 // 20$

$= \frac{5 \times 20}{5+20} = 4\Omega$



Q32 | Find(V_o) for the circuit ?

Solution:



① By K.L. take(abcd)

$(1+1)I = 4(6-I) + 18$

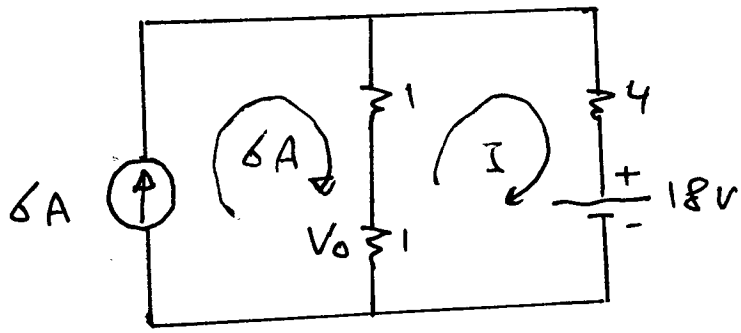
$2I = 24 - 4I + 18$

$\therefore 6I = 42.$

$\therefore I = 7$ Amp.

and $V_o = 7 \times 1 = \underline{7}$ Volt.

② By loop method.



loop(I):

$-18 = (4+1+1)I - 6 \times 2.$

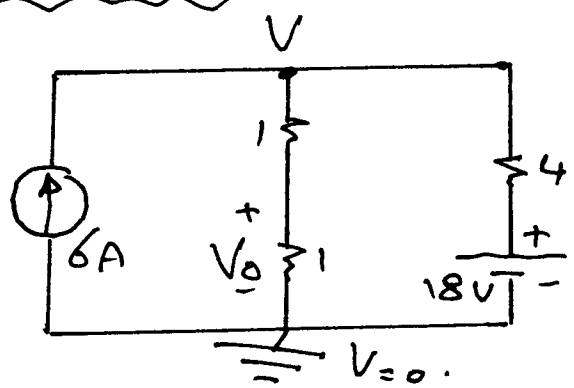
$-6 = 6I.$

$\therefore I = -1$ Amp.

But the current in $1\Omega = 6 - I = 6 - (-1) = 7$ Amp.

$\therefore V_o = \underline{7}$ Volt.

③ By Nodel method



Node(V):

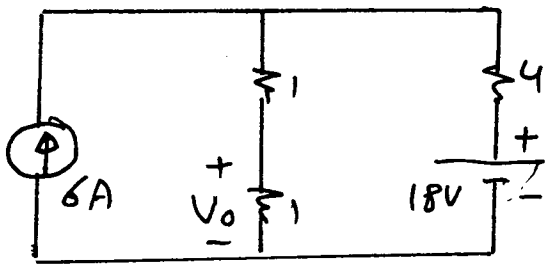
$\left(\frac{1}{2} + \frac{1}{4}\right)V - \frac{18}{4} = 6.$

$(2+1)V - 18 = 24$

$3V = 42.$

$\therefore V = 14$ Volt. $\longrightarrow \therefore V_o = \frac{14}{1+1} \times 1 = \underline{7}$ Volt (voltage Divider rule)

④ By superposition



* By 18V $\longrightarrow$ 6Amp (o/c).

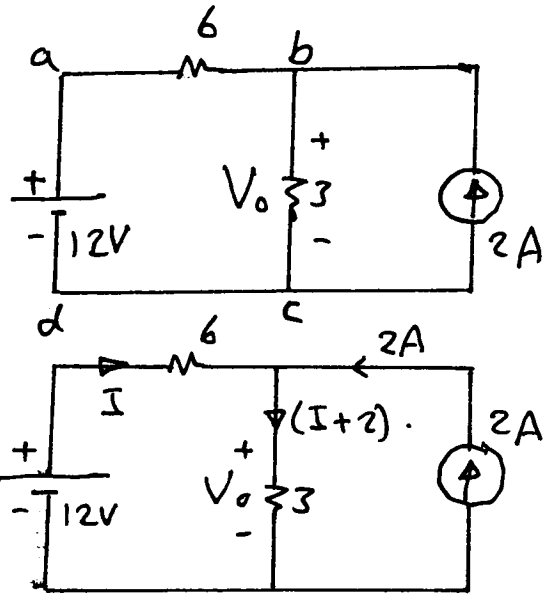
$V_o = \frac{18}{4+1+1} \times 1 = 3$ Volt. (V.D.R).

* By 6A $\longrightarrow$ 18V (S/C).

$V_o = \left(6 \frac{4}{4+1+1}\right) \times 1 = \frac{24}{6} = 4$ Volt

$\therefore V_o = 4 + 3 = \underline{7}$ Volt.

Q33 Find (V_o) for the circuit shown?



Solution

① By Kirchhoff Law take abcd

$12 = 6I + 3(I+2)$.

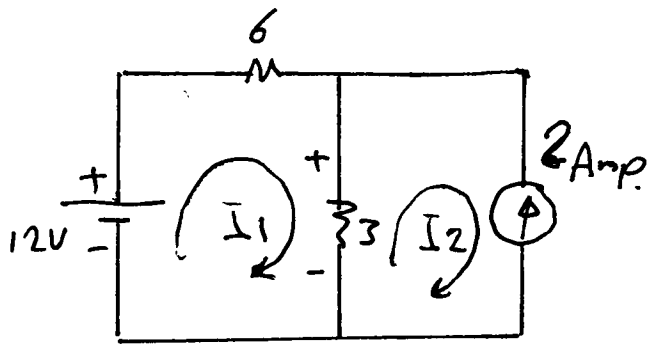
$6 = 9I$

$\therefore I = \frac{6}{9}$ Amp.

$\therefore V_o = 3(I+2) = 3\left(\frac{6}{9}+2\right)$

$= 8$ Volt.

② By loop method:



loop (1):

$12 = (6+3)I_1 - 3(-2)$.

$6 = 9I_1$

$\therefore I_1 = \frac{6}{9}$ Amp.

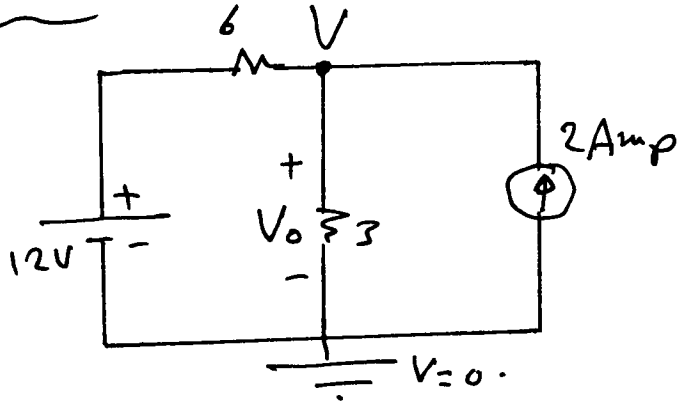
But $V_o = 3(I_1 - I_2)$

$= 3\left(\frac{6}{9} - (-2)\right)$

$= 8$ Volt

$I_2 = -2$ Amp.

③ By Nodal method.



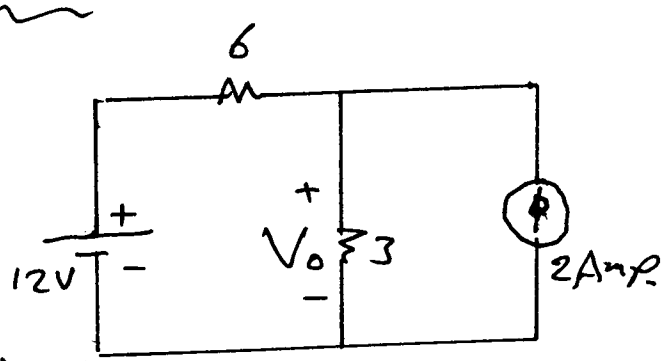
$\left(\frac{1}{3}+\frac{1}{6}\right)V - \frac{12}{6} = 2$.

$(2+1)V - 12 = 12$

$\therefore 3V = 24$

and $V = V_o = 8$ Volt.

④ By Superposition:



* By 12V $\longrightarrow$ 2Amp (o/c).

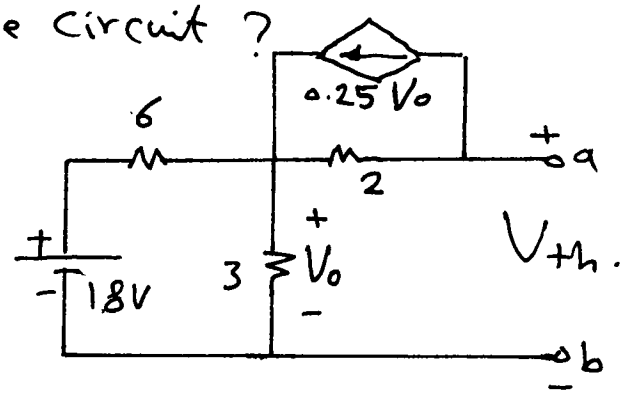
$V_o = \frac{12}{6+3} \times 3 = 4$ Volt. (V.D.R).

* By 2Amp $\longrightarrow$ 12V (S/c).

$V_o = \left(2\,\frac{6}{6+3}\right) \times 3 = 4$ Volt (C.D.R).

$\therefore V_o = 4+4 = 8$ Volt.

Q34 Find V_{th} between a & b for the circuit ?



Solution:

$V_o = V_{th} + V_{2\Omega}$.

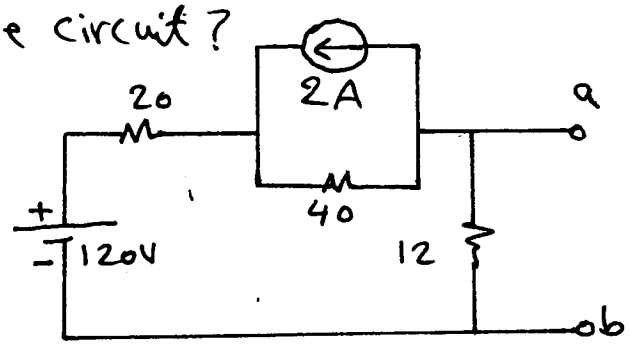
$\therefore V_{th} = V_o - V_{2\Omega}$.

$V_o = \left(\frac{18}{6+3}\right) \times 3 = 6$ Volt. (V.D.R).

$V_{2\Omega} = 2(0.25 V_o) = 2 \times (0.25 \times 6) = 2 \times 3 = 3$ Volt.

$\therefore V_{th} = 6 - 3 = 3$ Volt.

Q35 Find V_{th} between a & b for the circuit ?



Solution:

* By Kirchhoffs Law:

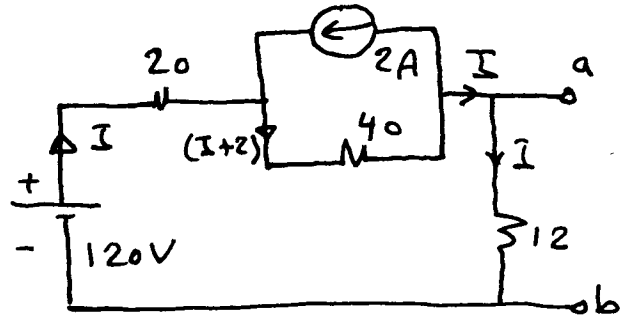
$120 = 20I + 40(I+2) + 12I$.

$= 20I + 80I + 80 + 12I$

$40 = 72I \longrightarrow I = \frac{40}{72}$ Amp.

But $V_{th} = V_{ab} = V_{12} = \frac{40}{72} \times 12 = \frac{40}{6}$

$V_{th} = 6.666$ Volt



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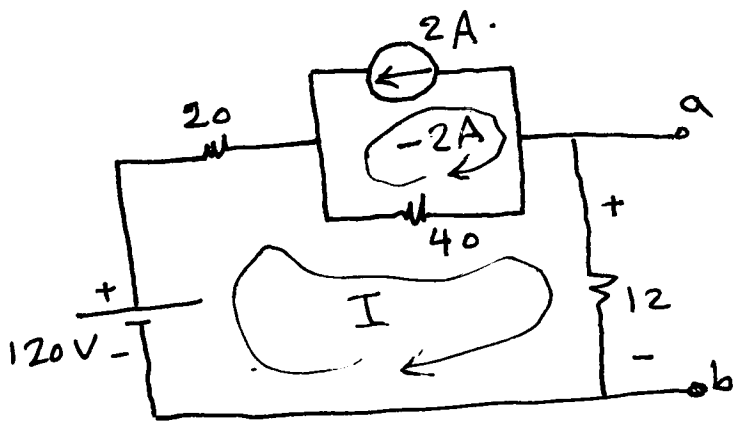
* By loop method:



Loop (I):

$120 = (20 + 40 + 12)I - 40(-2)$.

$40 = 72I$

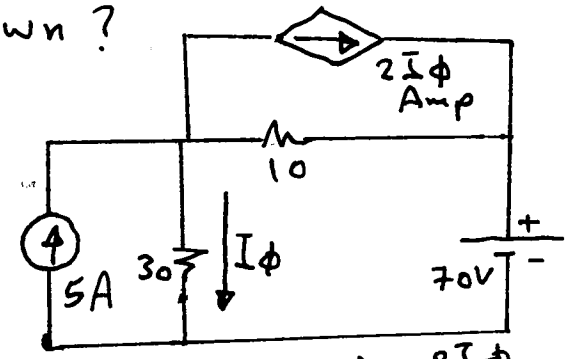
$\therefore I = \frac{40}{72}$ Amp.

$V_{th} = V_{ab} = V_{12\Omega}$.

$= \left(\frac{40}{72}\right) \times 12 = \frac{40}{6}$ Volt.

$= 6.666$ Volt.

Q36] Find $(I\phi)$ for the circuit shown?



Solution:

* By loop method.

$I\phi = 5 - I$.

loop(I):

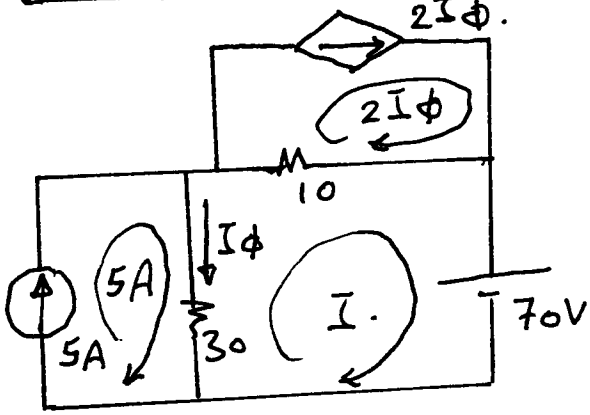
$-70 = (10+30)I - 30\times 5 - 10(2I\phi)$.

$80 = 40I - 20I\phi$.

$\therefore 80 = 40(5 - I\phi) - 20I\phi$.

$\therefore 60I\phi = 120$

$\therefore I\phi = 2$ Amp.



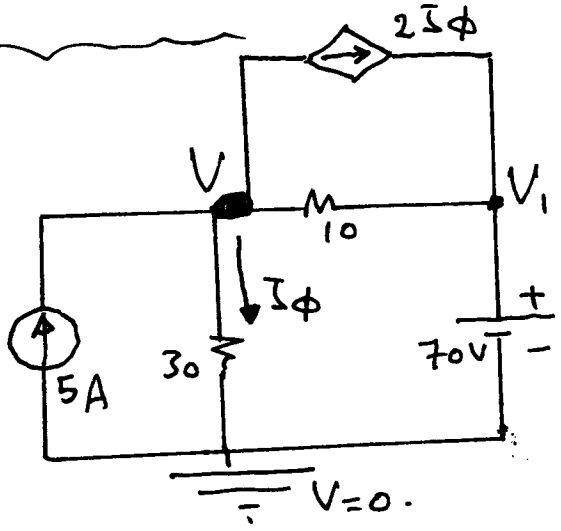
* By Node:

$V_1 = 70$ Volt.

Node(V).

$\left(\frac{1}{10}+\frac{1}{30}\right)V - \frac{70}{10} = 5 - 2I\phi$.

$(3+1)V - 210 = 150 - 60I\phi$.



But $I\phi = \frac{V}{30} \longrightarrow \therefore V = 30I\phi$.

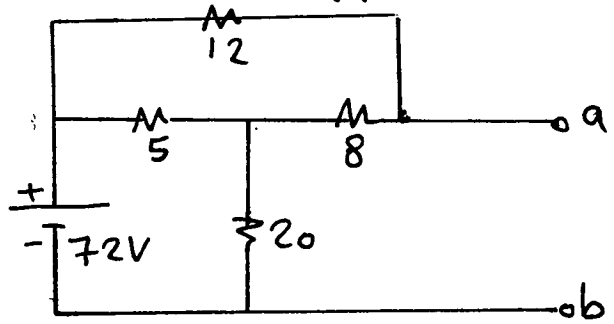
$\therefore 4(30I\phi) - 210 = 150 - 60I\phi$.

$180I\phi = 360$.

$\therefore I\phi = 2$ Amp.

Q37| Find V_{th} & R_{th} between a & b for the circuit?

Solution:

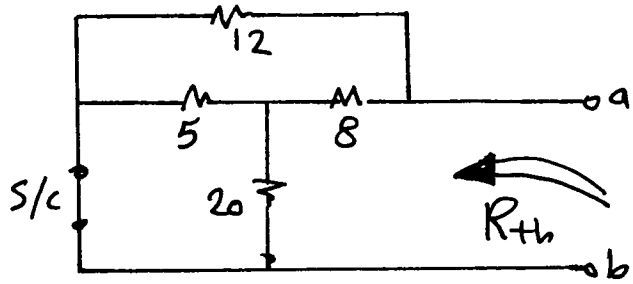


$R_{th} \longrightarrow$

$5 \| 20 = \frac{5 \times 20}{5+20} = 4\Omega$.

$\therefore R_{th} = (4+8) \| 12$

$= 6\Omega$.

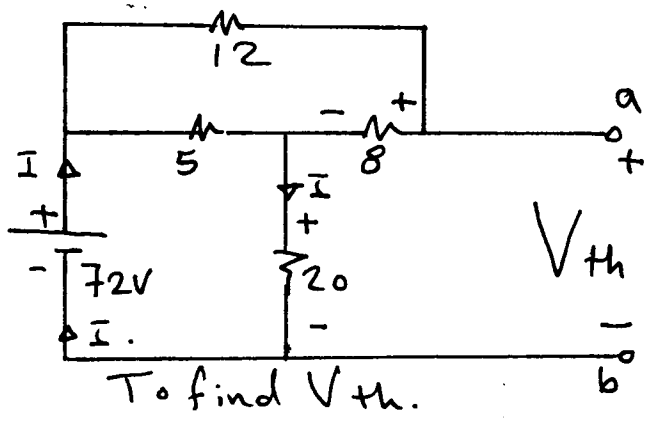


$V_{th} \longrightarrow$

$V_{th} = V_{20\Omega} + V_{8\Omega}$.

$I = \frac{72}{Req}$

$= \frac{72}{((12+8) \| 5) + 20}$



$= \frac{72}{4+20} = \frac{72}{24} = 3$ Amp.

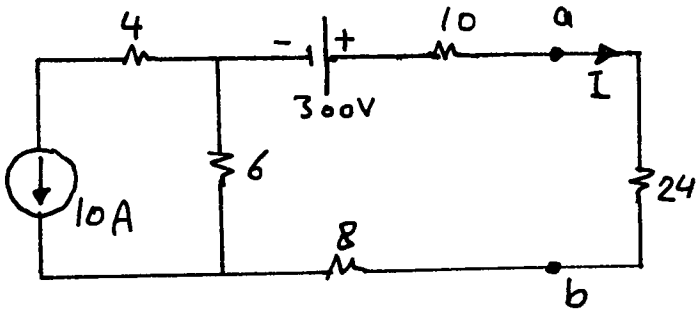
$\therefore V_{20\Omega} = 3 \times 20 = \underline{60 \text{ Volt}}$.

$V_{8\Omega} = 8\left(3 \frac{5}{5+12+8}\right) = 8 \frac{15}{25} = \frac{24}{5} = \underline{4.8 \text{ Volt}}$. (C.D.R)

$\therefore V_{th} = 60 + 4.8 = \underline{64.8 \text{ Volt}}$.

Q38 Find the current (I) and power consumed in (10Ω)?

Solution:



* By Superposition:

** By 300V → 10A (o/c).

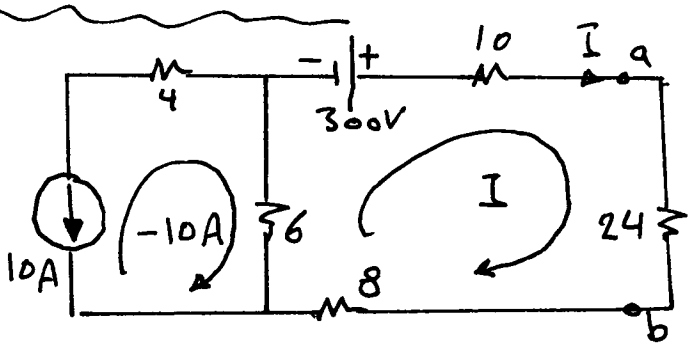
$$I_1 = \frac{300}{6+10+8+24} = \frac{300}{48} \text{ Amp}.$$

** By 10Amp → 300V (S/c).

$$I_2 = 10\,\frac{6}{6+10+8+24} = \frac{60}{48}$$

$$\therefore I = I_1 - I_2 = \frac{300}{48} - \frac{60}{48} = \frac{240}{48} = 5 \text{ Amp}.$$

* By loop:



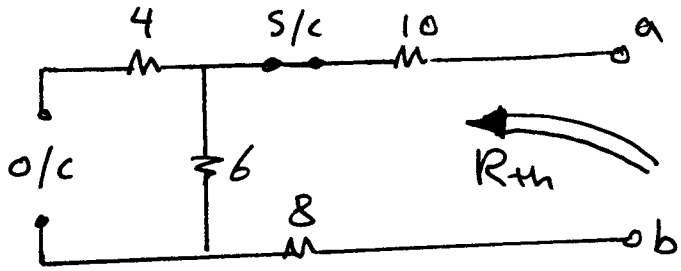
loop (I):-

$$300 = (6+8+10+24)I - 6(-10).$$

$$300 = 48I + 60.$$

$$\therefore I = \frac{300-60}{48} = \frac{240}{48} = 5 \text{ Amp}.$$

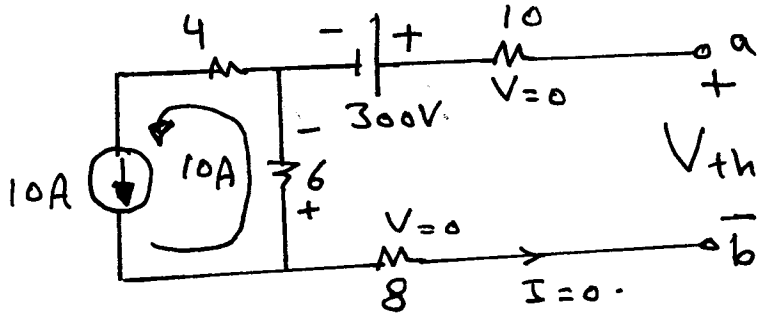
* By Thevenin's Theorem:



$$I = \frac{V_{th}}{R_{th}+24}.$$

R_{th} →

$$R_{th} = 6+8+10 = 24\Omega.$$



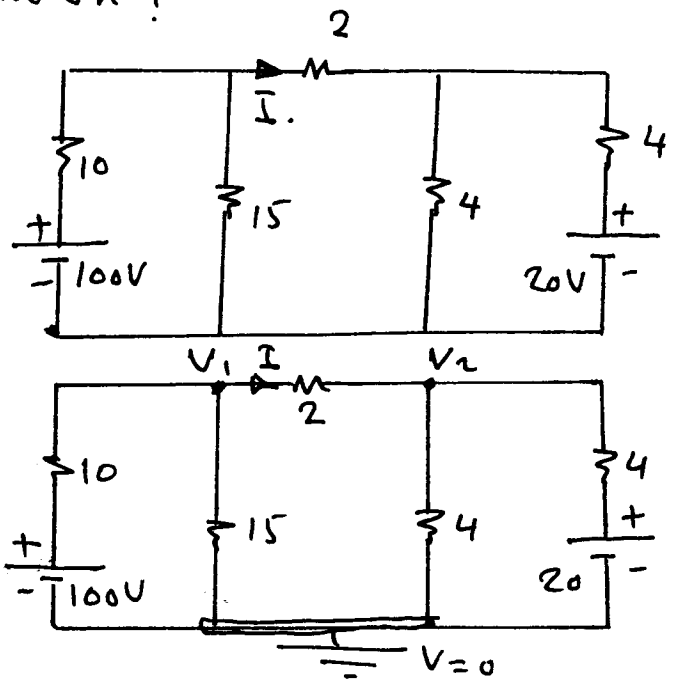
V_{th} →

$$V_{th} = 300 - V_{6\Omega}.$$

$$= 300 - 60 = 240 \text{ Volt}.$$

$$\therefore I = \frac{240}{24+24} = \frac{240}{48} = 5 \text{ Amp}.$$

Q39 Find (I) for the circuit shown?



Solution:

① By Nodal method:

$$I = \frac{V_1 - V_2}{2}$$

Node (V_1):

$$\left(\frac{1}{2}+\frac{1}{10}+\frac{1}{15}\right)V_1 - \frac{V_2}{2} - \frac{100}{10} = 0$$

$$(15+3+2)V_1 - 15V_2 - 300 = 0$$

$$20V_1 - 15V_2 = 300$$

$$4V_1 - 3V_2 = 60 \text{ ---------- } (1)$$

Node (V_2):

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right)V_2 - \frac{V_1}{2} - \frac{20}{4} = 0$$

$$(2+1+1)V_2 - 2V_1 = 20$$

$$4V_2 - 2V_1 = 20 \text{ ---------- } (2)$$

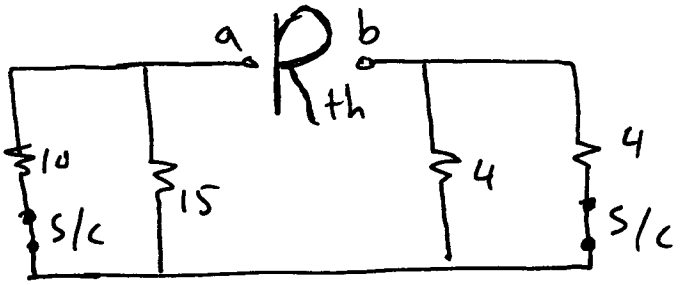
solving eq (1) & (2)

$V_1 = 30$ Volt & $V_2 = 20$ Volt.

$$\therefore I = \frac{30-20}{2} = 5 \text{ Amp}$$

② By Thevenin's Theorem:

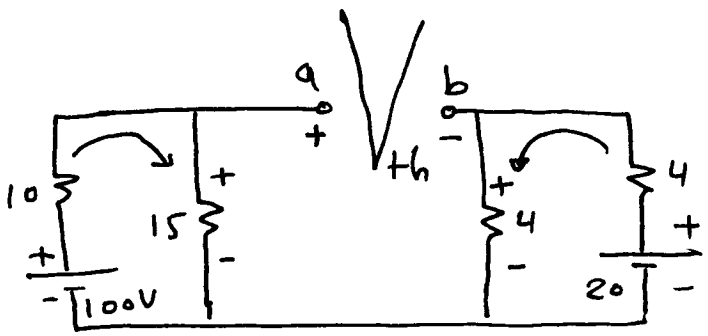
$$I = \frac{V_{th}}{R_{th}+2}$$



R_{th} →

$$R_{th} = (10 \| 15) + (4 \| 4)$$

$$= 6 + 2 = 8\ \Omega$$



V_{th} →

$$V_{th} = V_{15\Omega} - V_{4\Omega}$$

$$= \left(\frac{100}{10+15}\right)\times 15 - \left(\frac{20}{4+4}\right)\times 4$$

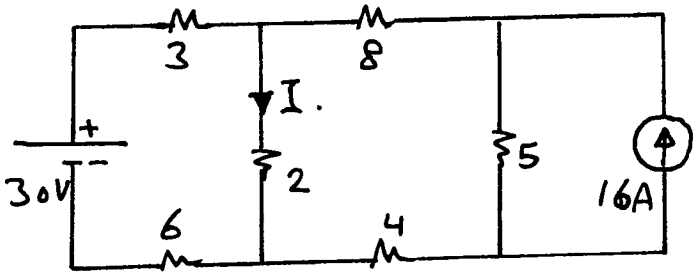
$$= 60 - 10 = 50 \text{ Volt}$$

$$\therefore I = \frac{50}{2+8} = 5 \text{ Amp}$$

Q.40 Find the current (I) for the circuit ?

Solution:

① By loop method :-



$I = I_1 - I_2$.

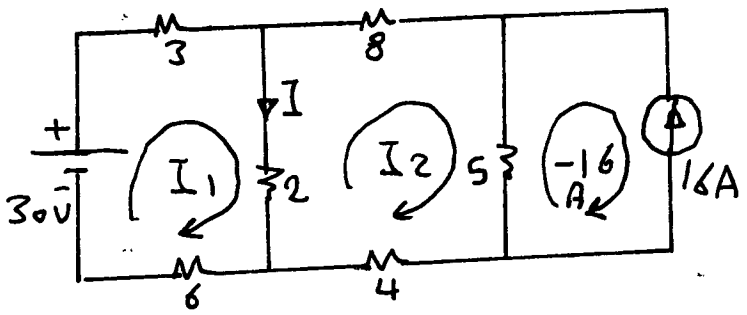
loop(1):

$30 = 11I_1 - 2I_2$ ------- ① .

loop(2):

$0 = 19I_2 - 2I_1 - 5(-16)$.

$-80 = 19I_2 - 2I_1$ ------ ②

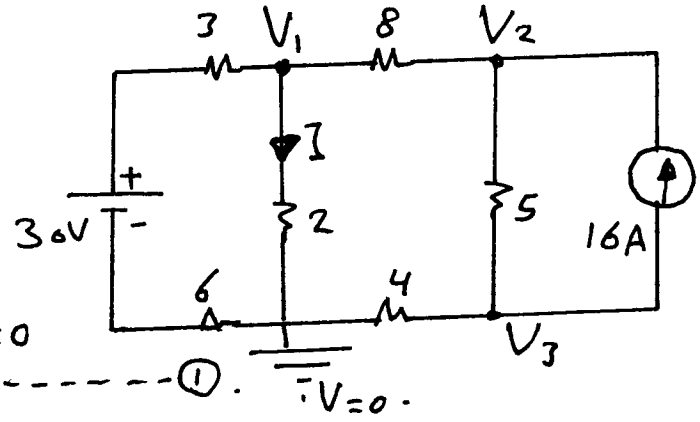


Solving eq(1) & (2)

$I_1 = 2$ Amp & $I_2 = -4$ Amp .

$\therefore I = 2 - (-4) = \underline{6 \text{ Amp}}$.

② By Nodal voltage method :



$I = \frac{V_1}{2}$. Amp .

Node(V_1):

$\left(\frac{1}{9} + \frac{1}{2} + \frac{1}{8}\right)V_1 - \frac{V_2}{8} - \frac{30}{9} = 0$ ------ ① .

$\therefore V = 0$.

Node(V_2): $\left(\frac{1}{8} + \frac{1}{5}\right)V_2 - \frac{V_1}{8} - \frac{V_3}{5} = 16$ ------ ② .

Node(V_3): $\left(\frac{1}{4} + \frac{1}{5}\right)V_3 - \frac{V_2}{5} = -16$ ------ ③ .

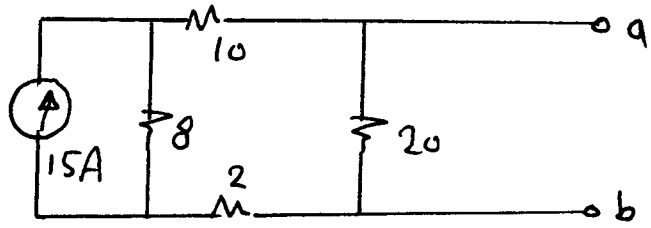
Solving eq(1), (2) & (3) .

$V_1 = 12$ Volt .

$\therefore I = \frac{12}{2} = \underline{6 \text{ Amp}}$.

Q41 Find V_{th} & R_{th} between a & b for the circuit?

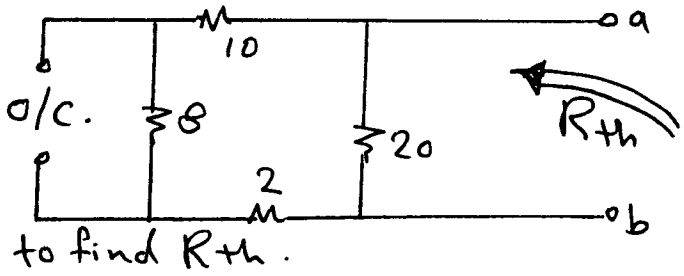
Solution:



R_{th} ⟶

$R_{th} = (2+8+10) \,//\, 20$

$= 10\,\Omega$.



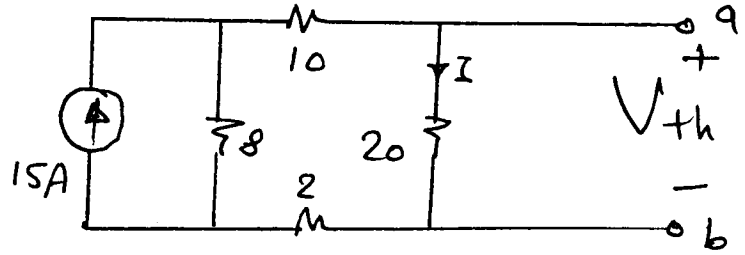
V_{th} ⟶

$V_{th} = V_{ab} = V_{20\Omega}$.

$= 20\,I$.

$= 20\left(15 \dfrac{8}{8+10+20+2}\right)$

$= 60$ Volt



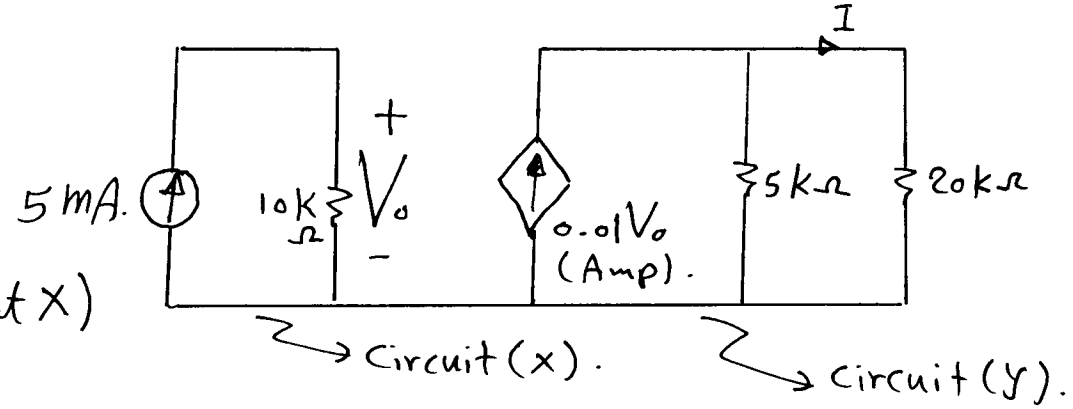
Q42 Find the current, Voltage & power consumed in (20kΩ)?

Solution:

$V_o = 5\times10^{-3}\times10\times10^{3}$

$= 50$ Volt.

(from the circuit X)



from circuit (y).

$I = (0.01V_o)\dfrac{5}{5+20} = 0.5\dfrac{5}{25}$.

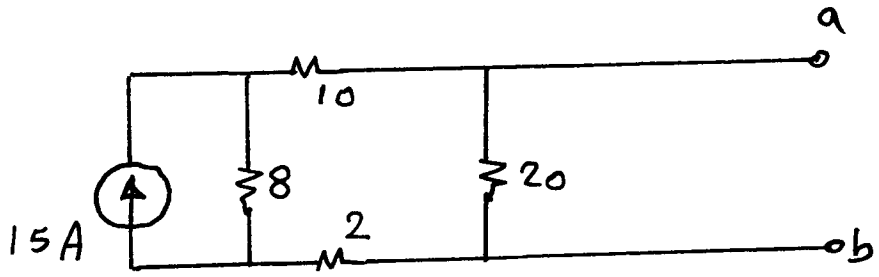
$= 0.1$ Amp.

$V_{20k\Omega} = 0.1\times20\times10^{3} = 2000$ Volt $= 2$ K volt.

$P_{20k\Omega} = I^2 \cdot R = (0.1)^2 \times 20\times1000 = 200$ Watt $= 0.2$ K Watt.

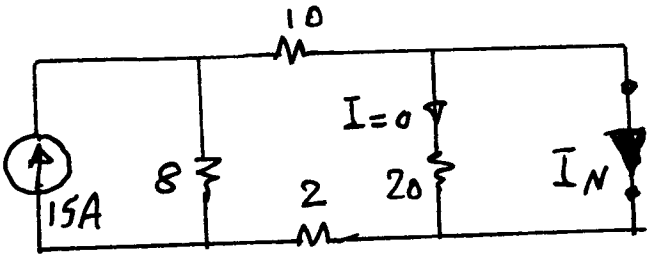
Q43 Find (I_N) between a & b ?

Solution:



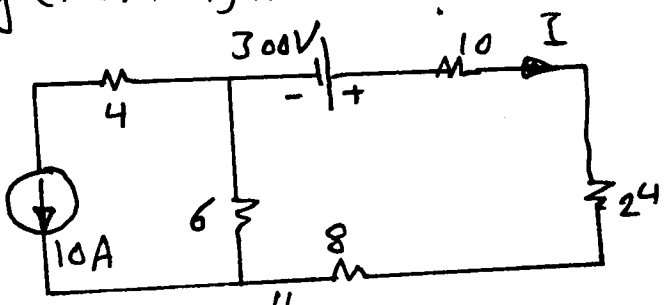
$$I_N = 15\,\frac{8}{8+10+2} = \frac{120}{20} = 6\ \text{Amp}.$$

(current Divide rule).



Q44 Find the current (I) by using (Norton) Theorem ?

Solution:

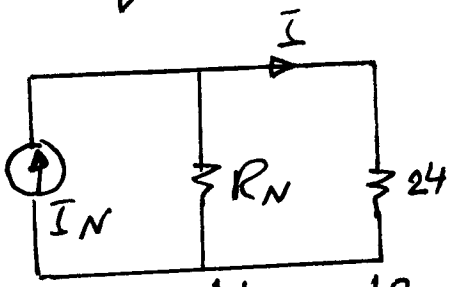


$$I = I_N\,\frac{R_N}{R_N+24}$$

$R_N = R_{th} = 6+8+10 = 24\,\Omega$.

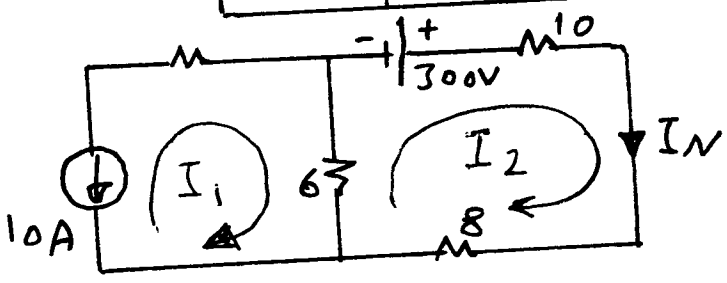
See Q(38).

$$\therefore I = I_N\,\frac{24}{24+24} = \frac{I_N}{2}\ \text{Amp}.$$



* To find (I_N) →

By loop:



$I_1 = -10\ \text{Amp}$.

$I_2 = I_N$.

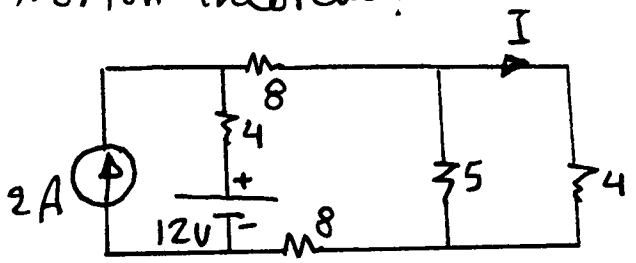
loop(2):

$$300 = (6+8+10)\,I_2 - 6\times I_1$$

$$300 = 24\,I_2 + 60 \longrightarrow I_2 = I_N = \frac{240}{24} = 10\ \text{Amp}$$

$$\therefore I = \frac{10}{2} = 5\ \text{Amp}.$$

Q45 Find the current (I) by Norton Theorem?



Solution:

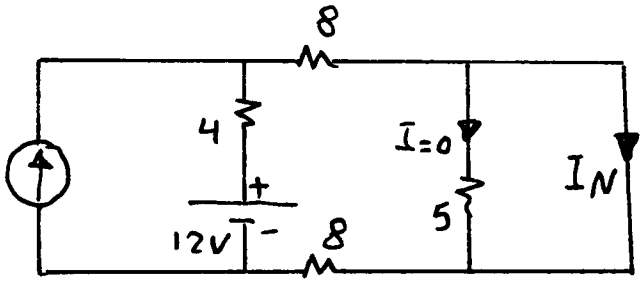
$$I = I_N \frac{R_N}{R_N + 4}.$$

$R_N = R_{th} = 5 // (8+4+8)$

$= 5 // 20 = 4\ \Omega$ (see Q 31).

$\therefore I = I_N \frac{4}{4+4} = \frac{I_N}{2}$ Amp.

$I_N \longrightarrow$



By using Superposition: 2A.

* By (2Amp) → 12V → (S/c).

$\therefore I_{N_1} = 2 \frac{4}{4+8+8} = \frac{8}{20}$ Amp.

* By (12Volt) → 2Amp → (O/c).

$I_{N2} = \frac{12}{4+8+8} = \frac{12}{20}$ Amp.

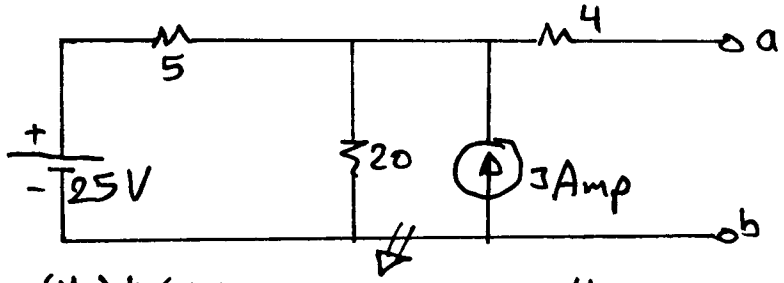
$\therefore I_N = I_{N_1} + I_{N2} = \frac{8}{20} + \frac{12}{20} = 1$ Amp.

Check $R_{th} = R_N = \frac{V_{th}}{I_N} = 4 = \frac{V_{th}}{1}$

$\therefore V_{th} = 4$ Volt. (see Q 31).

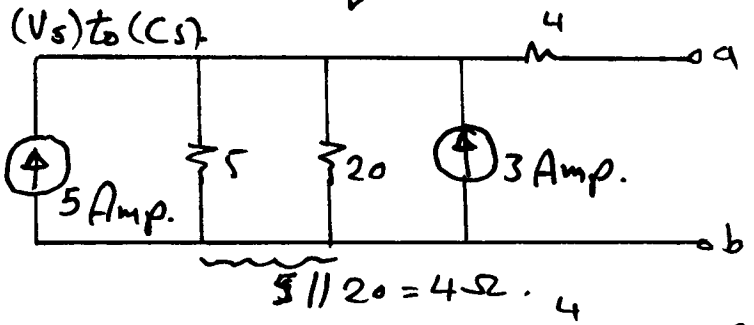
Q46 Find V_{th}, I_N & $R_N = R_{th}$ between a & b?

Solution:



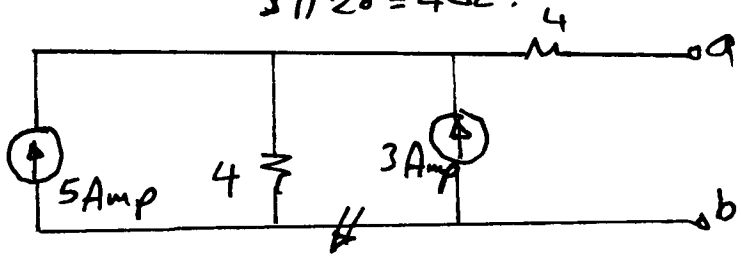
$R_{th} \rightarrow 4+4 = 8\Omega$.

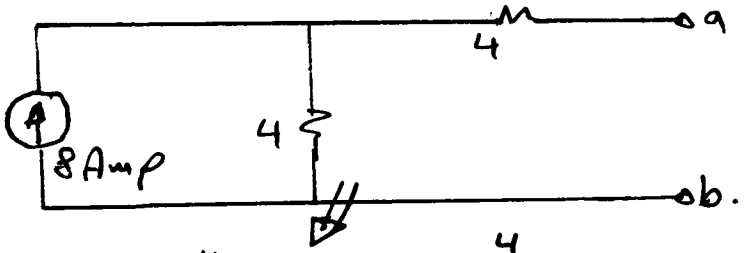
$V_{th} = 32$ Volt.

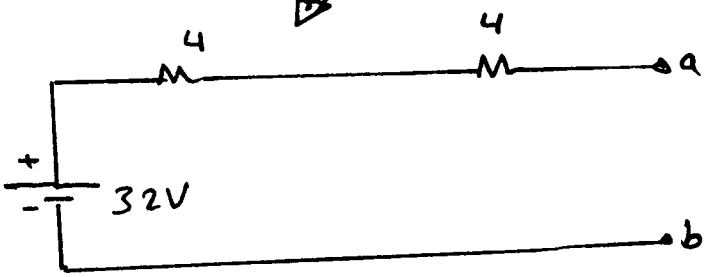


$5 \| 20 = 4\Omega$.

$I_N = \frac{V_{th}}{R_{th}} = \frac{32}{8} = 4$ Amp.

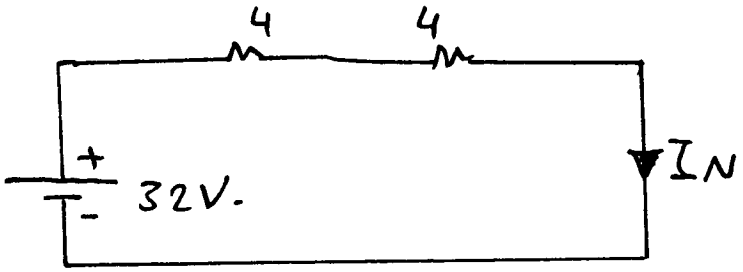






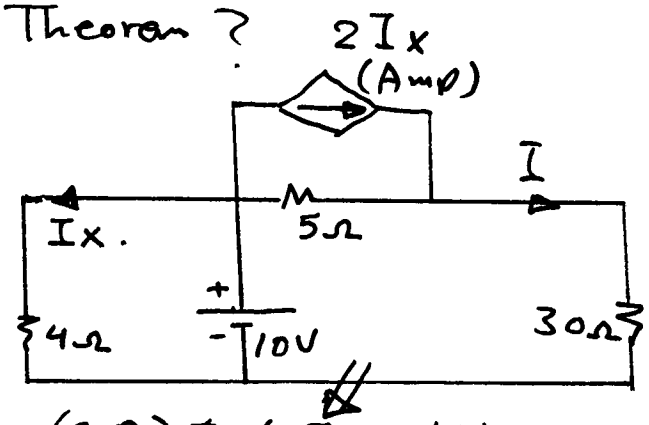
OR

$I_N \rightarrow$



$I_N = \frac{32}{4+4} = 4$ Amp.

Q47 Find (I) by using Norton Theoran ?

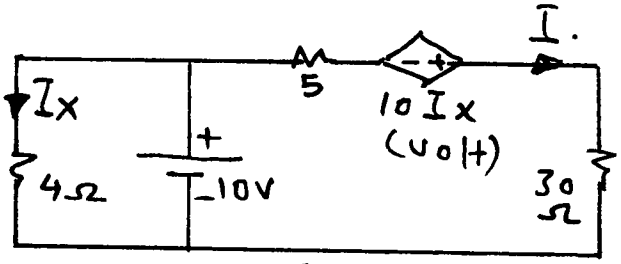


Solution:

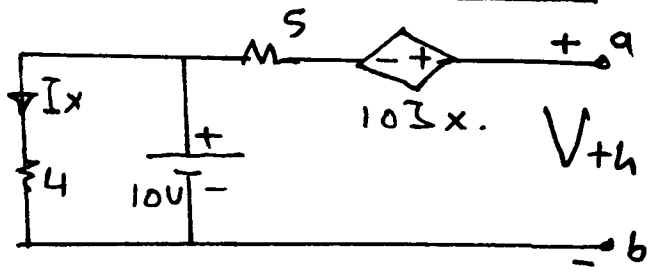
$R_N = R_{th} = \frac{V_{th}}{I_N}$.

(C.D) to ($I_s \rightarrow V_s$).

$I = I_N \frac{R_N}{R_N + 30}$.



* V_{th} →



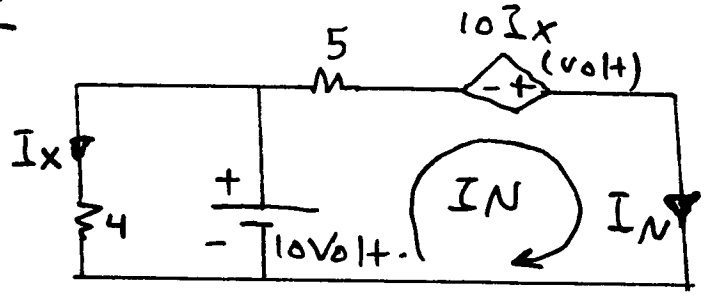
$V_{th} = 10 + 10I_x$. (Kirch. Voltage Law).

But;

$I_x = \frac{10}{4} = 2.5$ Amp.

$\therefore V_{th} = 10 + 10(2.5) = 35$ Volt.

* I_N →



loop(I_N):

$10 + 10I_x = 5I_N$.

But $I_x = \frac{10}{4} = 2.5$ Amp.

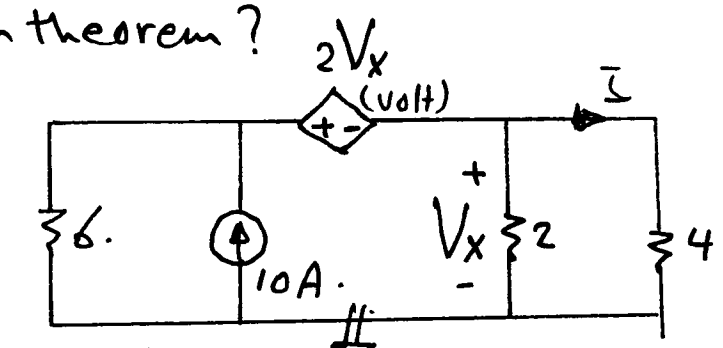
$\therefore 35 = 5I_N$

$\therefore I_N = 7$ Amp.

$\therefore R_N = R_{th} = \frac{35}{7} = 5\Omega$.

$I = 7 \frac{5}{5+30} = \frac{35}{35} = 1$ Amp.

Q48 Find the (I) by Norton theorem?

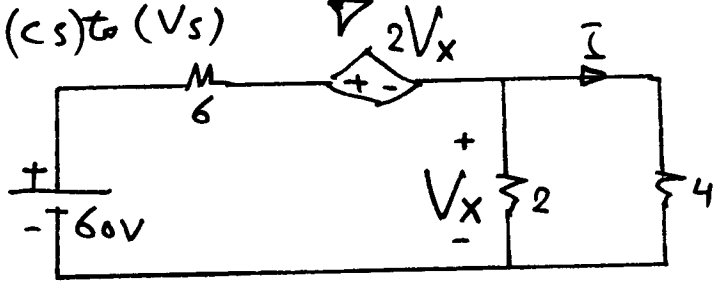


Solution

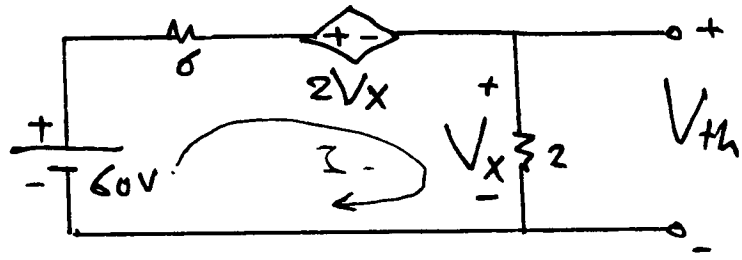
$$I = I_N \frac{R_N}{R_N + 4}.$$

$$R_N = R_{th} = \frac{V_{th}}{I_N}.$$

(cs) to (Vs)



* V_{th} →



$V_{th} = V_{2\Omega} = V_x$.

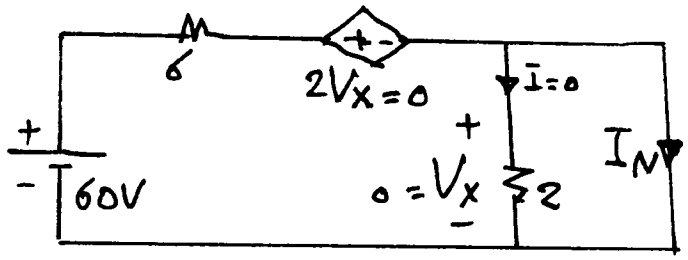
But $V_x = 2I$ (ohm's Law).

$$\therefore V_x = 2\left(\frac{60 - 2V_x}{6+2}\right)$$

$$V_x = \frac{120 - 4V_x}{8} \longrightarrow 8V_x = 120 - 4V_x.$$

$$\therefore 12V_x = 120 \longrightarrow V_x = V_{th} = \underline{10 \text{ Volt}}.$$

* I_N →

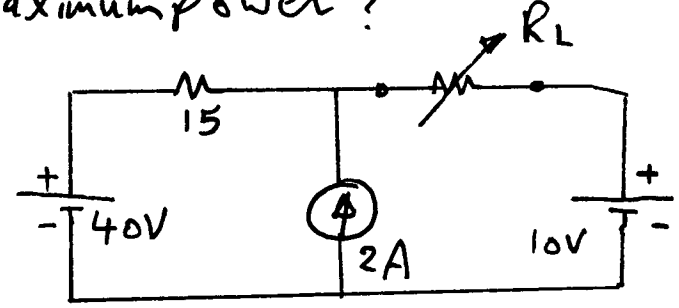


$$I_N = \frac{60}{6} = \underline{10 \text{ Amp}}. \quad \text{(ohms Law)}.$$

$$\therefore R_N = \frac{V_{th}}{I_N} = \frac{10}{10} = 1\Omega.$$

$$\therefore I = 10\,\frac{1}{1+4} = \underline{2 \text{ Amp}}.$$

Q49 Find the Load resistance (R_L) for maximum power to the Load, then find this maximum power?

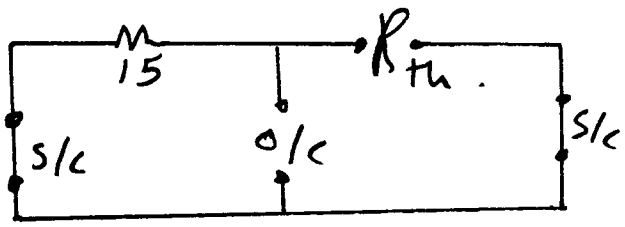


Solution:

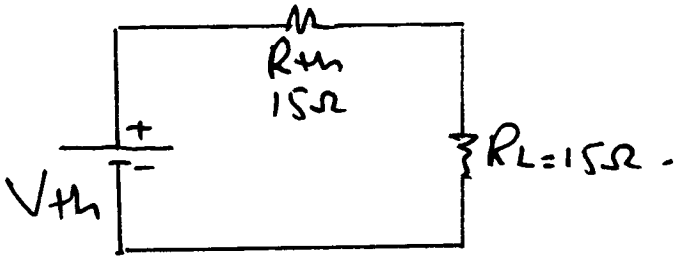
$R_L = R_{th}$

$= 15\Omega$.

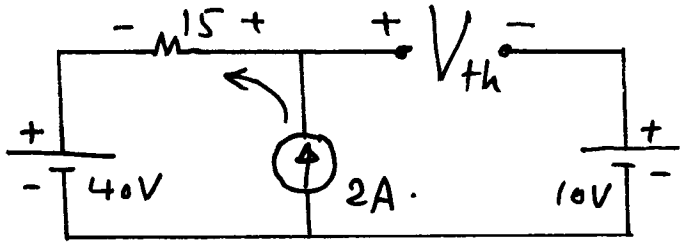
* R_{th} →



$$\therefore I = \frac{V_{th}}{R_{th}+R_L} = \frac{V_{th}}{15+15}.$$



* V_{th} →



By K.V.L.

$40 + 15 \times 2 = V_{th} + 10$

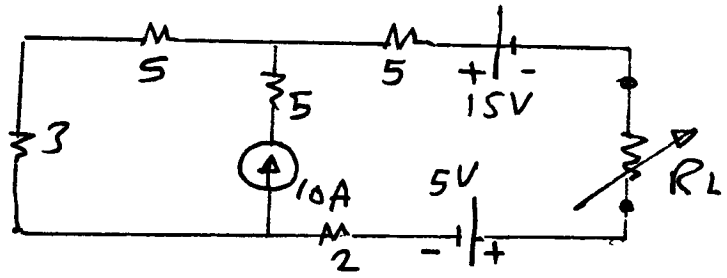
$70 = V_{th} + 10$

$\therefore V_{th} = 60$ volt.

$$\therefore I = \frac{V_{th}}{R_{th}+R_L} = \frac{60}{15+15} = 2 \text{ Amp}.$$

$$P = I^2 . R_L = (2)^2 \times 15 = 60 \text{ Watt}.$$

Q50] Find (R_L) for max. power & find this P_{max} ?



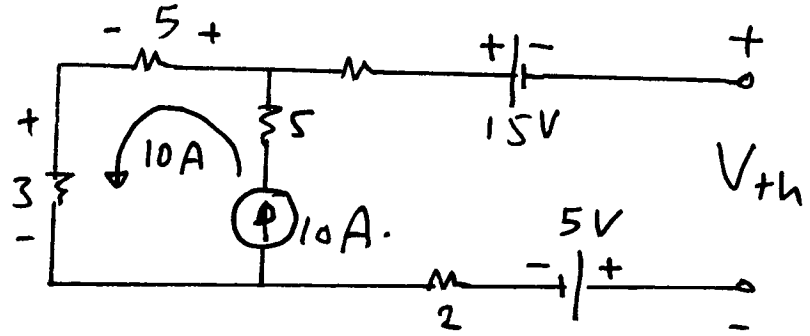
Solution:

$R_L = R_{th}$

$\therefore R_L = 5+5+3+2 = 15\Omega$.

$\therefore I = \frac{V_{th}}{R_{th}+R_L} = \frac{V_{th}}{15+15}$.

* $V_{th} \rightarrow$



$V_{5\Omega} + V_{3\Omega} = 15 + 5 + V_{th}$.

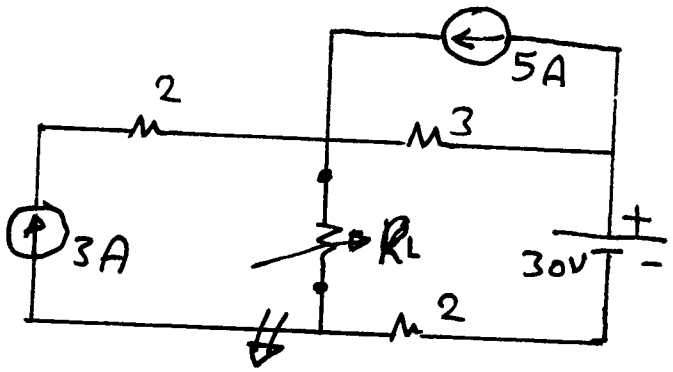
$10\times5 + 10\times3 = 20 + V_{th}$.

$80 = 20 + V_{th} \longrightarrow \therefore V_{th} = 60$ Volt.

$\therefore I = \frac{60}{15+15} = 2$ Amp.

$\therefore P = I^2 \cdot R_L = (2)^2 \times 15 = 60$ watt.

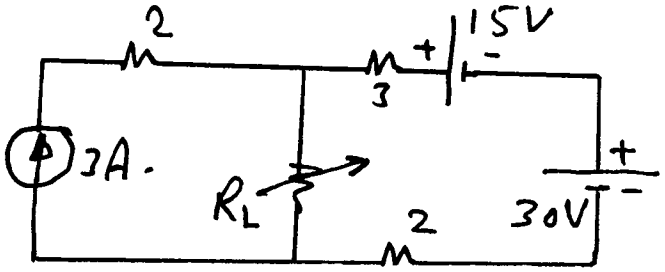
Q51 find (R_L) for max. power & find this P_{max}?



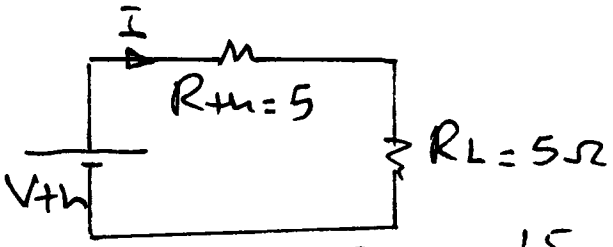
Solution

$R_L = R_{th}$.

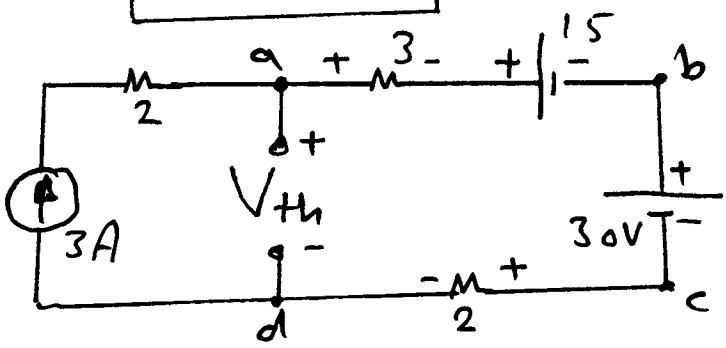
$\therefore R_L = 3 + 2 = 5\Omega$.



$$I = \frac{V_{th}}{R_{th}+R_L} = \frac{V_{th}}{5+5}.$$



* V_{th} → take abcd



$V_{th} = V_{3\Omega} + 15 + 30 + V_{2\Omega}$.

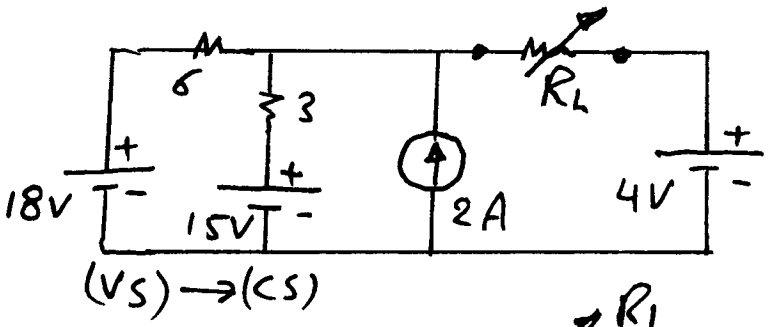
$= 3\times3 + 15 + 30 + 3\times2$.

$= 60$ Volt.

$\therefore I = \frac{60}{5+5} = 6$ Amp.

$P = I^2 \cdot R_L = (6)^2 \times 5 = 180$ watt.

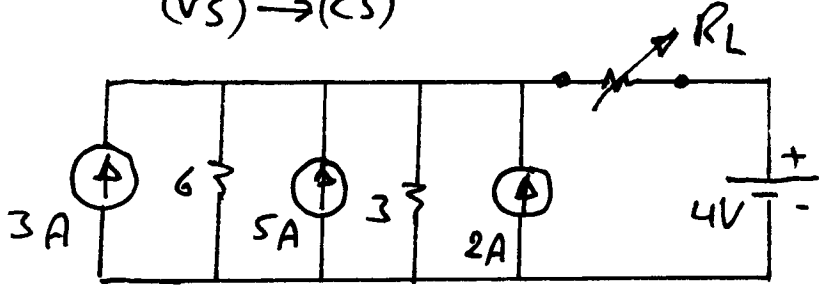
Q52 Find (R_L) for max. power & find this P_{max} ?



Solution:

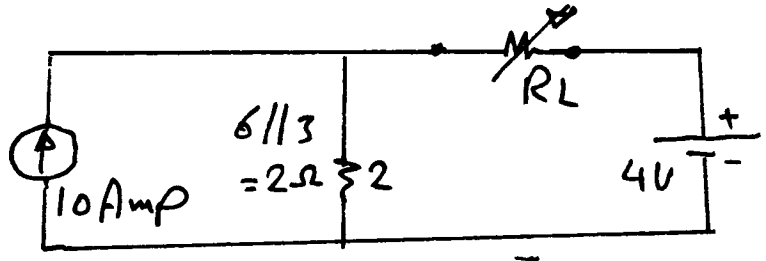
$R_L = R_{th}$.

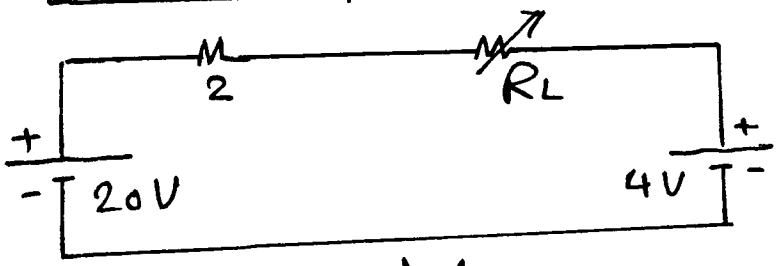
$\therefore R_L = 2\Omega$.



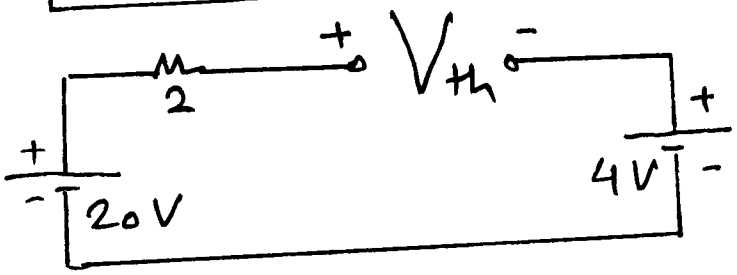
$$I = \frac{V_{th}}{R_{th}+R_L}$$

$$I = \frac{V_{th}}{2+2}$$





V_{th} →



$20 = V_{th} + 4$.

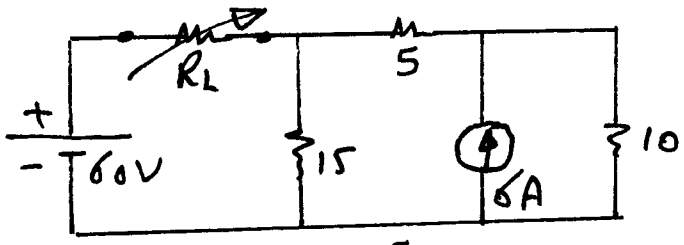
$\therefore V_{th} = 16$ volt.

$$I = \frac{16}{2+2} = 4 \text{ Amp}.$$

$$\therefore P = I^2 \cdot R_L = (4)^2 \times 2 = 32 \text{ watts}$$

Q53 | Find (R_L) for max. power, & find this P_{max}?

Solution:

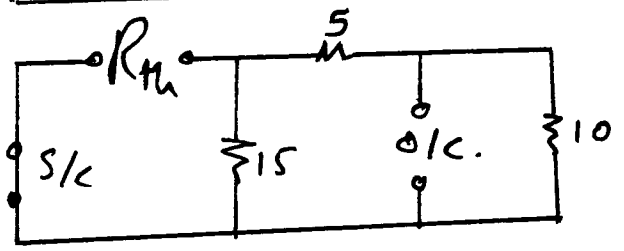


$R_L = R_{th}$

* $\therefore R_L \longrightarrow$

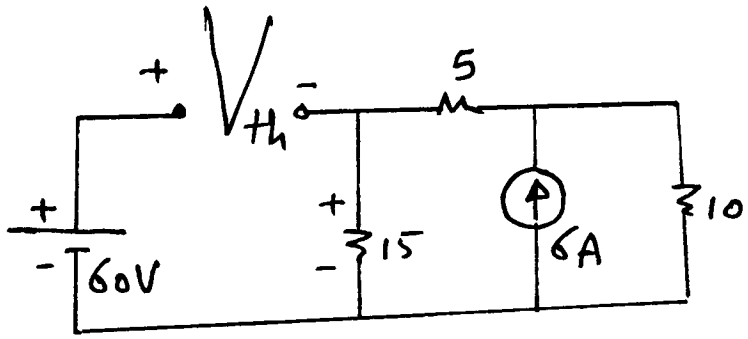
$= 15 // (5+10)$

$= 15 // 15 = \underline{7.5\ \Omega}$



$\therefore I = \dfrac{V_{th}}{R_{th}+R_L}$

$= \dfrac{V_{th}}{7.5+7.5}$.



* $V_{th} \longrightarrow$

$60 = V_{th} + V_{15\Omega}$.

$I_{in\,15\Omega} = 6\dfrac{10}{10+5+15} = \dfrac{60}{30} = 2\,Amp \longrightarrow (C.D.R)$.

$\therefore 60 = V_{th} + 15 \times 2$

$\therefore V_{th} = \underline{30\ volt}$.

$\therefore I = \dfrac{30}{7.5+7.5} = \underline{2\,Amp}$.

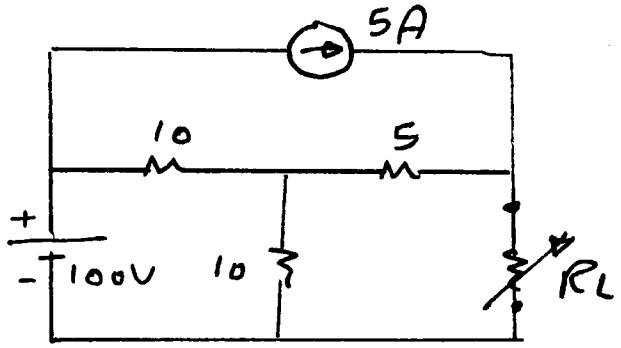
$P = I^2 . R_L$

$= (2)^2 \times 7.5 = \underline{30\ Watt}$.

Q54 Find (R_L) for max. power & find this P_{max} ?

Solution:

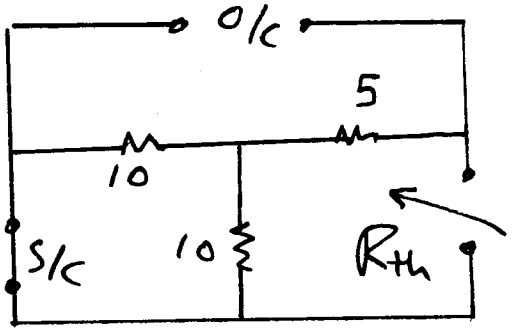
$R_L = R_{th}$.



* $R_L \longrightarrow$

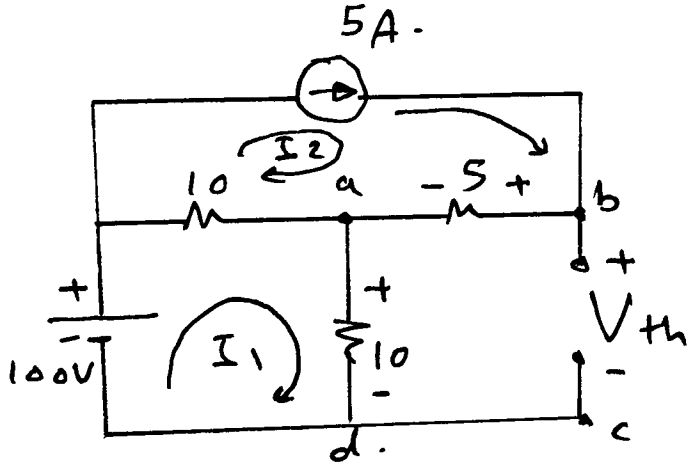
$\therefore R_L = R_{th} = 5 + (10 \| 10)$

$= 10\,\Omega$.



$\therefore I = \dfrac{V_{th}}{R_{th}+R_L}$

$= \dfrac{V_{th}}{10+10}$.



* $V_{th} \longrightarrow$

take abcd

$V_{th} = V_{5\Omega} + V_{10\Omega}$.

$V_{5\Omega} = 5 \times 5 = 25$ Volt.

$I_2 = 5$ Amp.

$\therefore$ loop (1):

$100 = (10+10) I_1 - 10 I_2$.

$100 = 20 I_1 - 10 \times 5$

$150 = 20 I_1 \longrightarrow \therefore I_1 = \dfrac{150}{20} = 7.5$.

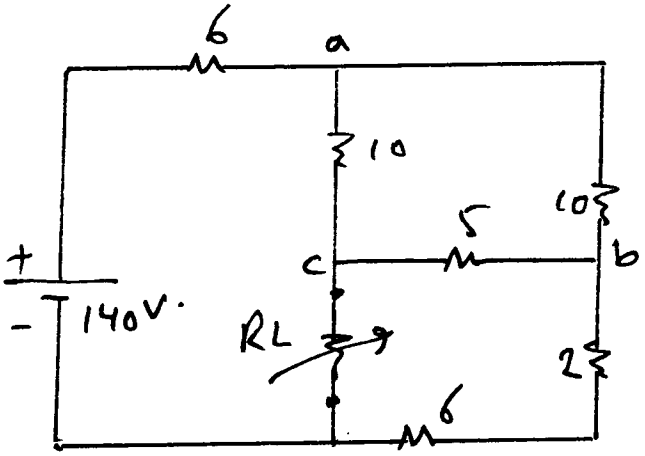
$V_{10\Omega} = 7.5 \times 10 = 75$ Volt.

$\therefore V_{th} = 25 + 75 = 100$ Volt.

$\therefore I = \dfrac{100}{10+10} = 5$ Amp.

$P = I^2 R_L = (5)^2 \times 10 = 250$ Watts.

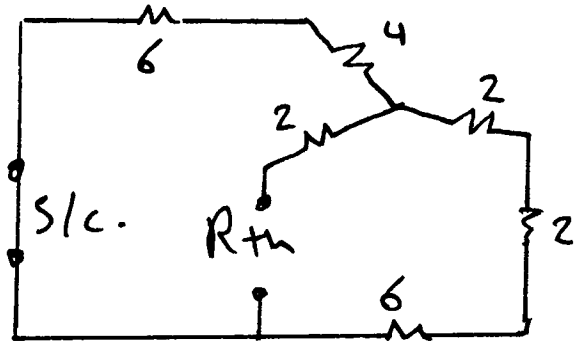
Q55 Find (R_L) for max. power ?



Solution:

$R_L = R_{th}$.

$R_L \rightarrow$



$$\therefore R_L = 2 + \left[(6+4)\,//\,(2+2+6)\right]$$

$$= 2 + (10\,//\,10)$$

$$= 7\ \Omega.$$

$\Delta \Rightarrow Y$

$$R_a = \frac{10 \times 10}{10+10+5} = 4\Omega.$$

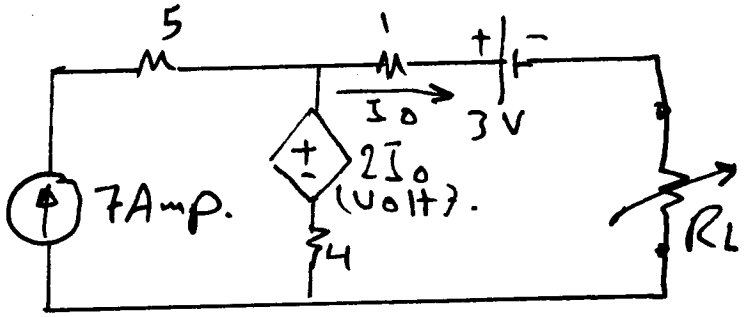
$$R_b = \frac{5 \times 10}{25} = 2\Omega.$$

$$R_c = \frac{5 \times 10}{25} = 2\Omega.$$

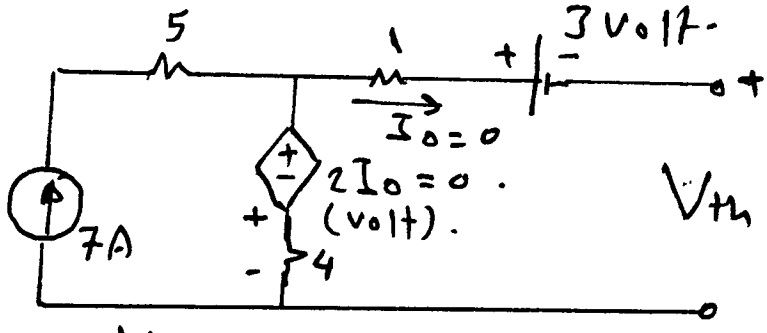
Q56 Find (R_L) for max. power then find this P_{max} ?

Solution:

$$R_L = R_{th} = \frac{V_{th}}{I_N}.$$



* V_{th} →



I_o = Zero (open circuit).

$\therefore 2I_o \to 0$.

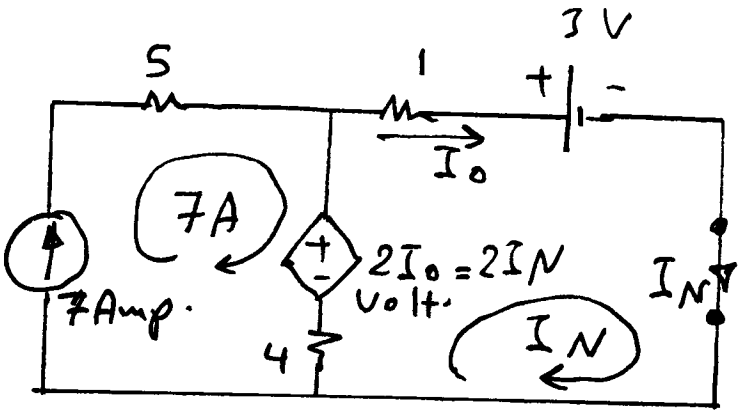
$V_{1\Omega} = 0$.

$\therefore V_{th} + 3 = 4 \times 7$.

$\therefore V_{th} = 25$ volt.

$V_{4\Omega} = 4 \times 7 = 28$ volt.

* I_N →



$I_N = I_o$.

$\therefore 2I_o = 2I_N$

loop (1) = 7 Amp.

loop (I_N):

$2I_N - 3 = 5I_N - 4 \times 7$.

$25 = 3I_N$.

$\therefore I_N = 25/3$ Amp.

$\therefore R_L = R_{th} = \frac{25}{25/3} = 3\Omega$.

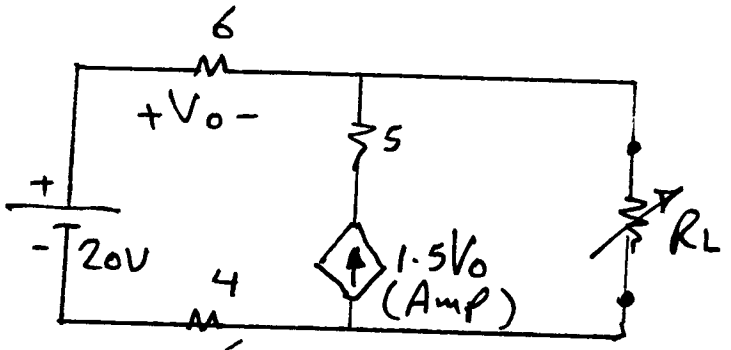
$\therefore I = \frac{V_{th}}{R_{th} + R_L} = \frac{25}{3+3}$ Amp.

$\therefore P = I^2 . R_L = \left(\frac{25}{6}\right)^2 \times 3 = 52$ watt.

Q57 | Find (R_L) for P_{max}. & Find this P_{max} ?

Solution

$$R_L = R_{th} = \frac{V_{th}}{I_N}$$

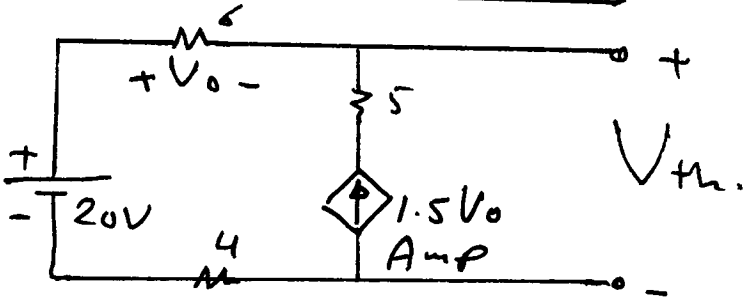


* $V_{th} \rightarrow$

$V_o = V_{6\Omega} = 6 \times 1.5 V_o$.

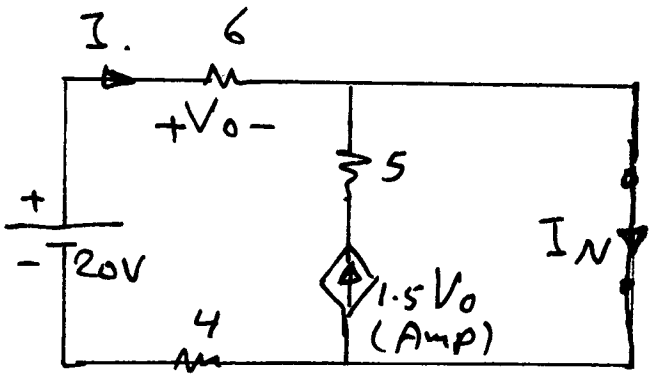
$\therefore V_o = 9 V_o$.

$\therefore$ Vo must be Zero



$\therefore$ $9.5 V_o \rightarrow = 0$.

$\therefore V_{th} = 20$ Volt.



* $I_N \rightarrow$

By K.C.L

$I + 1.5 V_o = I_N$.

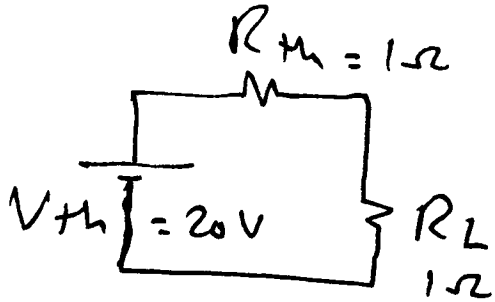
$\therefore I_N = \frac{20}{6+4} + 1.5 V_o$

$= 2 + 1.5 V_o$

But $V_o = 6I = 6 \times 2 = 12$ Volt.

$\therefore I_N = 2 + 1.5 \times 12 = 20$ Amp.

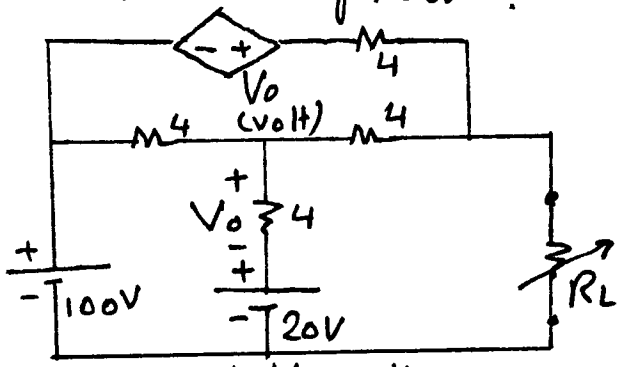
$\therefore R_L = R_{th} = \frac{20}{20} = 1\Omega$



$\therefore I = \frac{20}{1+1} = 10$ Amp.

$P = I^2 \cdot R_L = (10)^2 \times 1 = 100$ watt.

Q58 Find (R_L) for max. power then find the P_{max}?

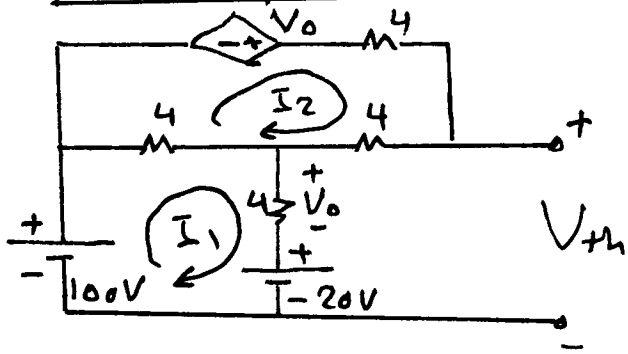


Solution:

$$R_L = R_{th} = \frac{V_{th}}{I_N}.$$

* $V_{th} \longrightarrow$

$$V_{th} = 20 + V_{4\Omega} + V_{4\Omega}.$$



loop(1):

$$100 - 20 = (4+4)I_1 - 4I_2$$

$$80 = 8I_1 - 4I_2.$$

$$\boxed{20 = 2I_1 - I_2} \quad \text{---(1)}.$$

loop(2):

$$V_o = (4+4+4)I_2 - 4I_1$$

But $V_o = 4I_1$

$$\therefore 4I_1 = 12I_2 - 4I_1$$

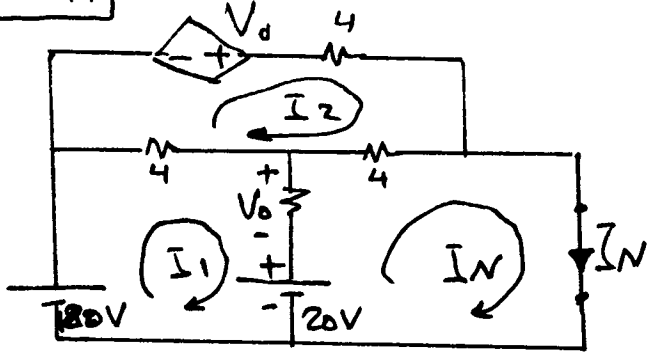
$$\boxed{8I_1 = 12I_2} \quad \text{---(2)} \longrightarrow I_1 = \frac{3I_2}{2}.$$

$$\therefore 20 = 2\left(\frac{3I_2}{2}\right) - I_2.$$

$$I_2 = 10\ Amp. \longrightarrow \therefore I_1 = \frac{3 \times 10}{2} = 15\ Amp.$$

$$\therefore V_{th} = 20 + 10 \times 4 + 10 \times 15 = \boxed{120\ Volt}$$

* $I_N \longrightarrow$



loop(1):

$$100 - 20 = 8I_1 - 4I_2 - 4I_N$$

$$80 = 8I_1 - 4I_2 - 4I_N$$

$$\boxed{20 = 2I_1 - I_2 - I_N} \quad \text{---(1)}.$$

loop(N):

$$20 = 8I_N - 4I_1 - 4I_2.$$

$$\boxed{5 = 2I_N - I_1 - I_2} \quad \text{---(2)}.$$

Loop (2):

$V_o = 12I_2 - 4I_1 - 4I_N$. But $V_o = 4(I_1 - I_N)$.

$\therefore 4(I_1 - I_N) = 12I_2 - 4I_1 - 4I_N$.

$4I_1 - 4I_N = 12I_2 - 4I_1 - 4I_N$.

$\therefore \boxed{8I_1 = 12I_2}$. ——→ (3).

$I_1 = \frac{3I_2}{2}$.

Solving: $I_1 = 45$ Amp.

$I_2 = 30$ Amp.

put I_1 & I_2 in eq (2).

$5 = 2I_N - 45 - 30$

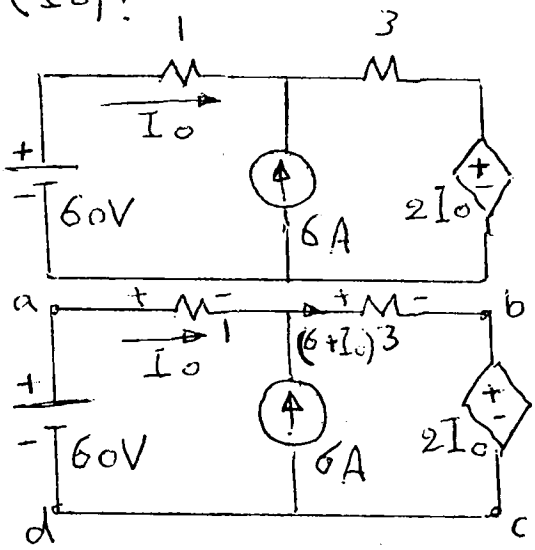
$80 = 2I_N$ ——→ $\boxed{I_N = 40 \text{ Amp}}$

$\therefore R_{th} = R_L = \frac{120}{40} = \underline{3\ \Omega}$

$\therefore I = \frac{120}{3+3} = \frac{120}{6} = \underline{20 \text{ Amp}}$.

$\therefore P = I^2 \cdot R_L = (20)^2 \times 3 = \underline{1200 \text{ watt}}$.

Q1 For the circuit shown, Find (I_o)?



a: By using Kirchhoffs Law:

abcd

$60 \pm I_o + 3(6+I_o) + 2I_o$.

$60 = I_o + 18 + 3I_o + 2I_o$

$42 = 6I_o$

$\therefore I_o = 7$ Amp

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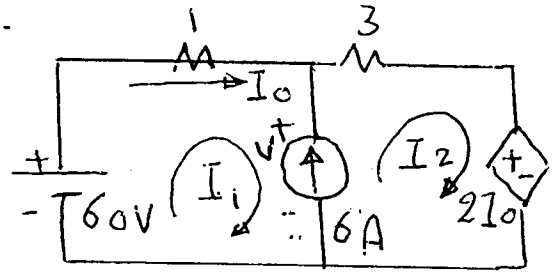
b: By mesh (loop current) method.



loop(1): $I_o = I_1$

$60 - V = I_o$ ——— (1).

loop(2):

$V - 2I_o = 3I_2$. ——— (2).

From eq(1): $V = 60 - I_o$.

$\therefore (60 - I_o) - 2I_o = 3I_2$

But: $6 = I_2 - I_1 = I_2 - I_o$.

$\therefore (60 - I_o) - 2I_o = 3(6 + I_o)$.

$\therefore 42 = 6I_o$

$\therefore I_o = 7$ Amp.

OR: take K.L for abcd

$\therefore 60 = I_1 + 3I_2 + 2I_o$

But $I_1 = I_o$ & $I_2 = 6 + I_1 = 6 + I_o$.

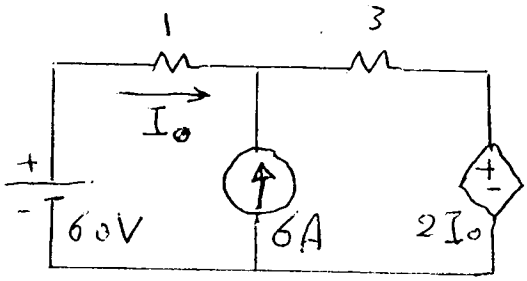
$\therefore 60 = I_o + 3(I_o + 6) + 2I_o$.

$42 = 6I_o$

$I_o = 7$ Amp

c: By Superposition method.

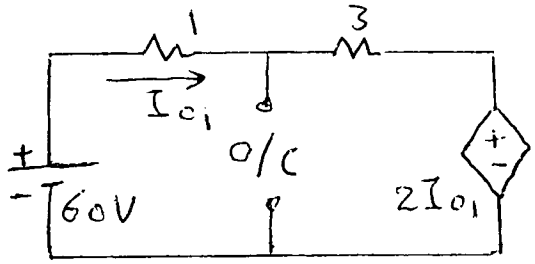
1- By effect of 60V → 6A (o/c).



$$\therefore I_{o_1} = \frac{60 - 2I_{o_1}}{4}$$

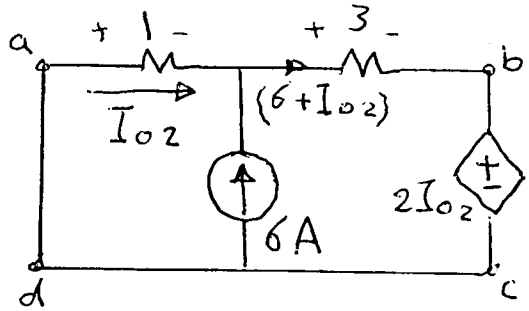
$$4I_{o_1} = 60 - 2I_{o_1}$$

$$\therefore I_{o_1} = 10 \text{ Amp}.$$



2- By effect of 6Amp → 60V (s/c).

take abcd



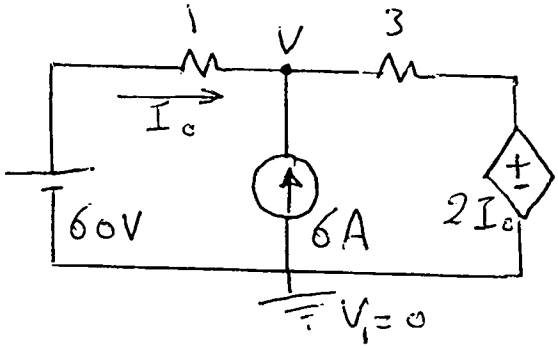
$$I_{o2} + 3(6 + I_{o2}) + 2I_{o2} = 0.$$

$$18 = -6I_{o2}.$$

$$\therefore I_{o2} = -3 \text{ Amp}.$$

But $I_o = I_{o_1} + I_{o2} = 7 \text{ Amp}$

d: By Nodal equation method.



Node (V):

$$\left(\frac{1}{3} + \frac{1}{1}\right)V - \frac{60}{1} - \frac{2I_o}{3} = 6.$$

$$(1+3)V - 180 - 2I_o = 18$$

$$4V - 2I_o = 198.$$

But $I_o = \frac{60 - V}{1}$

$$\therefore 4V - 2(60 - V) = 198.$$

$$6V = 318$$

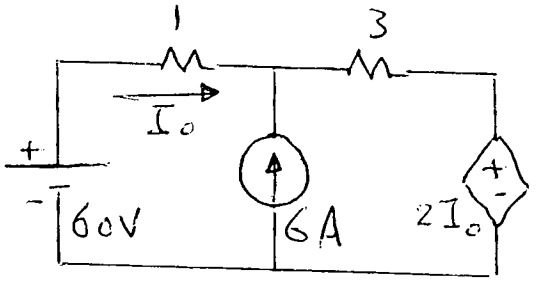
$$\therefore V = 53 \text{ Volt}.$$

$$\therefore I_o = 60 - V$$

$$I_o = 7 \text{ Amp}.$$

e: By Thevenin or Norton Theory

$$R_{th} = R_N = \frac{V_{th}}{I_N} = \frac{V_{o/c}}{I_{s/c}}$$



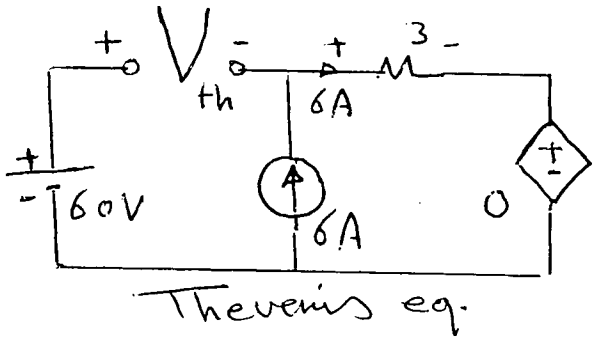
* V_{th}:

Since $I_o = 0$

$\therefore$ the depend source $(2I_o) = 0$.

$\therefore V_{th} = 60 - 3 \times 6$

$V_{th} = 42$ Volt.



Thevenin eq.

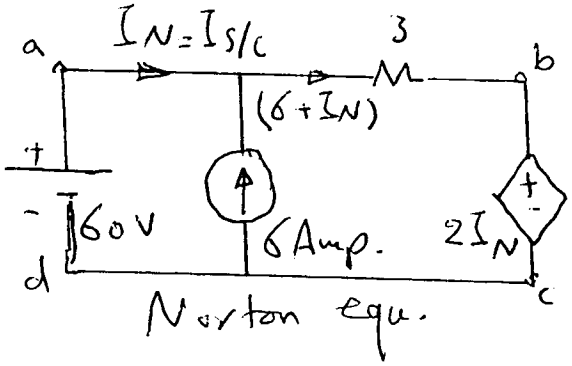
* I_N

abcd

$60 = 3(6 + I_N) + 2I_N$.

$42 = 5I_N$.

$\therefore I_N = \frac{42}{5}$ Amp.



Norton equ.

$\therefore R_{th} = R_N = \frac{42}{42/5} = 5\Omega$.

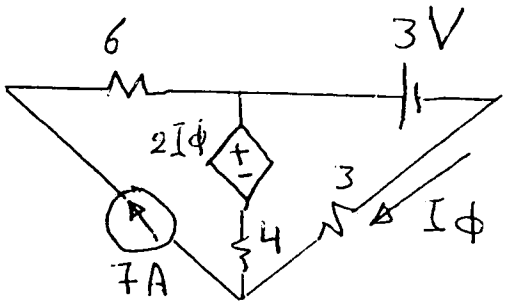
$\therefore$ By Thevenin equivalent

$$I = \frac{V_{th}}{R_{th}+1} = \frac{42}{5+1} = \underline{7 \text{ Amp}}$$

By Nortons equivalent

$$I = I_N \frac{R_N}{R_N+1} = \frac{42}{5} \times \frac{5}{5+1} = \underline{7 \text{ Amp}}$$

Q.2 Find (I_ϕ) for the circuit shown



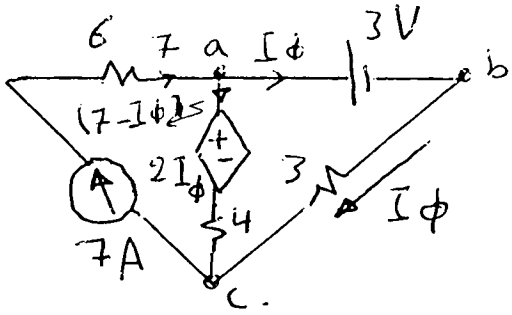
a: By K.L:

a b c

$4(7-I_\phi)+2I_\phi = 3+3I_\phi$

$25 = 5I_\phi$

$\therefore I_\phi = 5$ Amp.



b: By loop current method.

loop(1): $I_1 = 7$ Amp.

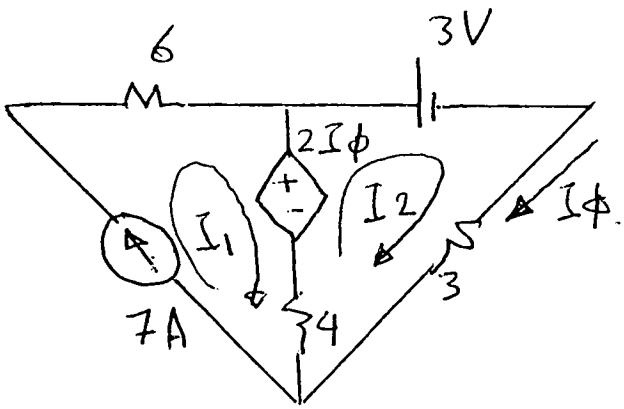
loop(2): $I_2 = I_\phi$

$2I_\phi - 3 = (3+4)I_2 - 4I_1$

$2I_\phi - 3 = 7I_\phi - 4\times 7$

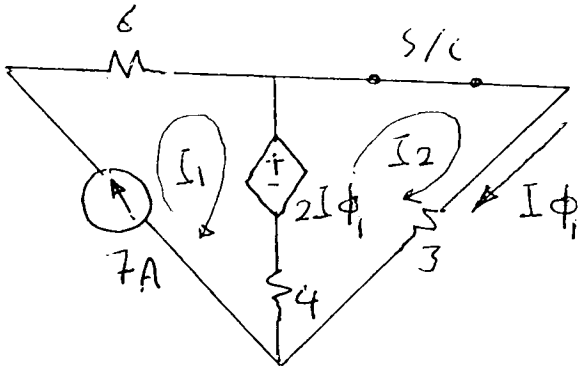
$25 = 5I_\phi$.

$\therefore I_\phi = 5$ Amp



c; By Superposition method

1- By effect of 7Amp → 3V (S/C).



loop(1) : $I_1 = 7$ Amp.

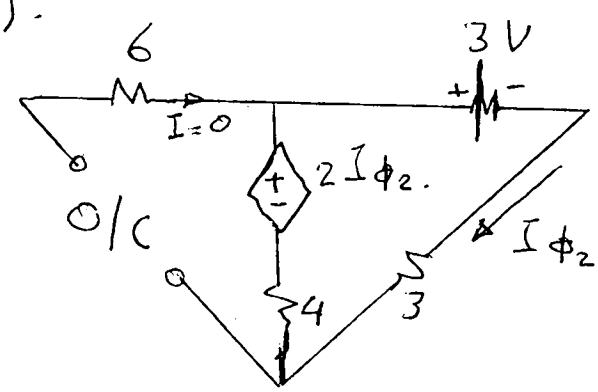
loop(2) : $I\phi_1 = I_2$.

$2I\phi_1 = 7I\phi_1 - 4I_1$

$\therefore 5I\phi_1 = 28$.

$I\phi_1 = 28/5$ Amp.

2. By effect of 3V → 7A (O/C).



$\therefore I\phi_2 = \frac{2I\phi_2 - 3}{7}$.

$7I\phi_2 = 2I\phi - 3$

$\therefore 5I\phi_2 = -3$.

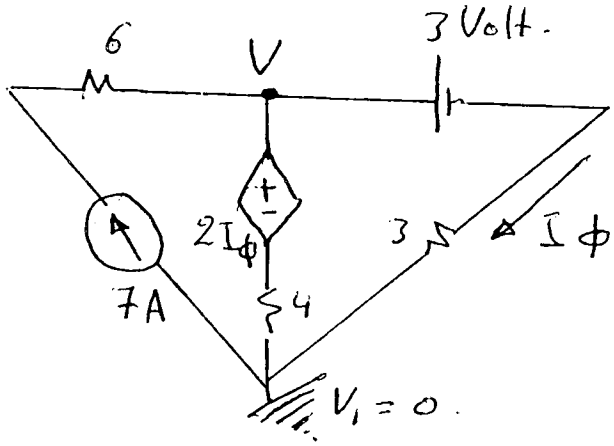
$\therefore I\phi_2 = -\frac{3}{5}$.

But $I\phi = I\phi_1 + I\phi_2$.

$= \frac{28}{5} + \left(-\frac{3}{5}\right)$.

$= \frac{25}{5} = 5$ Amp

d.: By Nodal Voltage method.



Node (v):

$$\left(\frac{1}{4}+\frac{1}{3}\right)V - \frac{2I\phi}{4} - \frac{3}{3} = 7.$$

$$(3+4)V - 6I\phi - 12 = 84$$

$$7V - 6I\phi = 96.$$

But $I\phi = \frac{V-3}{3}$.

$$\therefore 7V - 6\left(\frac{V-3}{3}\right) = 96.$$

$$7V - 2V + 6 = 96$$

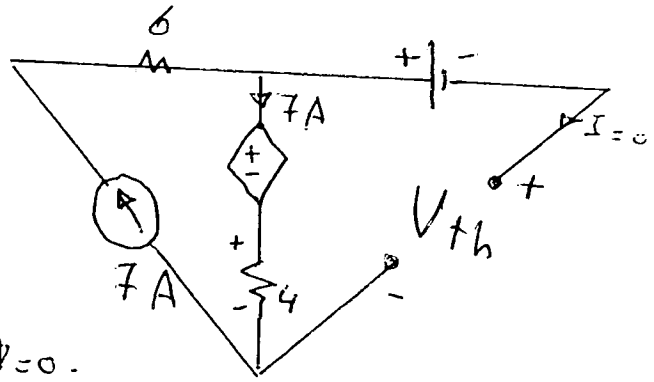
$$5V = 90.$$

$$\therefore V = 18 \text{ Volt}.$$

$$\therefore I\phi = \frac{18-3}{3} = 5 \text{ Amp}$$

e: By Thevenin or Norton Theory

$$R_{th} = R_N = \frac{V_{th}}{I_N}$$



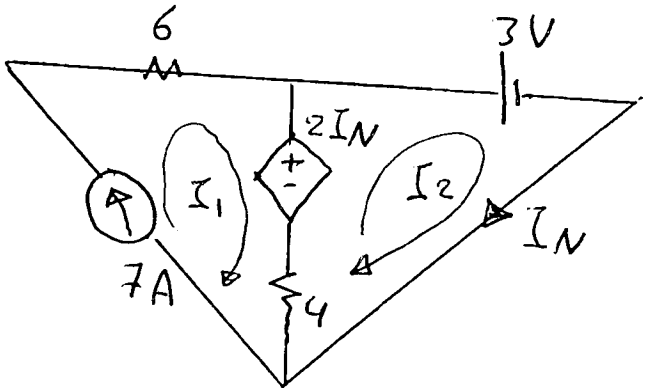
* V_{th} :

Since $I_\phi = 0$.

∴ the dependent source $= 2I_\phi = 0$.

∴ $V_{th} + 3 = 4 \times 7$.

$V_{th} = 25$ Volt.



* I_N :

$I_1 = 7$ Amp.

$I_2 = I_N$.

loop (2)

$2I_N - 3 = 4I_2 - 4I_1$

$2I_N - 3 = 4I_N - 4 \times 7$.

$25 = 2I_N$

∴ $I_N = \frac{25}{2}$ Amp.

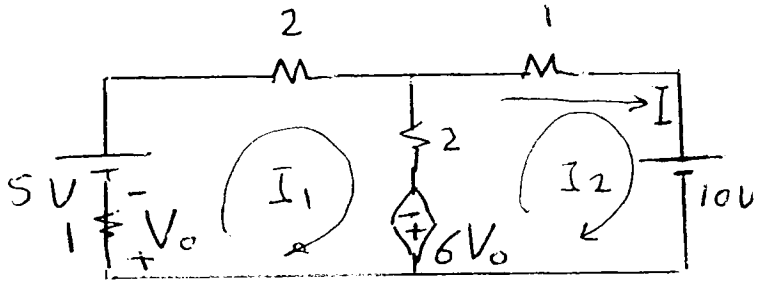
∴ $R_{th} = R_N = \frac{25}{25/2} = 2\ \Omega$.

∴ $I = \frac{V_{th}}{R_{th} + 3} = \frac{25}{2+3} = \underline{5 \text{ Amp}}$.

OR $I = I_N \frac{R_N}{R_N + 3}$

$= \underline{5 \text{ Amp}}$

Q3] Find (I) For the circuit shown.



a) By mesh (loop method).

loop (1):

$5 + 6V_o = (2+2+1)I_1 - 2I_2$. But $V_o = I_1 \times 1$

$\therefore 5 + 6I_1 = 5I_1 - 2I_2$.

$\therefore 5 + I_1 = -2I_2$. ——— (1) here $I_2 = I$.

loop (2):

$-6V_o - 10 = 3I_2 - 2I_1$

$-6I_1 - 10 = 3I_2 - 2I_1$

$-4I_1 - 3I_2 = 10$ ——— (2).

From eq (1) $I_1 = -2I_2 - 5$.

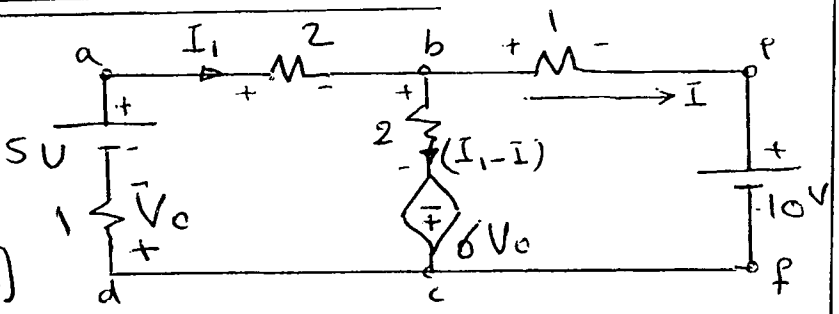
$\therefore -4(-2I_2 - 5) - 3I_2 = 10$.

$8I_2 + 20 - 3I_2 = 10$

$5I_2 = -10$.

$\therefore I_2 = I = -2$ Amp

B) By K.L.



abcd

$5 + 6V_o = (2+1)I_1 + 2(I_1 - I)$

also $V_o = I_1$

$\therefore 5 + 6I_1 = 3I_1 + 2I_1 - 2I$.

$5 = -I_1 - 2I$ ——— (1).

befc

$2(I_1 - I) = I + 10 + 6V_o$

$2(I_1 - I) = I + 10 + 6I_1$

$\therefore 10 = -3I - 4I_1$ ——— (2).

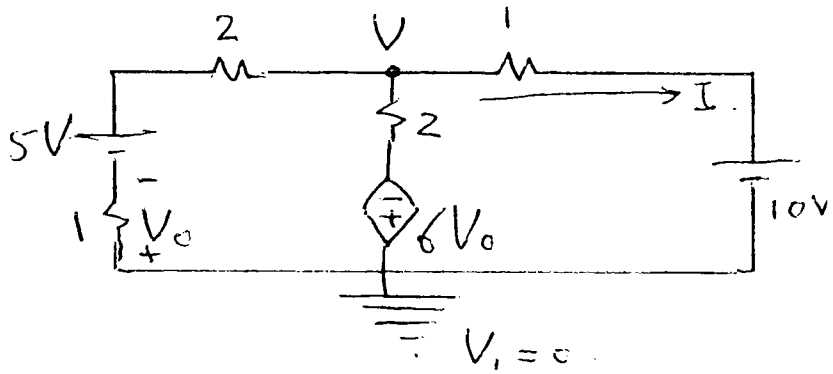
$\therefore 10 = -3I - 4(-5 - 2I)$.

$10 = 5I + 20$.

$\therefore 5I = -10$

$\therefore I = -2$ Amp

c) By Nodal equation method:



Node (V)

$$\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}\right)V-\frac{5}{3}-\frac{10}{1}+\frac{6V_o}{2}=0.$$

$$11V-10-60+18V_o=0.$$

$$11V+18V_o=70$$

But $V_o=\frac{5-V}{3}\times 1$.

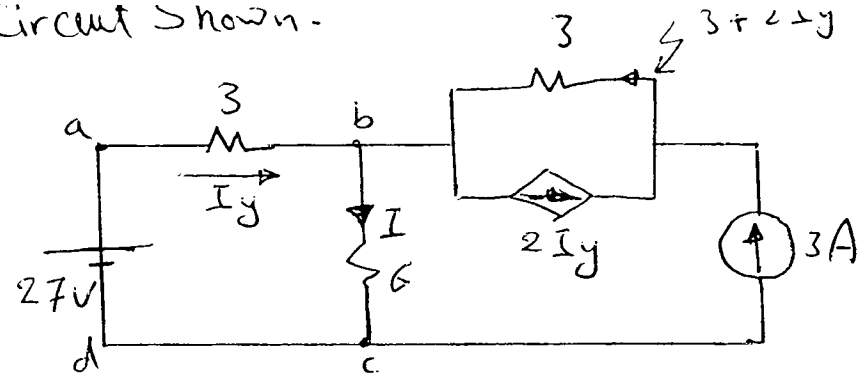
$$\therefore 11V+6(5-V)=70.$$

$$5V=40.$$

$$\therefore V=8 \text{ Volt}.$$

$$\therefore \underline{I=\frac{8-10}{1}=-2 \text{ Amp}}.$$

Q4/ Find (I) for the Circuit Shown.



a) By K.L.

abcd

$27 = 3I_y + 6I$.

at point C, we have $I = 3 + I_y$.

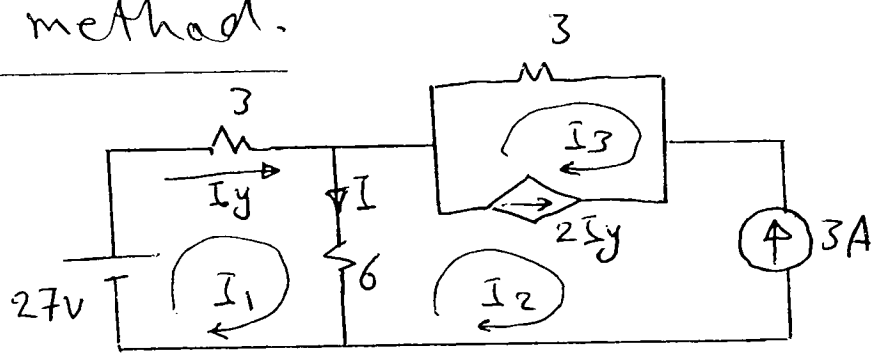
$\therefore I_y = I - 3$.

$\therefore 27 = 3(I-3) + 6I$.

$36 = 9I$.

$\therefore I = 4$ Amp

b) By loop current method.



loop (1):

$27 = (3+6)I_1 - 6I_2$.

$27 = 9I_1 - 6I_2$.

from loop (2) $I_2 = -3$ Amp.

$\therefore 27 = 9I_1 - 6(-3)$.

$9 = 9I_1$

$\therefore I_1 = 1$ Amp $= I_y$.

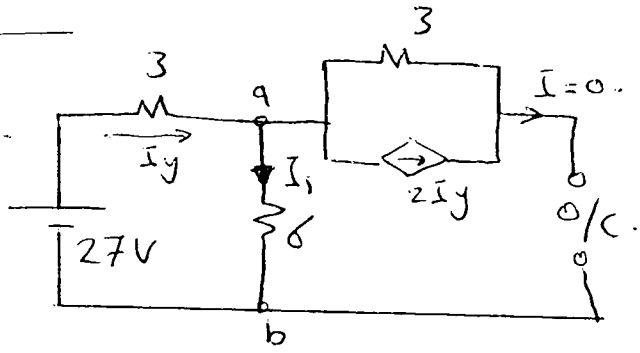
But $I = I_1 - I_2$.

$= 1 - (-3)$

$\therefore I = 4$ Amp.

c) By Superposition method.

1) By effect of 27V → 3A (o/c).

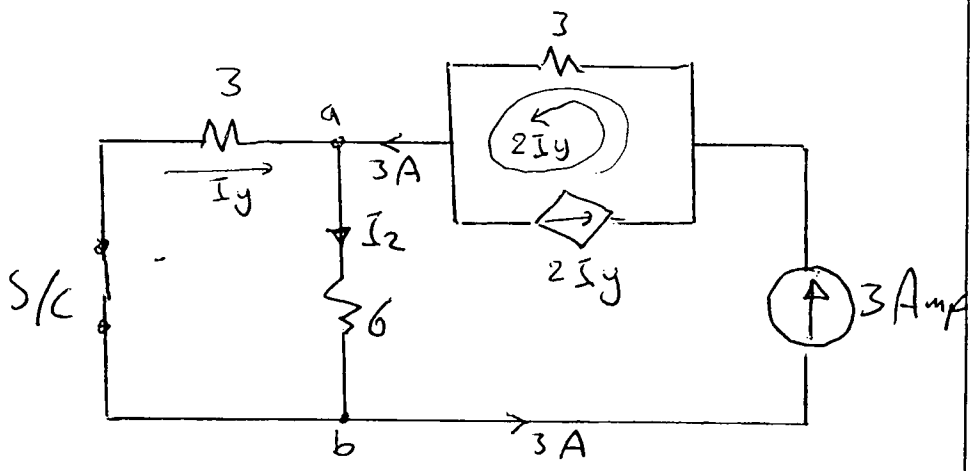


$\therefore I_1 = \frac{27}{3+6} = 3\,Amp\ (a \rightarrow b)$.

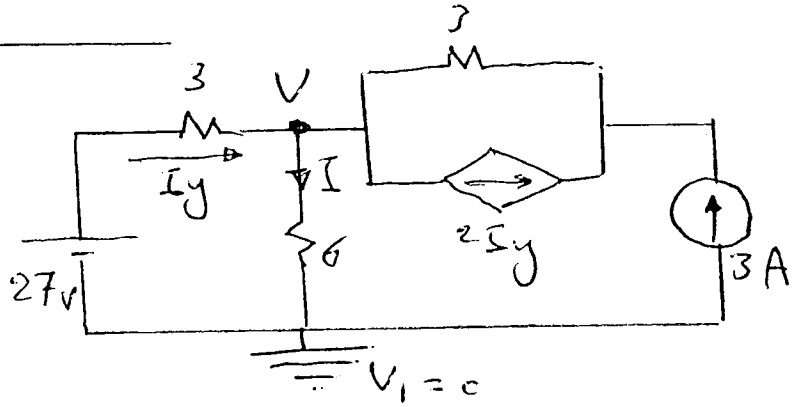
2) By effect of 3Amp → 27 (s/c).

$I_2 = 3\,\frac{3}{3+6} = 1\,Amp\ (a \rightarrow b)$.

$\therefore I = I_1 + I_2 = 4\,Amp$.



d) By Nodal Voltage method



Node (V):

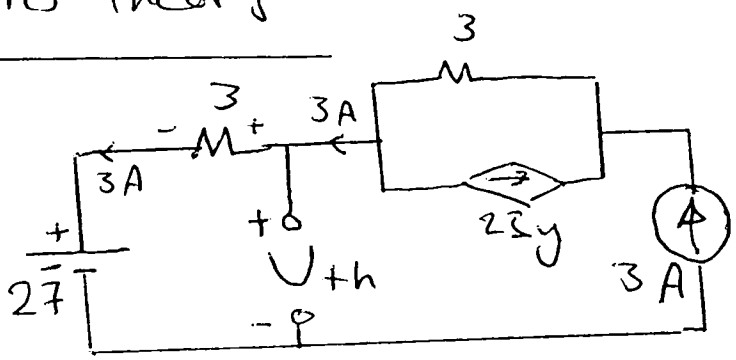
$$\left(\frac{1}{3}+\frac{1}{6}\right)V - \frac{27}{3} = 3$$

$$(2+1)V - 54 = 18$$

$$3V = 72.$$

$$\therefore V = 24 \text{ Volt}. \qquad \therefore I = \frac{V}{6} = 4 \text{ Amp}.$$

e) By Thevenin's or Norton's Theory



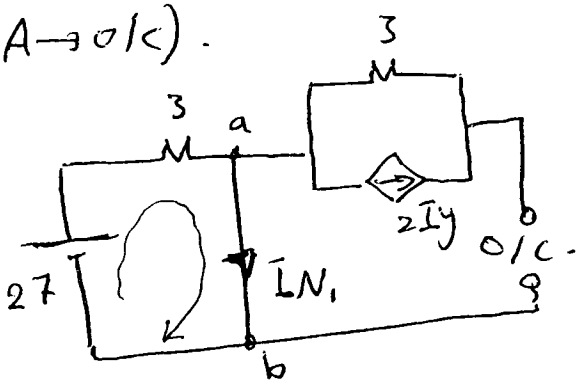
$$V_{th} = R_N = \frac{V_{th}}{I_N}$$

* $V_{th} = 27 + 3 \times 3$

$\therefore V_{th} = 36$ Volt

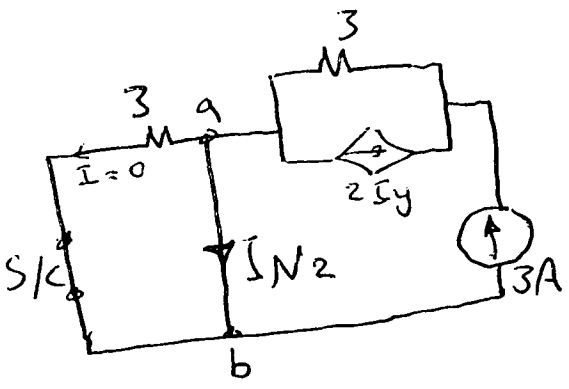
* I_N:

1) I_{N_1} → By effect of 27 Volt (3A → o/c).



$$I_{N_1} = \frac{27}{3} = 9 \text{ Amp } (a \to b).$$

2) I_{N_2} → By 3Amp → 27 → S/c



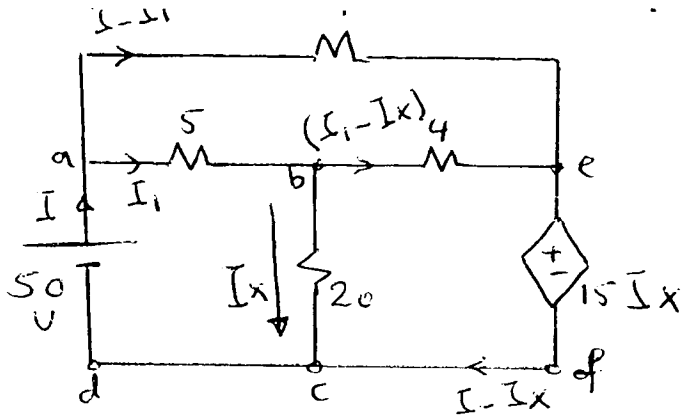
$$I_{N_2} = 3 \text{ Amp } (a \to b).$$

$\therefore I_N = I_{N_1} + I_{N_2} = 12 \text{ Amp}.$

$$\therefore R_{th} = \frac{36}{12} = 3\Omega.$$

$$\therefore I = \frac{V_{th}}{R_{th}+6} = \frac{36}{3+9} = 4 \text{ Amp}.$$

Q.5 Find the current (I_x).



a) By K.L.

abcd

$50 = 5I_1 + 20I_x$

$\therefore 10 = I_1 + 4I_x$ ——— (1).

befc

$20I_x = 15I_x + 4(I_1 - I_x)$.

$\therefore 9I_x = 4I_1$ ——— (2) $\therefore I_1 = \frac{9}{4}I_x$.

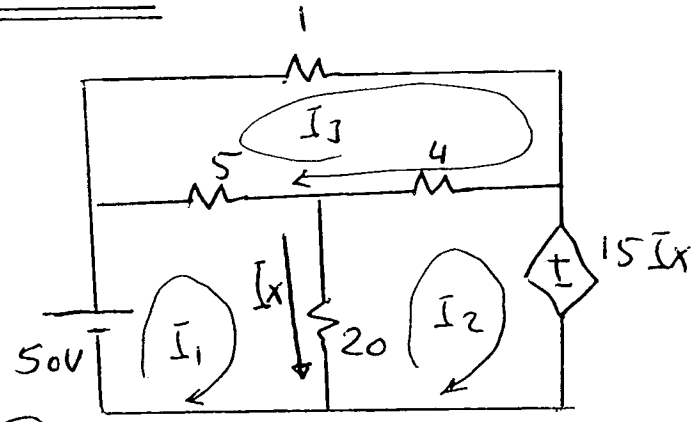
put I_1 in eq (1).

$\therefore 10 = \frac{9}{4}I_x + 4I_x$.

$40 = 25I_x$.

$\therefore I_x = \frac{40}{25} = 1.6$ Amp.

b) By loop current method



loop (1):

$50 = 25I_1 - 20I_2 - 5I_3$

$10 = 5I_1 - 4I_2 - I_3$ ——— (1).

loop (2):

$-15I_x = 24I_2 - 20I_1 - 4I_3$ ——— (2).

loop (3):

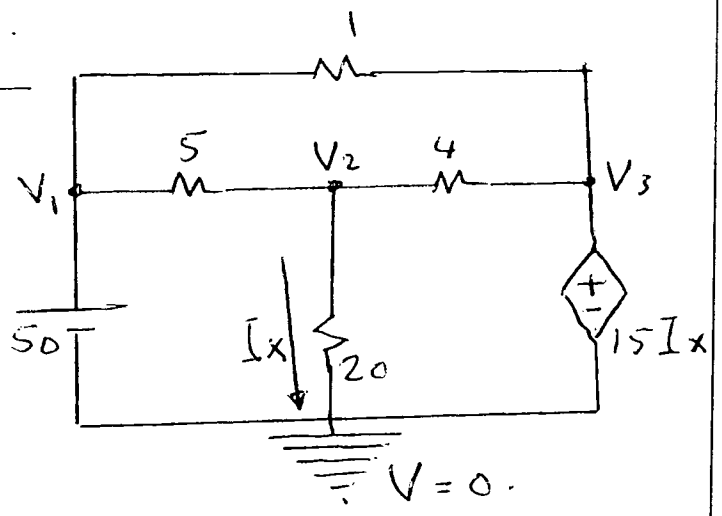
$0 = 10I_3 - 5I_1 - 4I_2$ ——— (3)

But $I_x = I_1 - I_2$

Solving eq (1), (2) & (3).

we get $I_x = 1.6$ Amp.

c) By Nodal Voltage method.



$V_1 = 50$ Volt.

$V_3 = 15\,I_x$

But $I_x = \frac{V_2 - 0}{20} = \frac{V_2}{20}$

Node (V_2):

$$\left(\frac{1}{4} + \frac{1}{5} + \frac{1}{20}\right)V_2 - \frac{50}{5} - \frac{15I_x}{4} = 0.$$

$$(5 + 4 + 1)V_2 - 200 - 75\,I_x = 0.$$

$$10V_2 - 75\,I_x = 200.$$

$$\therefore 10V_2 - 75\left(\frac{V_2}{20}\right) = 200.$$

$$40V_2 - 15V_2 = 800.$$

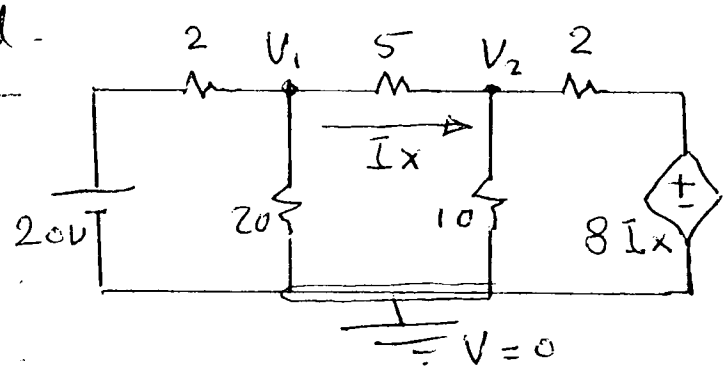
$$25V_2 = 800.$$

$$\therefore V_2 = \frac{800}{25} = 32 \text{ Volt.}$$

$$\therefore I_x = \frac{32}{20} = 1.6 \text{ Amp.}$$

Q.6] Find (I_x) for the Circuit

a) By Nodal Voltage method.



Node(V_1):

$$\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{20}\right)V_1-\frac{20}{2}-\frac{V_2}{5}=0$$

$$(10+4+1)V_1-200-4V_2=0$$

$$200=15V_1-4V_2 \quad \text{——— (1)}$$

Node(V_2):

$$\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}\right)V_2-\frac{V_1}{5}-\frac{8I_x}{2}=0$$

$$\therefore 4V_2-V_1-20I_x=0 \quad \text{——— (2)}$$

But $I_x=\frac{V_1-V_2}{5}$

$$\therefore 4V_2-V_1-20\left(\frac{V_1-V_2}{5}\right)=0$$

$$8V_2-5V_1=0$$

$$V_1=\frac{8}{5}V_2$$

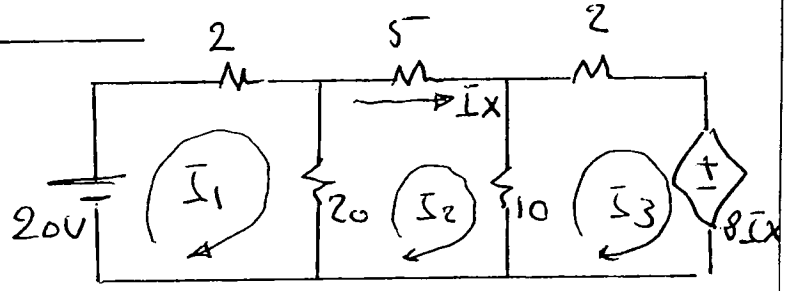
put V_1 in eq (1)

$$\therefore 200=15\left(\frac{8}{5}V_2\right)-4V_2$$

$$\therefore V_2=10 \text{ Volt} \quad \therefore V_1=16 \text{ Volt}$$

$$\therefore I_x=\frac{16-10}{5}=\frac{6}{5}=1.2 \text{ Amp}$$

b) By loop current method.



loop(1):

$$20=22I_1-20I_2 \quad \text{——— (1)}$$

loop(2):

$$I_2=I_x$$

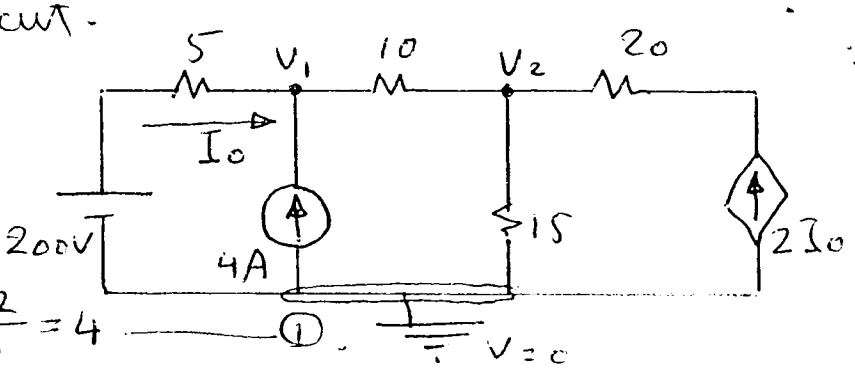
$$0=35I_2-20I_1-10I_3 \quad \text{——— (2)}$$

loop(3):

$$-8I_x=12I_3-10I_2 \quad \text{——— (3)}$$

Solving: $I_2=I_x=1.2$ Amp.

Q7/ Find (I_o) for the circuit.



a) By Nodal method

Node(V_1):

$$\left(\frac{1}{5}+\frac{1}{10}\right)V_1 - \frac{200}{5} - \frac{V_2}{10} = 4 \quad \text{——— (1)}$$

$$3V_1 - V_2 = 440 \quad \text{——— (1)}$$

Node(V_2):

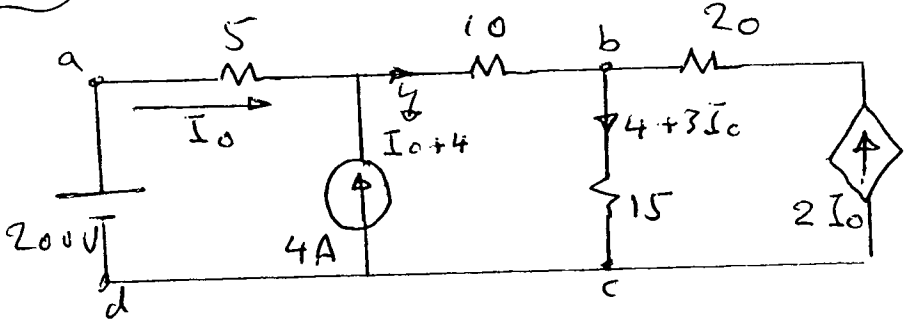
$$\left(\frac{1}{10}+\frac{1}{15}\right)V_2 - \frac{V_1}{10} = 2I_o \qquad \text{But } I_o = \frac{200-V_1}{5}$$

$$\therefore 5V_2 + 9V_1 = 2400 \quad \text{——— (2)}$$

Solving: $V_1 = 191.6667$ Volt.

$$\therefore I_o = \frac{200-191.6667}{5} = 1.6667 \text{ Amp.}$$

b) By K.L



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$$200 = 5I_o + 10(I_o+4) + 15(4+3I_o)$$

$$100 = 60\,I_o$$

$$\therefore I_o = \frac{100}{60} = 1.6667$$